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# Robust Artificial-Noise Aided Transmit Design for MultiUser MI SO Systems with I ntegrated Services <br> Weidong Mei, Zhi Chen, and Chuan Huang 

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## Background

- Physical-layer (PHY) security and multicasting > PHY security can overcome the inherent difficulties of cryptographic methods.
> PHY-multicasting transmits common messages in a way that all receivers can decode them.
> Traditionally they are independently investigated.
- PHY service integration
> merging multiple services into one integral service for one-time transmission.
> enable coexisting services to share the same resources, thereby significantly increasing the spectral efficiency.
- Motivation
> Many works focused on PHY service integration only from the viewpoint of information theory.
$\checkmark$ DMBC (Csiszar et al. '78)
$\checkmark$ MIMO (Ly et al. '10)
$\checkmark$ Bidirectional relay (Wyrembelski et al. '12)
$\checkmark$ Compound BC with uncertainties (Wyrembelski et al. '12)
$>$ How to derive certain transmit design to achieve the boundary points of the secrecy rate region?


## System Model

a A multi-antenna transmitter serves $K$ receivers, and each receiver has a single antenna.
$\square$ All receivers have ordered the multicast service and receiver 1 further ordered the confidential service.


- The received signal at receiver k

$$
y_{k}=\mathbf{h}_{k} \mathbf{X}+z_{k}
$$

$\mathbf{h}_{k}$--kth receiver's channel response $z_{k}$-AWGN $\square$ Transmitted components

$$
\mathbf{x}=\mathbf{x}_{0}+\mathbf{x}_{c}+\mathbf{x}_{a}
$$

$\mathbf{x}_{0}$-multicast message, $\mathbf{x}_{0} \sim C N\left(\mathbf{0}, \mathbf{Q}_{0}\right)$
$\mathbf{x}_{c}$-confidential message, $\mathbf{x}_{c} \sim C N\left(\mathbf{0}, \mathbf{Q}_{c}\right)$
$\mathbf{x}_{a}$-artificial noise, $\mathbf{x}_{a} \sim C N\left(\mathbf{0}, \mathbf{Q}_{a}\right)$
$\square$ Deterministically bounded CSI error model

$$
\mathbf{h}_{k}=\tilde{\mathbf{h}}_{k}+\mathbf{e}_{k}\| \| \boldsymbol{e}_{k} \|_{F}^{2} \leq \varepsilon_{k}^{2}
$$

## Robust Scheme

-Worst-case secrecy rate region
Under the above described deterministically bounded CSI error model, an achievable worst-case secrecy rate region is determined by [Ly et al. '10]

$$
\begin{aligned}
& R_{c} \leq \min _{k \in \mathcal{C}_{c}} \log \underset{\min _{\mathbf{h}^{\prime} \in B_{1}} 1+\left(1+\mathbf{h}_{1} \mathbf{Q}_{a} \mathbf{h}_{1}^{H}\right)^{-1} \mathbf{h}_{\mathbf{h}} \mathbf{Q}_{c} \mathbf{h}_{1}^{H}}{\operatorname{hax}_{k} \in B_{k}} 1+\left(1+\mathbf{h}_{k} \mathbf{Q}_{a} \mathbf{h}_{k}^{H}\right)^{-1} \mathbf{h}_{k} \mathbf{Q}_{c} \mathbf{h}_{k}^{H} \\
& R_{0} \leq \min _{\substack{k \in E \\
\mathbf{h}_{\in} \in B_{k}}} \log \left(1+\frac{\mathbf{h}_{k} \mathbf{Q}_{0} \mathbf{h}_{k}^{H}}{1+\mathbf{h}_{k}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a}\right) \mathbf{h}_{k}^{H}}\right)
\end{aligned}
$$

where $B_{k}=\left\{\mathbf{h}_{k} \mid \mathbf{h}_{k}=\tilde{\mathbf{h}}_{k}+\mathbf{e}_{k}\right\}, \mathcal{K}=\{1,2, \ldots, K\}, \mathcal{K}_{e}=\mathcal{K} /\{1\}$ -Problem Formulation
s.t. $\min _{\substack{k \in \mathcal{C}_{k} \\ \mathbf{h}_{k} \in \mathcal{B}_{k}}}\left\{\log \frac{1+\mathbf{h}_{k}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a}+\mathbf{Q}_{0}\right) \mathbf{h}_{k}^{H}}{1+\mathbf{h}_{k}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a}\right) \mathbf{h}_{k}^{H}}\right\} \geq \tau$,
$\operatorname{Tr}\left(\mathbf{Q}_{0}+\mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \leq P$, Deman
QoMS

This optimization problem also provides us a way to determine the boundary points of the secrecy rate region, by traversing all possible t's.

Further simplify (1) by introducing a slack variable $\beta$

$$
g^{*}(\tau)=\max _{\mathbf{Q}_{0}, \mathbf{Q}_{a} \mathbf{Q}_{e}, \beta} \min _{h_{1}, \mathcal{B}_{1}} \log \left(\frac{1+\mathbf{h}_{1}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a} \mathbf{h}_{1}^{H}\right.}{\beta\left(1+\mathbf{h}_{1} \mathbf{Q}_{a} h_{1}^{H}\right)}\right)
$$

s.t. $(\beta-1)\left(1+\mathbf{h}_{k} \mathbf{Q}_{a} \mathbf{h}_{k}^{H}\right)-\mathbf{h}_{k} \mathbf{Q}_{c} \mathbf{h}_{k}^{H} \geq 0, \forall \mathbf{h}_{k} \in B_{k}, k \in \mathcal{K}_{e}$, (2) $\mathbf{h}_{k} \mathbf{Q}_{\mathbf{0}} \mathbf{h}_{k}^{H}-\tau \mathbf{h}_{k} \mathbf{Q}_{a} \mathbf{h}_{k}^{H}-\tau \mathbf{h}_{k}^{\prime} \mathbf{Q}_{c} \mathbf{h}_{k}^{H}-\tau^{\prime} \geq 0, \forall \mathbf{h}_{k} \in B_{k}, k \in \mathcal{K}$,
$\operatorname{Tr}\left(\mathbf{Q}_{0}+\mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \leq P$,
$\mathbf{Q}_{0} \succeq \mathbf{0}, \mathbf{Q}_{a} \succeq \mathbf{0}, \mathbf{Q}_{c} \succeq \mathbf{0}$,
Since this problem is non-convex and challenging to solve directly. To deal with it, we recast it into a twostage optimization problem.
aProblem Re-Formulation
The outer-stage part is with regard to (w.r.t.) $\beta$

$$
\begin{aligned}
& \gamma^{*}\left(\tau^{\prime}\right)=\max _{\beta} \eta\left(\tau^{\prime}, \beta\right) \\
& \text { s.t. } 1 \leq \beta \leq 1+P_{h_{1}, \beta_{i}}^{\min }\left\|\mathbf{h}_{\|}\right\|^{2}
\end{aligned}
$$

where $\log \gamma^{*}\left(\tau^{\prime}\right)=\mathrm{g}^{*}\left(\tau^{\prime}\right)$. The inner-stage part calculates $\eta\left(\tau^{\prime}, \beta\right)$ for a fixed $\beta$

$$
\eta\left(\tau^{\prime}, \beta\right)=\max _{\mathbf{Q}_{0}, \mathbf{Q}_{o} \mathbf{Q}_{\mathrm{e}}, \min _{1} \in \boldsymbol{\beta}_{1}} \frac{1+\mathbf{h}_{1}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a}\right) \mathbf{h}_{1}^{H}}{\beta\left(1+\mathbf{h}_{\mathbf{Q}_{a}} \mathbf{h}_{1}^{H}\right)}
$$

s.t. $(\beta-1)\left(1+\mathbf{h}_{k} \mathbf{Q}_{a} \mathbf{h}_{k}^{H}\right)-\mathbf{h}_{k} \mathbf{Q}_{c} \mathbf{h}_{k}^{H} \geq 0, \forall \mathbf{h}_{k} \in B_{k}, k \in \mathcal{K}_{e}$, (3) $\mathbf{h}_{k} \mathbf{Q}_{0} \mathbf{h}_{k}^{H}-\tau^{\prime} \mathbf{h}_{k} \mathbf{Q}_{a} \mathbf{h}_{k}^{H}-\tau^{\prime} \mathbf{h}_{k} \mathbf{Q}_{c} \mathbf{h}_{k}^{H}-\tau^{\prime} \geq 0, \forall \mathbf{h}_{k} \in B_{k}, k \in \mathcal{K}$,
$\operatorname{Tr}\left(\mathbf{Q}_{0}+\mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \leq P$,
$\mathbf{Q}_{0} \succeq \mathbf{0}, \mathbf{Q}_{a} \succeq \mathbf{0}, \mathbf{Q}_{c} \succeq \mathbf{0}$.

## The inner-stage optimization

> By resorting to the $S$-procedure [1], we can recast the above optimization as follows
s.t. $\mathbf{T}_{k}\left(\beta, \mathbf{Q}_{c}, \mathbf{Q}_{Q}, t_{k}\right) \geq \mathbf{0}, t_{k} \geq 0, \forall k \in \mathcal{K}_{e}$,
$\mathbf{S}_{k}\left(\tau^{\prime}, \mathbf{Q}_{c}, \mathbf{Q}_{a}, \mathbf{Q}_{0}, \delta_{k}\right) \succeq \mathbf{0}, \delta_{k} \geq 0, \forall k \in \mathcal{K}$
$\operatorname{Tr}\left(\mathbf{Q}_{0}+\mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \leq P$,
$\mathbf{Q}_{0} \succeq \mathbf{0}, \mathbf{Q}_{a} \succeq \mathbf{0}, \mathbf{Q}_{c} \succeq \mathbf{0}$.
where
$\mathbf{T}_{k}\left(\beta, \mathbf{Q}_{c}, \mathbf{Q}_{a}, t_{k}\right)=$
$\left[t_{k} \mathbf{I}+(\beta-1) \mathbf{Q}_{a}-\mathbf{Q}\right.$
$\left((\beta-1) \mathbf{Q}_{a}-\mathbf{Q}_{c}\right) \tilde{\mathbf{h}}_{k}^{H}$
$\tilde{\mathbf{h}}_{k}\left((\beta-1) \mathbf{Q}_{a}-\mathbf{Q}_{c}\right) \quad \tilde{\mathbf{h}}_{k}\left((\beta-1) \mathbf{Q}_{a}-\mathbf{Q}_{c}\right) \tilde{\mathbf{h}}_{k}^{H}-t_{k} \varepsilon_{k}^{2}+\beta-1$
$\mathbf{S}_{k}\left(\tau^{\prime}, \mathbf{Q}_{c}, \mathbf{Q}_{a}, \mathbf{Q}_{0}, \delta_{k}\right)=$
$\left[\begin{array}{ll}\delta_{k} \mathbf{I}+\mathbf{Q}_{0}-\tau^{\prime}\left(\mathbf{Q}_{a}+\mathbf{Q}_{c}\right) & \left(\mathbf{Q}_{0}-\tau^{\prime}\left(\mathbf{Q}_{a}+\mathbf{Q}_{c}\right)\right) \tilde{\mathbf{h}}_{k}^{H} \\ \mathbf{h}_{k}\end{array}\right.$
$\left[\tilde{\mathbf{h}}_{k}\left(\mathbf{Q}_{0}-\tau^{\prime}\left(\mathbf{Q}_{a}+\mathbf{Q}_{c}\right)\right) \quad-\delta_{k} \varepsilon_{k}^{2}-\tau^{\prime}+\tilde{\mathbf{h}}_{k}\left(\mathbf{Q}_{0}-\tau^{\prime}\left(\mathbf{Q}_{a}+\mathbf{Q}_{c}\right)\right) \tilde{\mathbf{h}}_{k}^{H}\right]$
S-procedure
Let $\varphi_{k}(\mathbf{x})=\mathbf{x}^{H} \mathbf{A}_{k} \mathbf{x}+2 \mathfrak{R}\left\{\mathbf{b}_{k}^{H} \mathbf{x}\right\}+c_{k}$, where $\mathbf{A}_{k} \in \mathbb{H}^{n}$
$\mathbf{b}_{k} \in \mathbb{C}^{n}, c_{k} \in \mathbb{R}$. The implication $\varphi_{1}(\mathbf{x}) \leq 0 \Rightarrow \varphi_{2}(\mathbf{x}) \leq 0$ holds if and only if there exists a $\mu \geq 0$ such that

$$
\mu\left[\begin{array}{cc}
\mathbf{A}_{1} & \mathbf{b}_{1} \\
\mathbf{b}_{1}^{H} & c_{1}
\end{array}\right]-\left[\begin{array}{ll}
\mathbf{A}_{2} & \mathbf{b}_{2} \\
\mathbf{b}_{2}^{H} & c_{2}
\end{array}\right] \succeq \mathbf{0}
$$

The remaining difficulty in solving (4) lies in its objective function, especially the uncertainty therein.
[1] S. Boyd and L. Vandenberghe, Convex optimization. Cambridge university press, 2009..
> Identification of the quasi-concavity of (4)
Property 1: Let us define

$$
f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right)=\min _{\mathbf{h}_{1} \in B_{1}} \frac{1+\mathbf{h}_{1}\left(\mathbf{Q}_{c}+\mathbf{Q}_{a}\right) \mathbf{h}_{1}^{H}}{\beta\left(1+\mathbf{h}_{1} \mathbf{Q}_{a} \mathbf{h}_{1}^{H}\right)}
$$

Then $f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right)$ is a quasi-concave function on the problem domain of (4), and hence the maximization problem (4) is a quasi-concave problem.
Proof: We just need to verify the convexity of the $\alpha$-superlevel set of $f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right)$

$$
S_{\alpha}=\left\{f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right) \mid \mathbf{Q}_{a} \succeq \mathbf{0}, \mathbf{Q}_{c} \succeq \mathbf{0}, f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right) \geq \alpha\right\}
$$

By using the $S$-procedure again, one can easily obtain

$$
f\left(\mathbf{Q}_{c}, \mathbf{Q}_{a}\right) \geq \alpha \Leftrightarrow \mathbf{X}\left(\beta, \mathbf{Q}_{c}, \mathbf{Q}_{a}, \rho\right) \succeq \mathbf{0}
$$

where
$\mathbf{X}\left(\beta, \mathbf{Q}_{c}, \mathbf{Q}_{a}, \rho\right)=$
$\left[\rho \mathbf{I}+(1-\alpha \beta) \mathbf{Q}_{a}+\mathbf{Q}_{c}\right.$
$\left((1-\alpha \beta) \mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \tilde{\mathbf{h}}_{k}^{H}$
$\left[\begin{array}{cc}\tilde{\mathbf{h}}_{k}\left((1-\alpha \beta) \mathbf{Q}_{a}+\mathbf{Q}_{c}\right) & \tilde{\mathbf{h}}_{k}\left((1-\alpha \beta) \mathbf{Q}_{a}+\mathbf{Q}_{c}\right) \tilde{\mathbf{h}}_{k}^{H}-\rho \varepsilon_{1}^{2}-\alpha \beta+1\end{array}\right]$ The proof is completed

Consequently, the optimization problem (4) can be efficiently solved by combining a bisection search [1] with a convex optimization solver, e.g., CVX.

## Numerical Results

$>\mathrm{N}_{t}=2, K=5$
>Channel responses
$\tilde{\mathbf{h}}_{1}=[2,0.4], \tilde{\mathbf{h}}_{k}=[0.9-0.1 k, 0.5+0.1 k], k \in \mathcal{K}_{e}$
$\Rightarrow P=20 \mathrm{~dB}$
$>\varepsilon_{k}=0.2$ for all $k$

- Worst-case secrecy rate regions

$>$ The existence of channel uncertainty dramatically diminishes the achievable secrecy rate region
> AN indeed enhances the security performance without compromising the QoMS.
$>$ The gap tends to be reduced, which implies that AN is prohibitive at high QoMS region.

Secrecy rate versus \#unauthorized receivers

$>$ The worst secrecy rates drops with the number of unauthorized receivers
> Incorporating service integration restrains the maximum worst-case secrecy rates

## Concluding Remarks

$\square$ Considered the optimal robust AN-aided transmit design for multiuser MISO broadcast channel with confidential service and multicast service
-By resorting to a two-stage reformulation, the problem can be handled by solving a sequence of fractional SDPs.

■AN can effectively fortify the transmission security, but high demand for QoMS will confine its use in turn.

