

# On Massive MIMO Cellular Systems Resilience to Radar Interference

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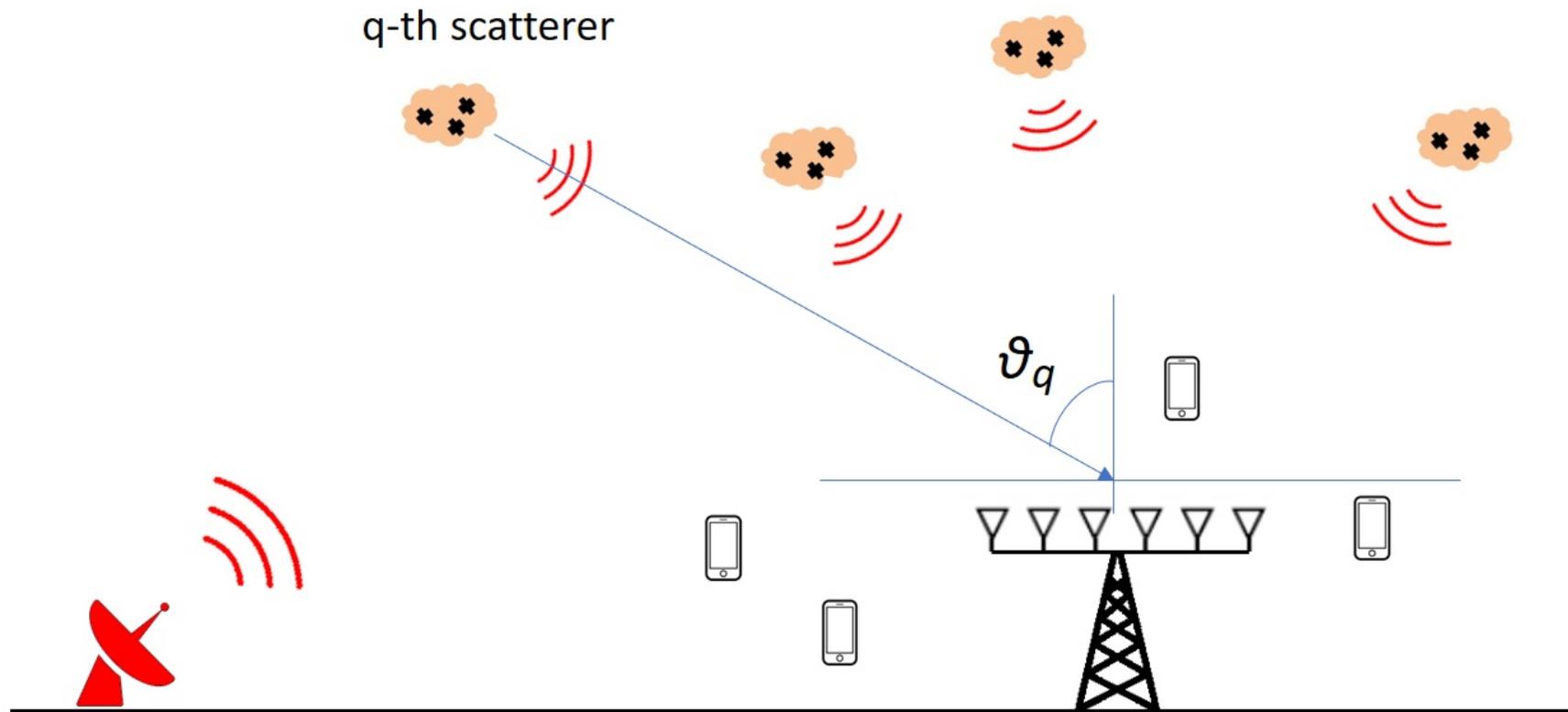


# Summary of the paper

This paper considers a single-cell massive MIMO communication system in uplink coexisting with a radar system using the same frequency band. In particular,

- (a) We consider a system model taking into account the clutter produced by the radar system at the massive MIMO receiver
- (b) We provide a theoretical analysis, in terms of UL spectral efficiency and in terms of mutual information
- (c) We show that for large number of antennas at the BS the radar clutter effects can be suppressed

# System model (Part I)



## References

- [1] S. Buzzi, M. Lops, C. D'Andrea, and C. D'Elia, "Co-existence between a radar system and a massive MIMO wireless cellular system," in *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jun. 2018, pp. 1–5

# System model (Part II)

- We denote by
  - $N$  the number of subcarriers of the SC-FDMA system
  - $M$  the number of elements of the ULA at the BS
  - $C$  the number of consecutive subcarriers where the channel is constant
  - $K$  the number of users of the cellular system

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- The radar system transmits every  $T_{\text{PRT}}$  a coded waveform whose baseband equivalent is

$$s_R(t) = \sqrt{P_T} \sum_{\ell=0}^{L-1} c_\ell \psi(t - \ell T_s)$$

# Signal model (Part I)

- **UL data transmission:** The observable corresponding to the  $N$  subcarriers for the  $\ell$ -th data packet is the  $(M \times N)$ -dimensional matrix

$$\mathbf{Y}(\ell) = \sum_{k=1}^K \sqrt{p_k} \left( \left[ \mathbf{h}_k^{(1)} \dots \mathbf{h}_k^{(Q)} \right] \otimes \mathbf{1}_{1 \times C} \right) \text{diag}(\mathbf{X}_k(\ell)) + \mathbf{W}(\ell) + \mathbf{C}(\ell),$$

with  $Q = N/C$ .

# Signal model (Part II)

- **UL training:** Let  $T$  denote the number of consecutive packets devoted to training, and let  $\mathbf{p}_k(1), \dots, \mathbf{p}_k(T)$  denote  $N$ -dimensional vectors containing the  $k$ -th MS pilots.

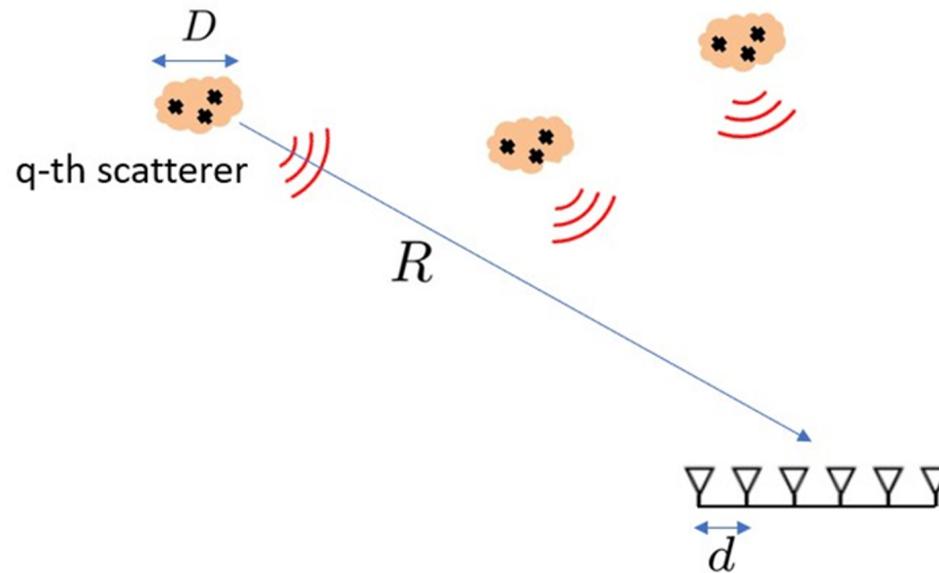
Assuming that the  $M$ -dimensional channel vectors  $\mathbf{h}_k^{(q)}$ ,  $\forall k = 0, \dots, K - 1$ , are to be estimated we focus on the observable

$$\mathcal{Y}_q = \sum_{k=1}^K \sqrt{p_{p,k}} \mathbf{h}_k^{(q)} \mathbf{P}_k^{(q)T} + \mathcal{W}_q + \mathcal{C}_q,$$

where  $\mathbf{P}_k^{(q)}$  is a  $(TC)$ -dimensional vector containing FFT samples of the  $k$ -th MS pilots

# Clutter modeling (Part I)

- Given the BS array dimension, it is reasonable to assume that these scatterers are seen by the BS as "**colocated**", i.e.  $d \ll R \frac{\lambda}{D}$



## References

- [2] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 106–114, 2007

# Clutter modeling (Part II)

- The radar-to-BS channel can be modeled as a LTI channel

$$\mathbf{h}(t) = \sum_{q=0}^{N_s-1} \sum_{m=0}^{Q-1} \beta_{q,m} \mathbf{b}(\theta_q) \delta(t - \tau_q - m/W)$$

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- Let  $\mathcal{S}(\ell)$  denote the set of the scatterers corrupting the reception of the  $\ell$ -th data packet; the clutter  $(M \times N)$ -dimensional matrix is

$$\mathbf{C}(\ell) = \sum_{q \in \mathcal{S}(\ell)} \sum_{m=0}^{Q-1} \sum_{p=0}^{L-1} \sqrt{P_T} \beta_{q,m} c_p \mathbf{b}(\theta_q) \mathbf{r}_{q,p,m}^T(\ell) \mathbf{W}_{N,FFT}$$

# Receiver processing

- **UL channel estimation**
  - Pilot matched (PM)
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- **UL channel estimation**
  - Pilot matched (PM)
  - Minimum-mean-square-error (MMSE)
- **UL data detection**
  - Channel-matched (CM)
  - Zero-forced clutter (ZF)
  - Linear minimum mean square (LMMSE)
  - Full zero-forcing (FZF)

## References

- [1] S. Buzzi, M. Lops, C. D'Andrea, and C. D'Elia, "Co-existence between a radar system and a massive MIMO wireless cellular system," in *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jun. 2018, pp. 1–5

# Uplink spectral efficiency derivation

- We derive closed-form expressions for a LB to the UL SE under the hypothesis of CM detection and both PM and MMSE channel estimation strategies with the **UatF** bounding technique
- The UatF bound exploits the channel estimates only for computing the receive combining vectors, while this information is not exploited in the signal detection phase
- This assumption is reasonable when there is substantial **channel hardening**

$$SE_k^{(n)} \geq \frac{N_{\text{pkt}} - T}{N_{\text{pkt}}} \log_2 \left( 1 + \text{SINR}_k^{(n)} \right) \text{ [bit/s/Hz]},$$

## References

- [3] T. L. Marzetta, E. G. Larsson, H. Yang, and H. Q. Ngo, *Fundamentals of Massive MIMO*. Cambridge University Press, 2016

# Information Theoretic Analysis

- The clutter contribution has no effect on the single-user mutual information

$$I\left(\mathbf{y}; \mathbf{X}^{(0)}, \mathbf{X}^{(1)}, \dots, \mathbf{X}^{(Q-1)} \mid \mathbf{h}^{(0)}, \dots, \mathbf{h}^{(Q-1)}\right) = \log \left| \mathbf{I}_{NM} + \mathbf{K}' (N_0 \mathbf{I}_{NM} + \mathbf{K}_c)^{-1} \right|$$

- We show that, in the **large number of antennas regime**, the product  $[\mathbf{K}'^\dagger \mathbf{K}_c]$  vanish **almost surely**
- The radar clutter effect at the BS is suppressed and single-user capacity is restored

## References

- [4] W. F. Stout, *Almost sure convergence*. Academic Press, 1974
- [5] P. Billingsley, *Convergence of probability measures*, 2nd ed. Wiley, 1999

# Simulation Setup<sup>1</sup>

Name	Value	Description
$f_c$	3 GHz	carrier frequency
$M$	16, 64, 128	number of antennas at the BS
$d$	$\frac{\lambda}{2}$	antenna spacing
$K$	1,10	number of users in the cellular system uniformly distributed in the range [20,500] m
$p_k$	100 mW	MSs transmit power in training and data transmission phases
$N$	4096	number of subcarriers
$\Delta_f$	30 kHz	subcarrier spacing
$C$	16	number of consecutive subcarriers where the channel is considered constant
$Q$	$\frac{N}{C} = 256$	number of scalar coefficients representing the amount of channel state information for each user and for each BS antennas
$N_s$	100	number of total scatterers in the system uniformly distributed in the range [1,150] km
$N_{CP}$	288	discrete length of the cyclic-prefix
$T_{PRT}$	1 ms	radar pulse repetition time
$N_{pkt}$	14	number of packets into a 0.5 ms timeslot
$T_s$	8.146 ns	symbol time
$T$	7	number of packets used for the channel estimation
$L$	32	discrete length of the radar coded waveform
$F$	3 dB	noise figure at the receiver
$\mathcal{N}_0$	-174 dBm/Hz	power spectral density of the noise

<sup>1</sup>These numbers are inspired by the December 2017 3GPP first release of the 5G New Radio standard.



# Numerical Results: SINR versus CNR

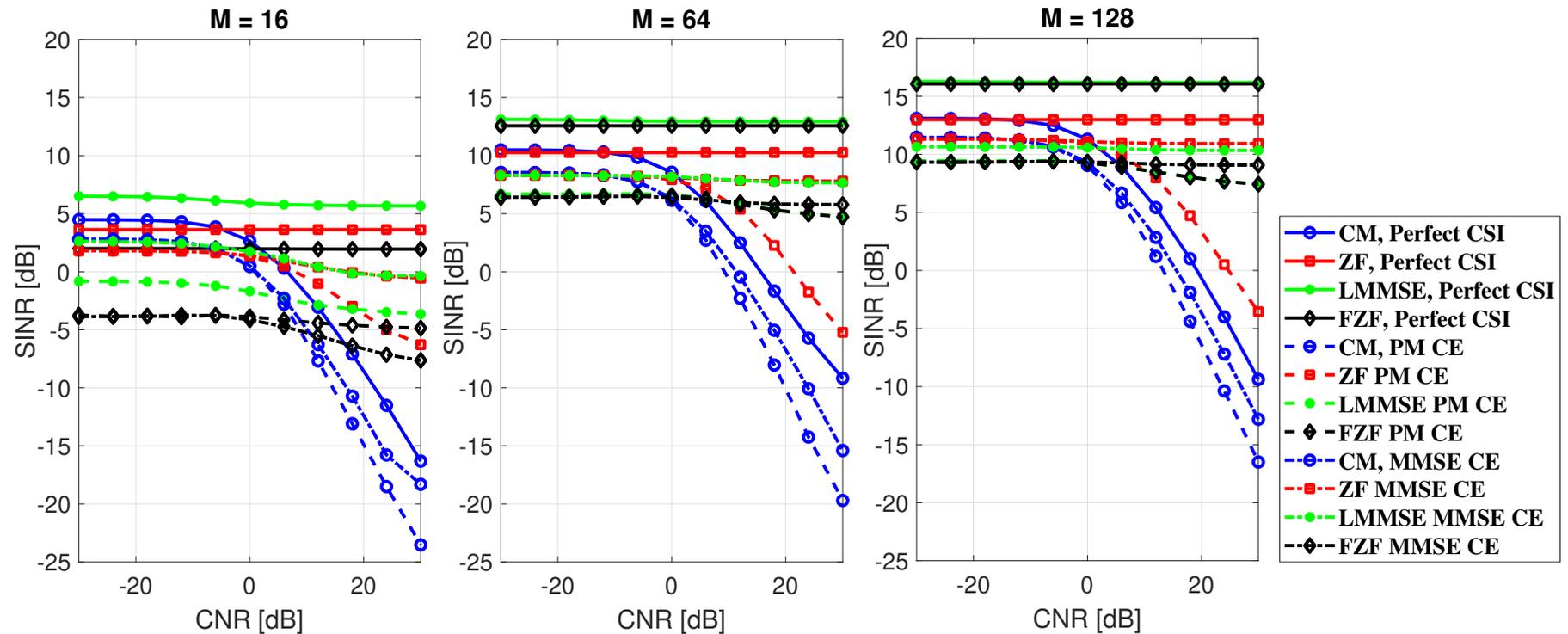
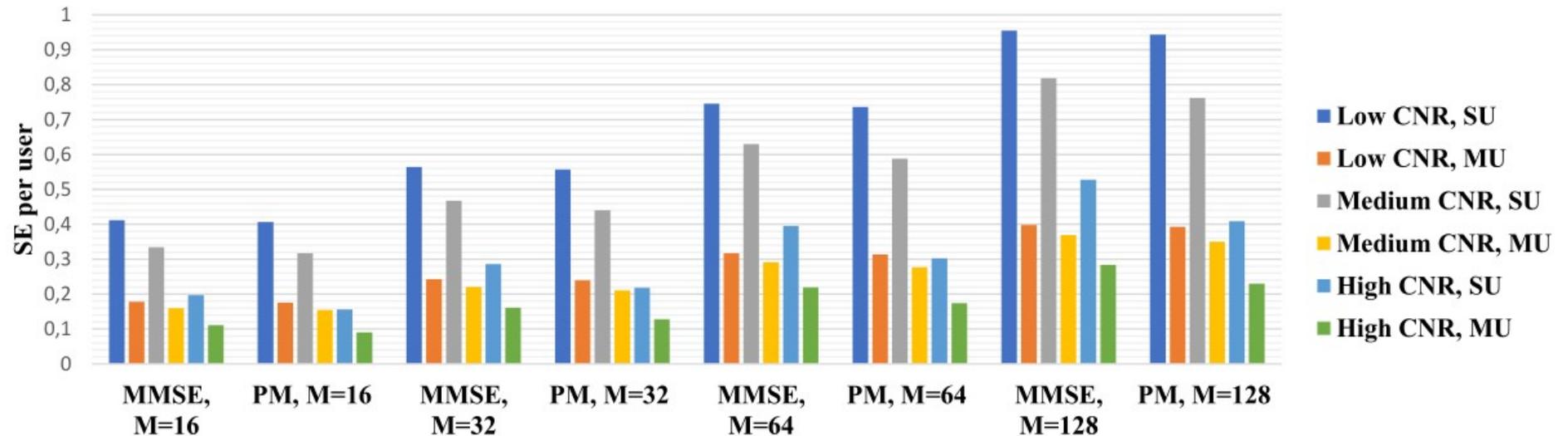


Figure: SINR versus CNR of four detection strategies in the cases of perfect CSI, PM CE and MMSE CE, with  $K = 10$  and different values of  $M$ .

# Numerical Results: Spectral efficiency lower bounds



**Figure:** SE per user lower bounds with CM beamforming, in SU and MU scenarios, for increasing values of  $M$ , for  $\text{CNR} = -20$  dB (Low CNR), for  $\text{CNR} = 0$  dB (Medium CNR), for  $\text{CNR} = 10$  dB (High CNR), and with PM and MMSE CE techniques.

# Numerical Results: Key insights

- Performance steadily improves increasing the number of BS antennas
- CM beamforming is the most vulnerable combining scheme to interference, while other strategies exhibit much better performance
- Ascending performance order: CM, ZF, FZF, LMMSE
- If *clutter second order statistics* are known at the BS, several strategies exist to tackle the interference from the co-existing radar system, with better and better performance increasing the number of BS antennas

# Conclusions

- This paper has been focused on an uplink massive MIMO communication system which co-exists with a radar system using the same frequency band
- We derived spectral efficiency lower bound expressions for the considered scenario
- We showed that in the large number of antennas regime the clutter contribution has no effect on the single-user mutual information
- The results confirmed that increasing the number of antennas at the BS provides increased robustness against the clutter disturbance originating from the radar system

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## Acknowledgement

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# *Thank you for listening!*



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