Particle Flow Particle Filter for Gaussian Mixture Noise Models

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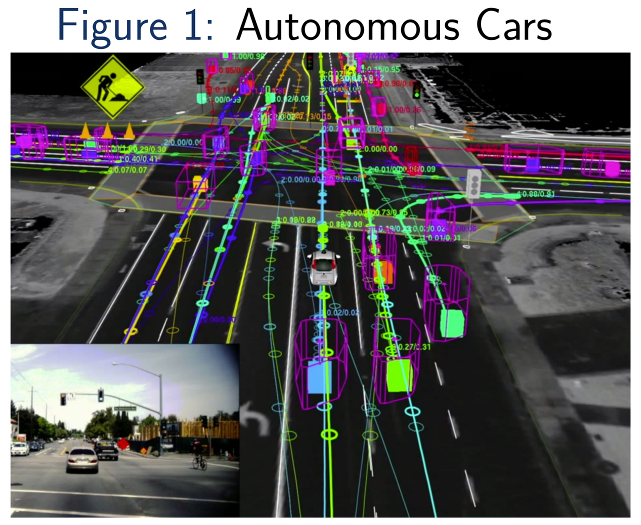
Introduction and Motivation

- **Goal** : state estimation from sequential measurements
- **A Motivating Example :**

Figure 1: Autonomous Cars



Reproduced from [3]

- **States** : positions and velocities of cars
- **Measurements** : LIDAR and/or short-range RADAR
- **Major challenges :**
  - Higher dimensional states, informative measurements
  - Multimodality in process and measurement noises

Problem Formulation

- **Dynamic Model :**

$$p(x_k|x_{k-1}) = \sum_{m=1}^{M} \alpha_{k,m} \mathcal{N}(x_k|g_k(x_{k-1})+\psi_{k,m}, Q_{k,m}) = \sum_{m=1}^{M} P(d_k=m)p(x_k|x_{k-1},d_k=m)$$

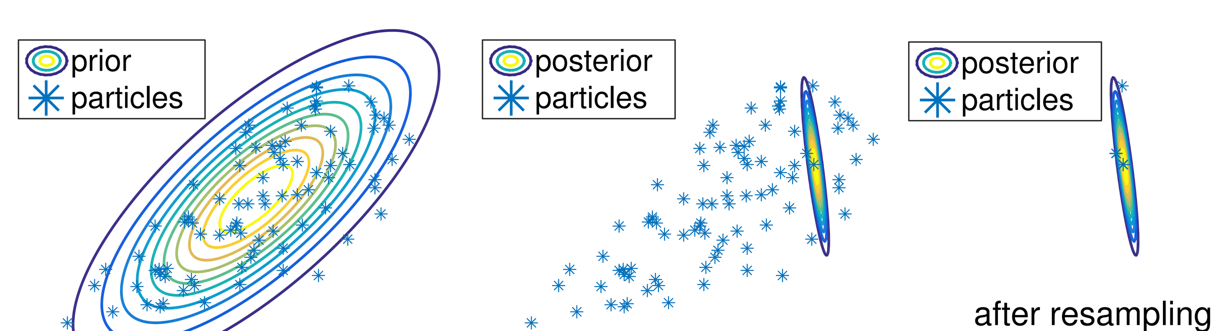
- **Measurement Model :**

$$p(z_k|x_k) = \sum_{n=1}^{N} \beta_{k,n} \mathcal{N}(z_k|h_k(x_k)+\zeta_{k,n}, R_{k,n}) = \sum_{n=1}^{N} P(c_k=n)p(z_k|x_k,c_k=n)$$

Particle Filter

- Employs sequential importance sampling and resampling.

Figure 2: Particle filtering

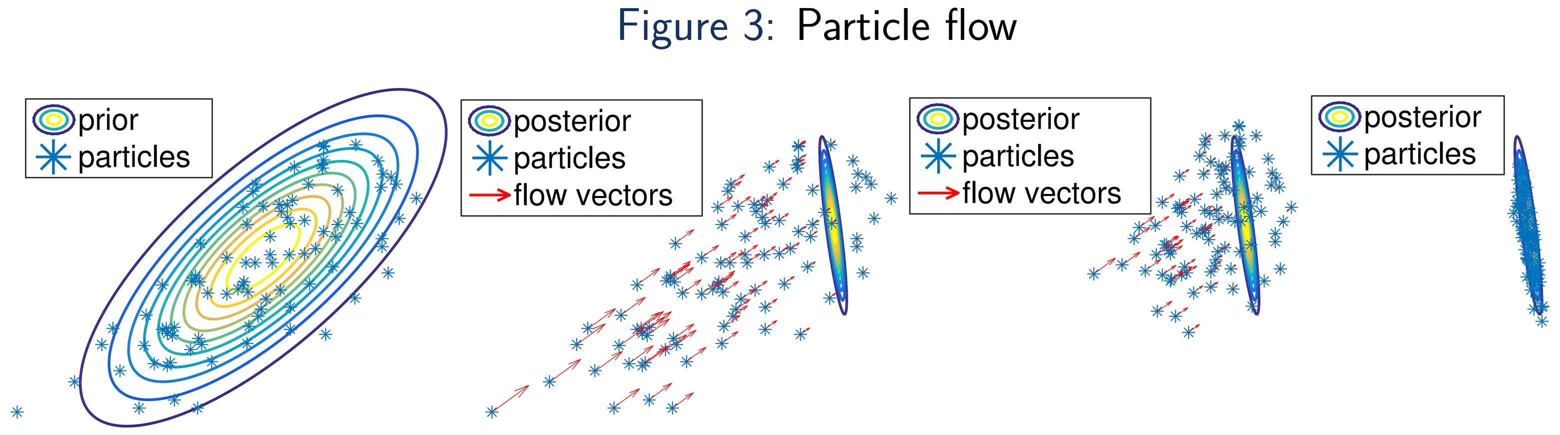


- Poor representation of posterior distribution for high dimensionality of state and/or informative measurements.

Particle Flow

- Particle flow [1] involves solution of a differential equation to migrate particles form the prior distribution to the posterior distribution.

Figure 3: Particle flow



Particle Flow Particle Filter (PFPF)

- PFPF of [2] constructs its proposal distribution based on a modified deterministic particle flow applied to the samples from the prior.
- The deterministic mapping $\eta_1^i = T^i(\eta_0^i, x_{k-1}^i, z_k)$ is invertible, so the proposal can be evaluated as:

$$q(\eta_1^i|x_{k-1}^i, z_k) = \frac{p(\eta_0^i|x_{k-1}^i)}{|\det(\dot{T}^i(\eta_0^i; x_{k-1}^i, z_k))|},$$

where, $\dot{T}^i(\cdot)$ is the Jacobian function of the mapping $T^i(\cdot)$.

Proposed Algorithm (PFPF-GMM)

- Our design of joint proposal distribution of (x_k, d_k, c_k):

$$q(x_k^i, d_k^i, c_k^i|x_{0:k-1}^i, d_{1:k-1}^i, c_{1:k-1}^i, z_{1:k}) = P(d_k^i)P(c_k^i)q(x_k^i|x_{k-1}^i, d_k^i, c_k^i, z_k)$$

- Conditioned on the auxiliary variables (d_k, c_k), invertible particle flow of [2] is used to construct $q(x_k^i|x_{k-1}^i, d_k^i, c_k^i, z_k)$.
- Importance weights for the joint posterior:

$$\omega_k^i = \frac{p(x_{0:k}^i, d_{1:k}^i, c_{1:k}^i|z_{1:k})}{q(x_{0:k}^i, d_{1:k}^i, c_{1:k}^i|z_{1:k})} \propto \omega_{k-1}^i \frac{p(x_k^i|x_{k-1}^i, d_k^i)p(z_k|x_k^i, c_k^i)}{q(x_k^i|x_{k-1}^i, d_k^i, c_k^i, z_k)}$$

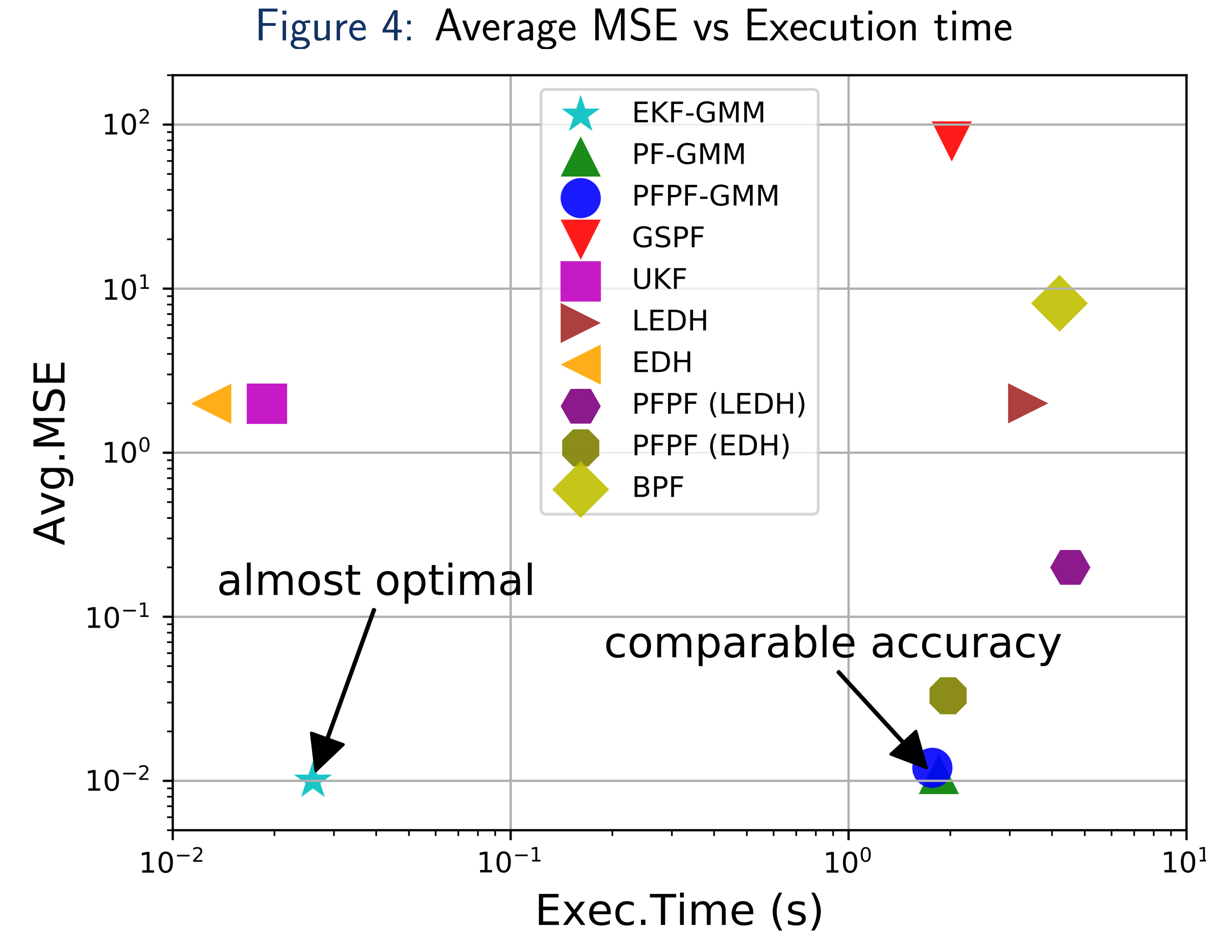
- Estimation via importance sampling :

$$p(x_k|z_{1:k}) \approx \sum_{i=1}^{N_p} \omega_k^i \delta(x_k - x_k^i)$$

Numerical Experiments and Results

- We compare the novel PFPF-GMM algorithm with
  - Kalman filter type algorithm for unimodal posterior (UKF)
  - Particle flow algorithms for unimodal posterior (LEDH, EDH)
  - Particle flow particle filters for unimodal posterior (PFPF (LEDH), PFPF (EDH))
  - Bootstrap Particle Filter (BPF)
  - Filtering algorithms for multimodal posteriors (EKF-GMM, PF-GMM, GSPF)
- **Linear Dynamic and Measurement Models :**
  - Dimension of state, $d = 64$, $x_k = \alpha x_{k-1} + v_k$, $z_k = x_k + w_k$.
  - v_k and w_k are drawn from Gaussian mixtures with three components.

Figure 4: Average MSE vs Execution time

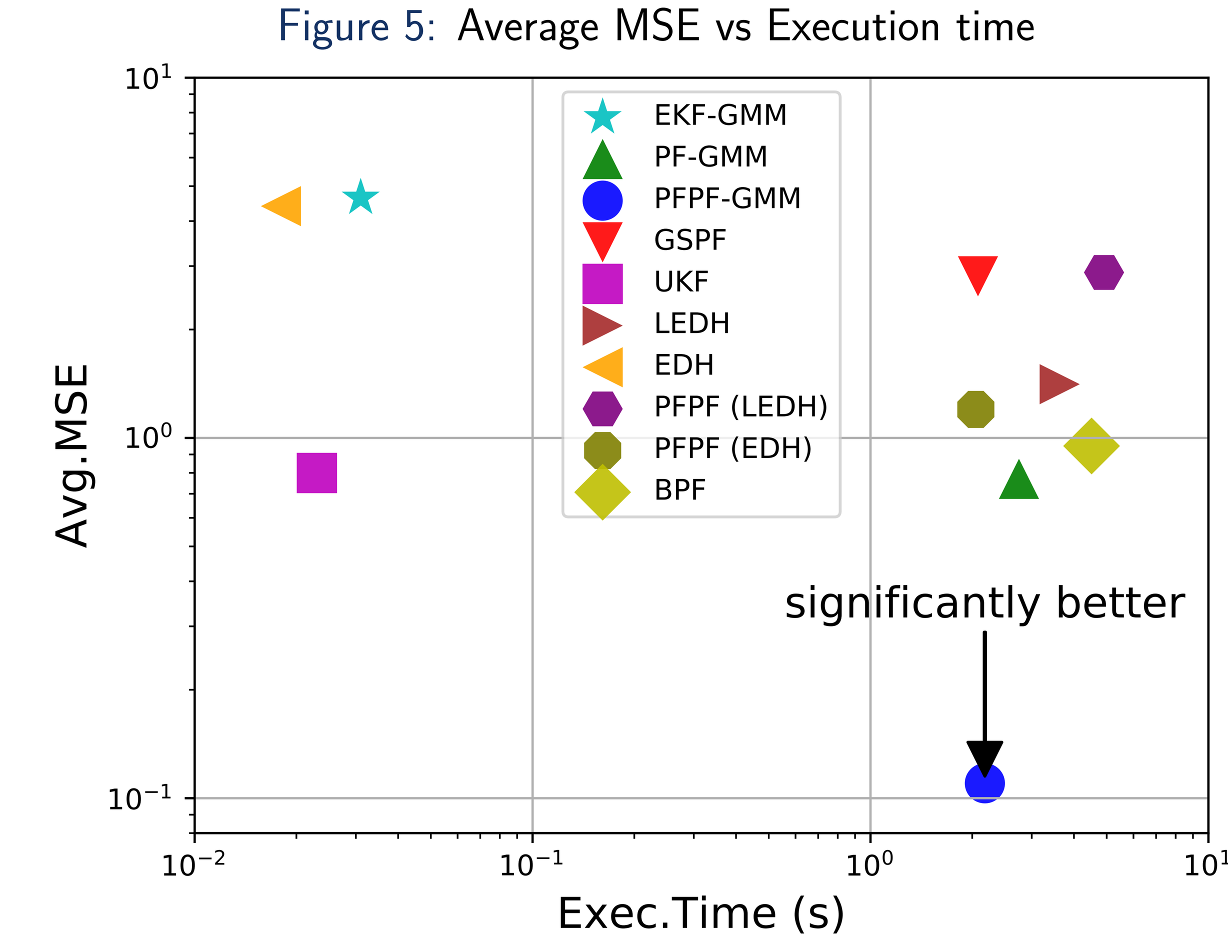


- **Nonlinear Dynamic and Measurement Models :**
  - Dimension of state, $d = 64$.
  - Dynamic model:

$$g_k^c(x_{k-1}) = 0.5x_{k-1}^c + 8\cos(1.2(k-1)) + \begin{cases} 2.5\frac{x_{k-1}^{c+1}}{1+(x_{k-1}^{c})^2} & \text{, if } c=1 \\ 2.5\frac{x_{k-1}^{c+1}}{1+(x_{k-1}^{c-1})^2} & \text{, if } 1<c<d \\ 2.5\frac{x_{k-1}^{c}}{1+(x_{k-1}^{c-1})^2} & \text{, if } c=d \end{cases}$$

  - Measurement model : $h_k^c(x_k) = \frac{(x_k^c)^2}{20}$, $1 \le c \le d$.
  - v_k and w_k are drawn from Gaussian mixtures with three components.

Figure 5: Average MSE vs Execution time



- Algorithms suitable for unimodal posterior distributions perform poorly in both experiments.
- BPF and GSPF suffer form weight degeneracy in high dimensions.
- In the linear example, both PFPF-GMM and PF-GMM achieve comparable MSE to almost optimal EKF-GMM.
- In the nonlinear example, PFPF-GMM outperforms all other algorithms significantly.

Conclusion

- We presented a novel particle filter for Gaussian mixture noise models.
- Successfully tracks multiple modes of the posterior distribution.
- The proposed filter offers impressive performance in higher dimensions and in settings with low measurement noise.

References

[1] F. Daum, J. Huang and A. Noushin, "Exact particle flow for nonlinear filters", in *Proc. SPIE Conf. Signal Proc., Sensor Fusion, Target Recog.*, Orlando, FL, USA, April 2010.

[2] Y. Li and M. Coates, "Particle filtering with invertible particle flow", *IEEE Trans. Signal Processing*, vol. 65, no. 15, pp. 4102–4116, August 2017.

[3] Traffic Safety Store, "What self driving cars see", https://www.trafficsafetystore.com/blog/wp-content/uploads/what-se lf-driving-cars-see.jpg, Retrieved: 2017/12/14.