PoGaIN: Poisson-Gaussian Image Noise Modeling from Paired Samples

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Motivation

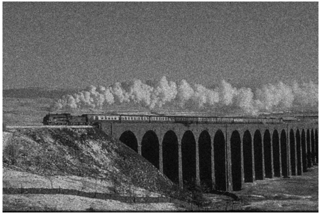
Figure: Noise-free and noisy image from BSD300 (Martin *et al.* 2001)

Problem at hand

- Denoising
- Noise estimation
- Restricted setting: paired samples
- Applications:
  - Dataset modelling, deep learning
  - Imaging system analysis

Results

- Statistical approach based on cumulant expansion
- **Exploiting paired samples**
- Outperforming baselines



Noise Model

Inspired by Foi *et al.* 2008, where x is the noise-free image:

$$y = \eta_p + \eta_g, \tag{1}$$

$$\eta_p \sim \frac{1}{a}\mathcal{P}(ax), \quad \eta_g \sim \mathcal{N}(0, b^2), \quad a \in (0, 100], \tag{2}$$

$$\mathbb{E}[y] = x, \quad \mathbb{V}[y] = \frac{x}{a} + b^2. \tag{3}$$

Explored approaches

- Noisy image only:
  - *FOI*, algorithm introduced by Foi *et al.* 2008
  - *CNN*, Convolutional Neural Net
- Paired samples:
  - Maximum log-likelihood
  - *VAR*, method based on variance
  - ***OURS***, approach using the cumulant expansion

OURS

Given a random variable $X \sim \mathcal{X}$

Definition

Cumulant-generating function:

$$K_{\mathcal{X}}(t) = \log(\mathbb{E}[e^{Xt}]). \tag{4}$$

Definition

r-th cumulant of $\mathcal{X}$:

$$\kappa_r[\mathcal{X}] = K_{\mathcal{X}}^{(r)}(0). \tag{5}$$

OURS

Unbiased estimator via k-statistics (Weisstein n.d.[a]), given n samples:

$$\kappa_2[\mathcal{X}] = \frac{n}{n-1} m_2(x), \quad \kappa_3[\mathcal{X}] = \frac{n^2}{(n-1)(n-2)} m_3(x), \tag{6}$$

using the sample central moments (Weisstein n.d.[b])

$$m_2(x) = \frac{n-1}{n} \sum_i (x_i - \overline{x})^2, \quad m_3(x) = \frac{(n-1)(n-2)}{n^2} \sum_i (x_i - \overline{x})^3, \tag{7}$$

where $\overline{x}$ denotes the mean.

OURS

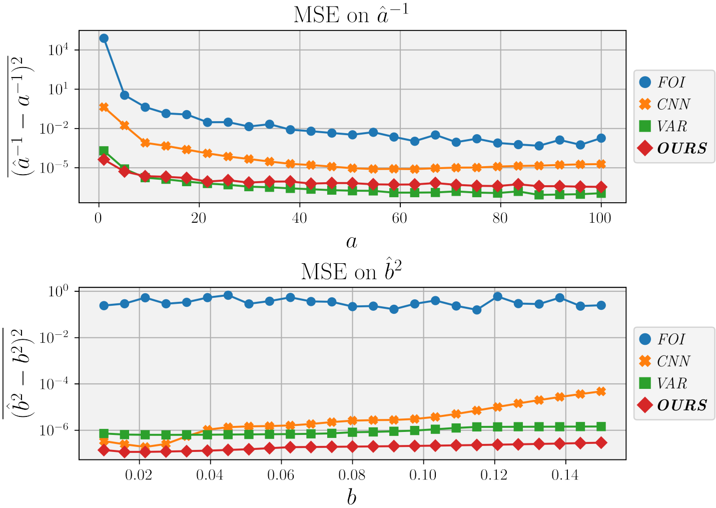
Let x_i be n pixels of a noise-free image, and X and Y be two random variables such that:

$$X \sim \mathcal{X}, \quad \mathbb{P}[X = x_i] = \frac{|\{\, k : x_k = x_i \,\}|}{n}, \quad Y \sim \mathcal{Y} = \frac{\mathcal{P}(a\mathcal{X})}{a} + \mathcal{N}(0, b^2) \tag{8}$$

We show that:

$$\begin{cases} \kappa_2[\mathcal{Y}] = \frac{\overline{x}}{a} + \overline{x^2} - \overline{x}^2 + b^2 \\ \kappa_3[\mathcal{Y}] = \overline{x^3} - 3\overline{x^2}\,\overline{x} + 2\overline{x}^3 + 3\frac{\overline{x^2}}{a} - 3\frac{\overline{x}^2}{a} + \frac{\overline{x}}{a^2} \end{cases} . \tag{9}$$

Estimation error



Thanks

Thank you very much for your attention!

ICASSP's poster presentation session on Denoising:

6/9/2023 at 14:00 (EEST)

`https://github.com/IVRL/PoGaIN`

References I

[1] D. Martin, C. Fowlkes, D. Tal, and J. Malik, "A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics," in *Proc. 8th Int'l Conf. Computer Vision*, vol. 2, Jul. 2001, pp. 416–423.

[2] A. Foi, M. Trimeche, V. Katkovnik, and K. Egiazarian, "Practical Poissonian-Gaussian noise modeling and fitting for single-image raw-data," *IEEE Transactions on Image Processing*, vol. 17, no. 10, pp. 1737–1754, 2008.

References II

[3] E. W. Weisstein, *K-statistic from mathworld–a wolfram web resource*, [Online]. Available: https://mathworld.wolfram.com/k-Statistic.html.

[4] E. W. Weisstein, *Sample central moment. from mathworld–a wolfram web resource*, [Online]. Available: https://mathworld.wolfram.com/SampleCentralMoment.html.