Track-before-detect for sub-Nyquist radar

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Overview

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- Based on compressed sensing (CS) that leverages the sparsity of the target scene, sub-Nyquist radar systems attain target recovery performance close to the traditional Nyquist radar [Na, 2018].

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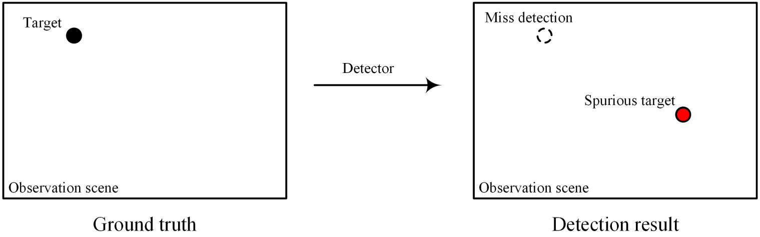

图: An example of miss detection and spurious target.

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- TBD jointly processes a plurality of frames [Tonissen, 1996], and provides tracks of targets and their detection results simultaneously.
- By combining the multi-frame information, TBD improves the detection performance.

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- However, in low SNR situations, since no prior information of new targets is available, the weighted sparse recovery still has poor performance in discovering newly emerged weak targets.
- Thus we simultaneously perform the traditional unweighted and weighted sparse recovery methods to make it more suitable for low SNR cases.

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$$h(t) = \frac{1}{\tau} \sum_{n=1}^{N} H(2\pi n/\tau) e^{-j2\pi n t/\tau}, \tag{2}$$

  where $H(2\pi n/\tau) = 0$ for some $n \notin \Phi$.

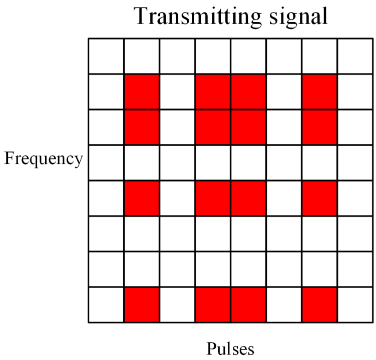

图: An example of the sub-Nyquist transmitting waveform in one CPI where $P = N = 8$, $\Psi = 2, 4, 5, 7$ and $\Phi = 2, 3, 5, 8$.

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$$y_p[n] = \sum_{l=1}^{L} \beta_l' e^{-j\frac{2\pi}{\tau} n\tau_l} e^{-j2\pi f_l^D p\tau}, \tag{4}$$

  where $p \in \Psi$ and $n \in \Phi$.

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$$\mathbf{Y} = \mathbf{R}\mathbf{X}\mathbf{V}^T + \mathbf{N}, \tag{6}$$
where $\mathbf{R} \in \mathbb{C}^{N_1 \times N_2}$ and $\mathbf{V} \in \mathbb{C}^{P_1 \times P_2}$ are the steering matrices of range and velocity, respectively, and $[\mathbf{R}]_{i,j} = e^{-j2\pi(\Phi_i - 1)(j-1)/N_2}$, $[\mathbf{V}]_{i,j} = e^{-j2\pi(\Psi_i - 1)(j-1)/P_2}$. The last term $\mathbf{N} \in \mathbb{C}^{N_1 \times P_1}$ is the i.i.d. additive white Gaussian noise (AWGN).

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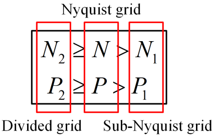

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$$B = \lambda \max_{i,j} \{[\mathbf{J}]_{i,j} + \varepsilon\}. \tag{10}$$

Here the parameter B is set to limit the minimum value of the elements in $\mathbf{W}$ not less than λ.

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- As derived above, in the kth frame, the Fourier coefficients of the echo, denoted by $\mathbf{Y}_k$, is given by

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  where $\mathbf{R}_k$ and $\mathbf{V}_k$ can change over frames.
- Through recovering $\mathbf{X}_k$ from $\mathbf{Y}_k$, we can obtain the range and velocity of the targets, which is considered as an estimate of the true state for tracking.

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- Motion model: describes the movement of the target, indicates the propagation of target states between adjacent frames.
- Measurement model: represents the function of measurements with respect to their ground truth.
- We define the state vector as:

$$\mathbf{s}_k^l = \left[r_k^l, v_k^l, a_k^l \right]^T \tag{12}$$

which refers to the range, velocities and acceleration of the lth target at the kth frame.

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$$\mathbf{s}_k^l = \mathbf{A}\mathbf{s}_{k-1}^l + \mathbf{u}_k^l, \quad 1 \le k \le T, \tag{13}$$

where $\mathbf{A}$ is often referred to the state transition matrix, given by

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The random vector $\mathbf{u}_k^l \sim \mathcal{N}(0, \mathbf{Q})$ is the zero-mean additive Gaussian noise, and the covariance matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} = \begin{bmatrix} \frac{1}{4}P^4\tau^4 & \frac{1}{2}P^3\tau^3 & \frac{1}{2}P^2\tau^2 \\ \frac{1}{2}P^3\tau^3 & P^2\tau^2 & P\tau \\ \frac{1}{2}P^2\tau^2 & P\tau & 1 \end{bmatrix}\rho, \tag{15}$$

where ρ indicates the disturbance that the acceleration is subjected to and is chosen empirically.

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- We then denote the measurement vector by

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- The measurement model, which links between the ground truth $\mathbf{s}_k^l$ and the recovery result $\mathbf{z}_k^l$, is given by a linear model as

$$\mathbf{z}_k^l = \mathbf{M}\mathbf{s}_k^l + \mathbf{w}_k^l, \quad 1 \le k \le T. \tag{17}$$

Here, $\mathbf{M}$ is called the tracking measurement matrix defined as

$$\mathbf{M} = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right], \tag{18}$$

noise vector $\mathbf{w}_k^l \sim \mathcal{N}\left(0, \sigma_n^2 \mathbf{I}_2\right)$ is the zero-mean additive Gaussian with $\mathbf{I}_n$ being a n dimensional unit matrix.

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  - Constructing weighting matrix: Weighting matrix is supposed to sufficiently reflect the prior provided by tracking procedure.

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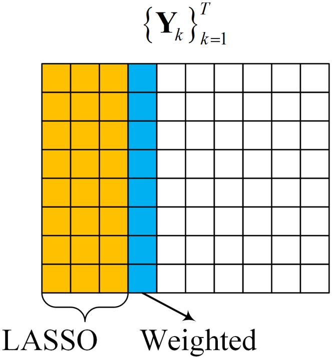

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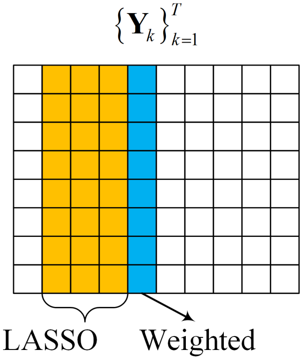

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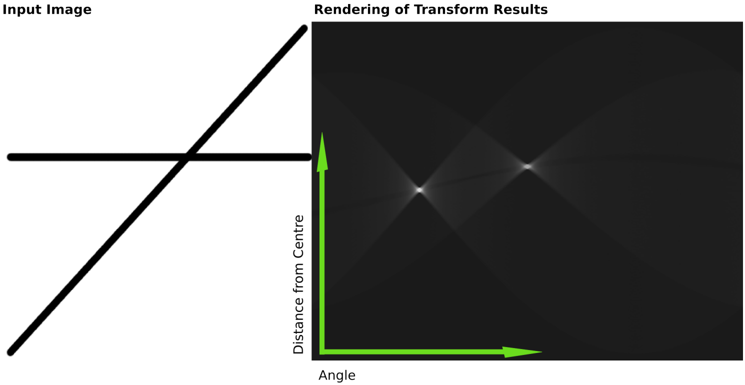

图: An example showing the results of a Hough transform on a raster image containing two thick lines [Wikipedia].

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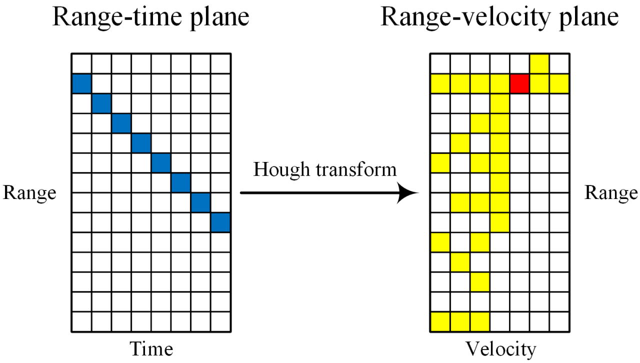

图: An example of a Hough transform which transforms range-time plane into range-velocity plane.

WL1NM-TBD: Tracking portion

- Generating new tracks: Hough transform.
- Updating tracks: Kalman filter and track association.

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$$
\begin{aligned}
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&\mathbf{P}^{p}_{k|k-1} = \mathbf{A}\mathbf{P}^{p}_{k-1|k-1}\mathbf{A}^{T} + \mathbf{Q},
\end{aligned}
\tag{19}
$$

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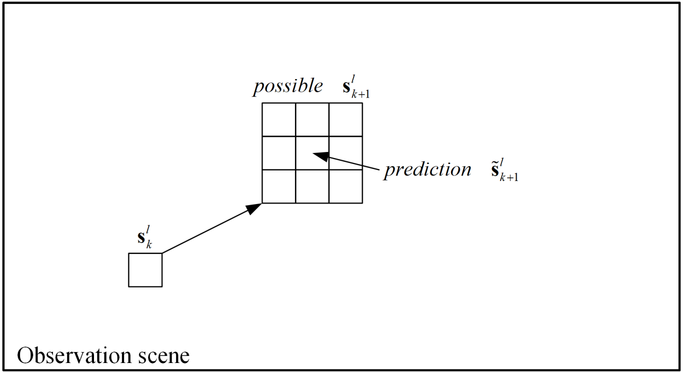

图: An example for track association.

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$$\begin{aligned} \mathbf{K}^p_k &= \mathbf{P}^p_{k|k-1}\mathbf{H}^T\left(\mathbf{U} + \mathbf{H}\mathbf{P}^p_{k|k-1}\mathbf{H}^T\right)^{-1}, \\ \mathbf{s}^p_{k|k} &= \mathbf{s}^p_{k|k-1} + \mathbf{K}^p_k(\mathbf{z}^p_k - \mathbf{H}\mathbf{s}^p_{k|k-1}), \\ \mathbf{P}^p_{k|k} &= (\mathbf{I} - \mathbf{K}^p_k\mathbf{H})\mathbf{P}^p_{k|k-1}. \end{aligned} \tag{21}$$

WL1NM-TBD: Tracking portion

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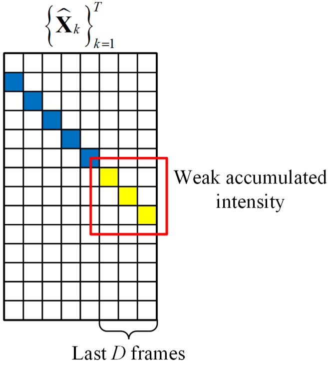

图: An example of deleting old tracks.

WL1NM-TBD: Constructing weighting matrix

- Recall our weighted ℓ_1 norm minimization problem

$$\min_{\mathbf{X}} \left\{ \frac{1}{2} \left\| \mathbf{Y} - \mathbf{R}\mathbf{X}\mathbf{V}^T \right\|_F^2 + \left\| \mathrm{vec}\left(\mathbf{W} \circ \mathbf{X} \right) \right\|_1 \right\}, \tag{8}$$

where the weighting matrix $\mathbf{W}$ is given by

$$[\mathbf{W}]_{i,j} = B/([\mathbf{J}]_{i,j} + \varepsilon), \tag{9}$$

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$$[\mathbf{J}_k]_{i,j} = \sum_{l=1}^{L} A^l e^{-q^l \left(\frac{1}{\sigma_r^{l\,2}} \left(i - r_0^l \right)^2 - c^l \left(i - r_0^l \right) \left(j - v_0^l \right) + \frac{1}{\sigma_v^{l\,2}} \left(j - v_0^l \right)^2 \right)}, \tag{22}$$

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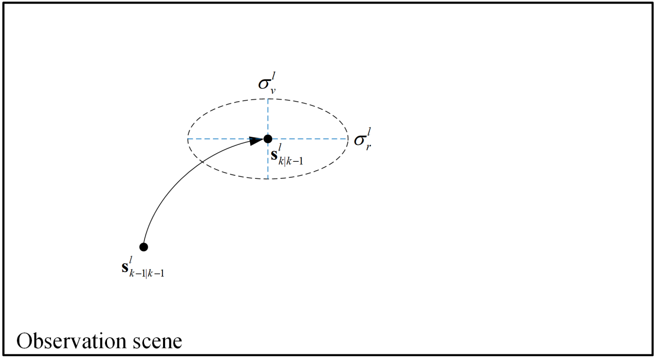

图: Formulating likelihood matrix $\mathbf{J}$ with the prediction of Kalman filter.

WL1NM-TBD: Summary

- Now we summarize our WL1NM-TBD in flow diagram.

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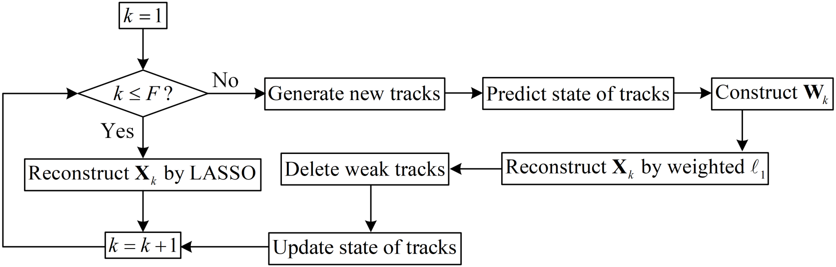

图: Flow diagram of WL1NM-TBD.

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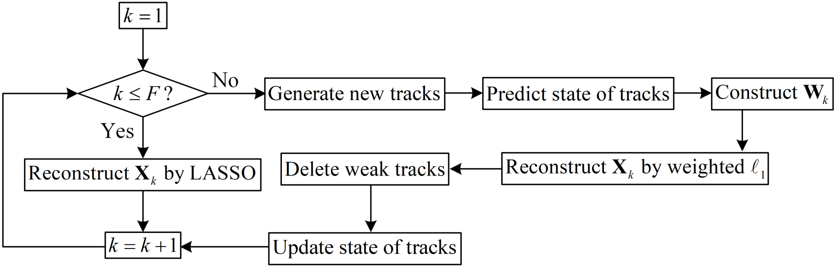

图: Flow diagram of WL1NM-TBD.

- Finally, the detection result is provided by tracks.

Numerical result

Experiment parameters

- The "full" transmitting waveform: $N = P = 16$.

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- In the first experiment, we provide an example of the proposed WL1NM-TBD comparing to LASSO and MF, where the SNR is 7dB and both targets move at a radial velocity of 1.5km/s.

Numerical result

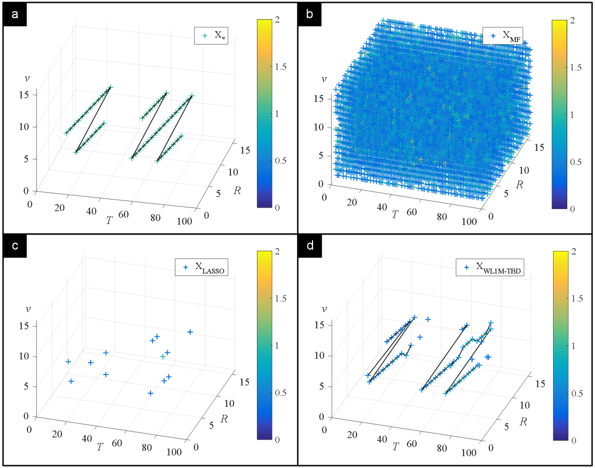

图: (a) The ground truth, and recovery result of (b) MF, (c) LASSO and (d) WL1NM-TBD.

Numerical result

- Next, we place $L = 1$ target in the scene and observe it with $T = 10$ CPIs, and the probability of false alarm is set to no more than 0.01.

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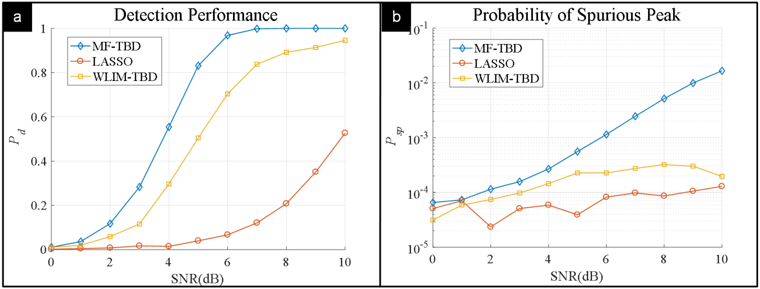

图: (a) The probability of detection. (b) The probability of spurious peak.

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Thank you!