Image Source Method Based On the Directional Impulse Responses

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The authors would like to thank Prof. Dr.-Ing. Sascha Spors, Dr.-Ing. Frank Schultz and Nara Hahn from Universität Rostock for their contribution.

Overview

This paper presents **a wideband image source method** to simulate the time-domain signal on the boundary of the spherical listening region. The proposed method considers the loudspeaker directional impulse responses (DIRs).

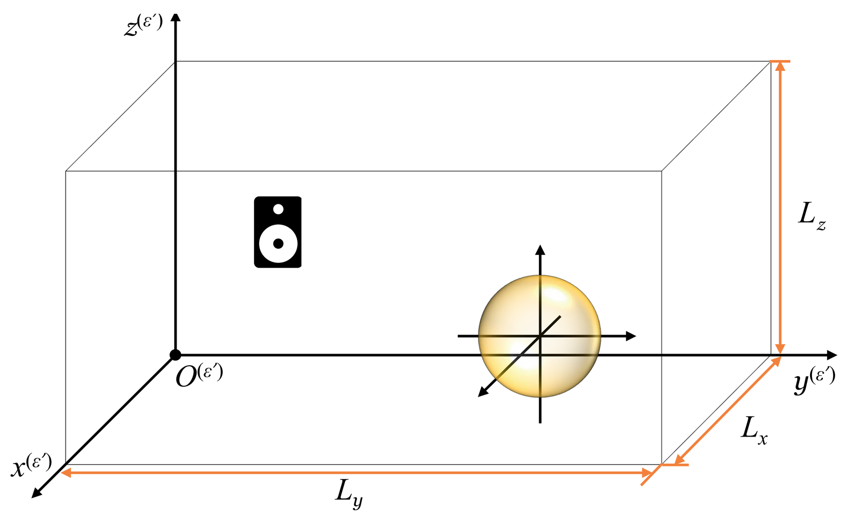

Figure 1: Problem setup. The ISM simulates the time-domain signal on the yellow boundary of the spherical listening region.

Loudspeaker DIRs

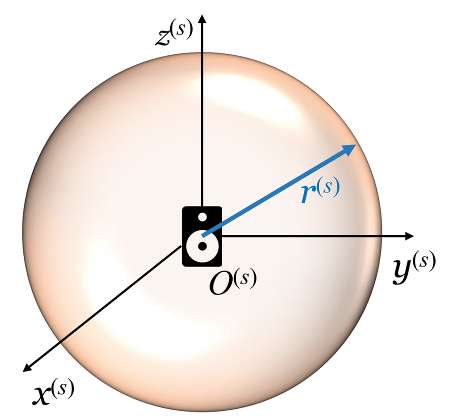

Figure 2: Measurement setup. The loudspeaker DIRs are measured at sampling points on the light red spherical surface of radius $r^{(s)}$.

Loudspeaker DIRs are treated as a sequence of propagating spherical wave fronts with direction-dependent amplitude.

Far-field DIRs measured on a spherical surface of radius $r^{(s)}$

$$h(t, r^{(s)}, \theta^{(s)}, \phi^{(s)}) = \int_\tau h(\tau, r^{(s)}, \theta^{(s)}, \phi^{(s)})\delta(t-\tau)d\tau. \quad (1)$$

Replace with an ideal source at $O^{(s)}$ that emits

$$d(t, \theta^{(s)}, \phi^{(s)}) = \int_\tau d(\tau, \theta^{(s)}, \phi^{(s)})\delta(t-\tau)d\tau.$$

Let

$$d(\tau, \theta^{(s)}, \phi^{(s)}) = 4\pi r^{(s)} h(\tau + r^{(s)}/c, r^{(s)}, \theta^{(s)}, \phi^{(s)}).$$

Assume each spherical wave front only experiences uniform attenuation related to the traveled distance, the DIRs of this ideal source measured on the spherical surface of radius $r^{(s)}$ should follow exactly (1).

Free-Field

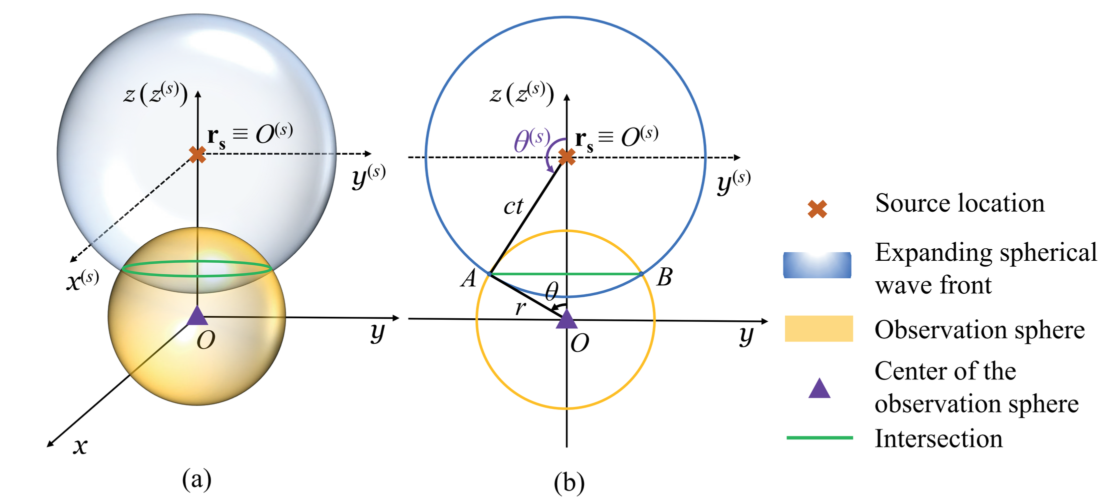

Figure 3: (a) 3D view and (b) cross section view of the observation sphere in yellow and the expanding spherical wave front in light blue. The source is at $\mathbf{r}_s$. Line AB is the cross section of the circle, which is the intersection of the observation sphere and the expanding spherical wave front.

▶ The source emits a single spherical wave front

$$d(t, \theta^{(s)}, \phi^{(s)}) = Y_v^u(\theta^{(s)}, \phi^{(s)})\delta(t).$$

At $t = t_0$, $\theta = \theta_0$. The observed signal

$$g(t_0, r, \theta, \phi) = \frac{c}{4\pi r r_s}\delta(\cos\theta - \cos\theta_0)Y_v^u(\theta_0^{(s)}, \phi)$$

in which $\phi = \phi^{(s)}$, and $\theta_0^{(s)}$ is w.r.t. the $x^{(s)}y^{(s)}z^{(s)}$ coordinate system [1].

The spherical harmonic (SH) coefficients

$$\zeta_n^m(t_0, r) = \frac{c}{2r r_s}\mathcal{P}_v^u(\cos\theta_0^{(s)})\mathcal{P}_n^m(\cos\theta_0)\,\delta_{m,u}\,\Xi(t_0)$$

- $\mathcal{P}_n^m(\cdot) = \sqrt{(2n+1)/4\pi}\sqrt{(n-m)!/(n+m)!}P_n^m(\cdot)$
- $\Xi(t_0)=1$ if $r_s - r \le c t_0 \le r_s + r$; else, $\Xi(t_0)=0$.
- $\zeta_n^m(t_0, r) \neq 0$ only when $n \ge |u|$ and $m = u$.
- $\cos\theta_0 = (r^2 + r_s^2 - c^2 t_0^2)/(2 r r_s)$.
- $\cos\theta_0^{(s)} = -(c^2 t_0^2 + r_s^2 - r^2)/(2 c t_0 r_s)$.

▶ The source emits a sequence of spherical wave fronts

$$d(t, \theta^{(s)}, \phi^{(s)}) = \int_\tau \underbrace{\sum_{v=0}^{V}\sum_{u=-v}^{v}\gamma_v^u(\tau)Y_v^u(\theta^{(s)}, \phi^{(s)})}_{d(\tau, \theta^{(s)}, \phi^{(s)})}\,\delta(t-\tau)d\tau,$$

Using the superposition principle, the SH coefficients of the observed signal

$$\zeta_n^m(t_0, r) = \frac{c}{2r r_s}\int_\tau \sum_{v=0}^{V}\sum_{u=-v}^{v}\gamma_v^u(\tau)\,\mathcal{P}_v^u[\cos\theta_0^{(s)}(\tau)]\,\mathcal{P}_n^m[\cos\theta_0(\tau)]\delta_{m,u}\,\Xi(t_0, \tau)\,d\tau. \quad (2)$$

- $\Xi(t_0,\tau)=1$ if $r_s - r \le c(t_0-\tau) \le r_s + r$; else, $\Xi(t_0,\tau)=0$.
- $\cos\theta_0(\tau) = [r^2 + r_s^2 - c^2(t_0-\tau)^2]/[2 r r_s]$.
- $\cos\theta_0^{(s)}(\tau) = -[c^2(t_0-\tau)^2 + r_s^2 - r^2]/[2c(t_0-\tau)\,r_s]$.

ISM

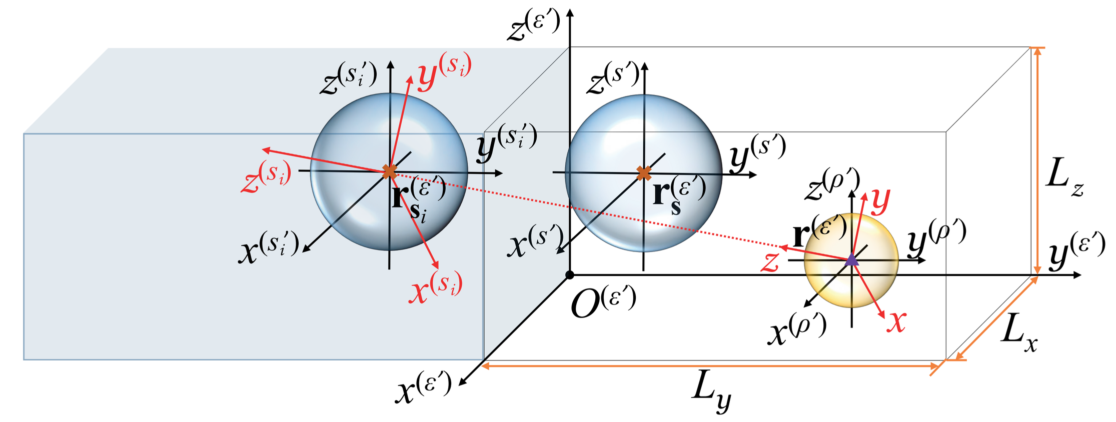

Figure 4: Setup of the ISM. The image source at $\mathbf{r}_{\mathrm{s}_i}^{(\varepsilon')}$ is the mirror reflection of the original source at $\mathbf{r}_{\mathrm{s}}^{(\varepsilon')}$. The observation sphere in yellow is centered at $\mathbf{r}^{(\varepsilon')}$.

Principles

- Wall reflections replaced by image sources in a grid of mirrored rooms.
- For each image source
  - The amplitudes of the emitted spherical wave fronts are the mirror reflections of those emitted by the original source.
  - The red coordinate systems are introduced so that the image source is on the positive z-axis, i.e., the setup becomes similar to Figure 3.

Procedure

The source emits a sequence of spherical wave fronts $d(t, \theta^{(s')}, \phi^{(s')})$.

Step 1 - Calculate the SH coefficients of the spherical wave fronts $d(t, \theta^{(s_i')}, \phi^{(s_i')})$ emitted by the image sources by using the parity properties of the SH functions.

Step 2 - Calculate the SH coefficients of $d(t, \theta^{(s_i)}, \phi^{(s_i)})$. The rotation of the coordinate system is achieved by using the Wigner D-matrix [2] in the SH domain.

Step 3 - Calculate the SH coefficients of the observed signal w.r.t. the xyz coordinate system by following (2). Also incorporate the attenuation due to wall reflections.

Step 4 - Calculate the SH coefficients of the observed signal w.r.t. the $x^{(\rho')}y^{(\rho')}z^{(\rho')}$ coordinate system by using the Wigner D-matrix.

Step 5 - Add the contributions of all image sources.

Simulations

- Room dimension [4, 6, 3] m, wall reflection coefficients [0.45, 0.7, 0.8, 0.5, 0.6, 0.75].
- Source location $\mathbf{r}_{\mathrm{s}}^{(\varepsilon')} =$ [1.5, 3.4, 2.4] m.
- The observation sphere of radius $r = 0.2$ m is centered at $\mathbf{r}^{(\varepsilon')} =$ [1.5, 3.4, 1] m.
- $d(t, \theta^{(s')}, \phi^{(s')}) = Y_0^0(\theta^{(s')}, \phi^{(s')})\delta(t) + Y_1^0(\theta^{(s')}, \phi^{(s')})\delta(t-0.01)$.
- 24 image sources are considered.
- The sampling frequency is 16 kHz.
- SH truncation order of the observed signal is 10.

To reduce the effect of aliasing, uniformly sampled version of (2) is convolved with a low-pass filter with 257 samples and cut-off frequency at 2 kHz. [3] and [4] cover more advanced sampling and bandlimitation methods.

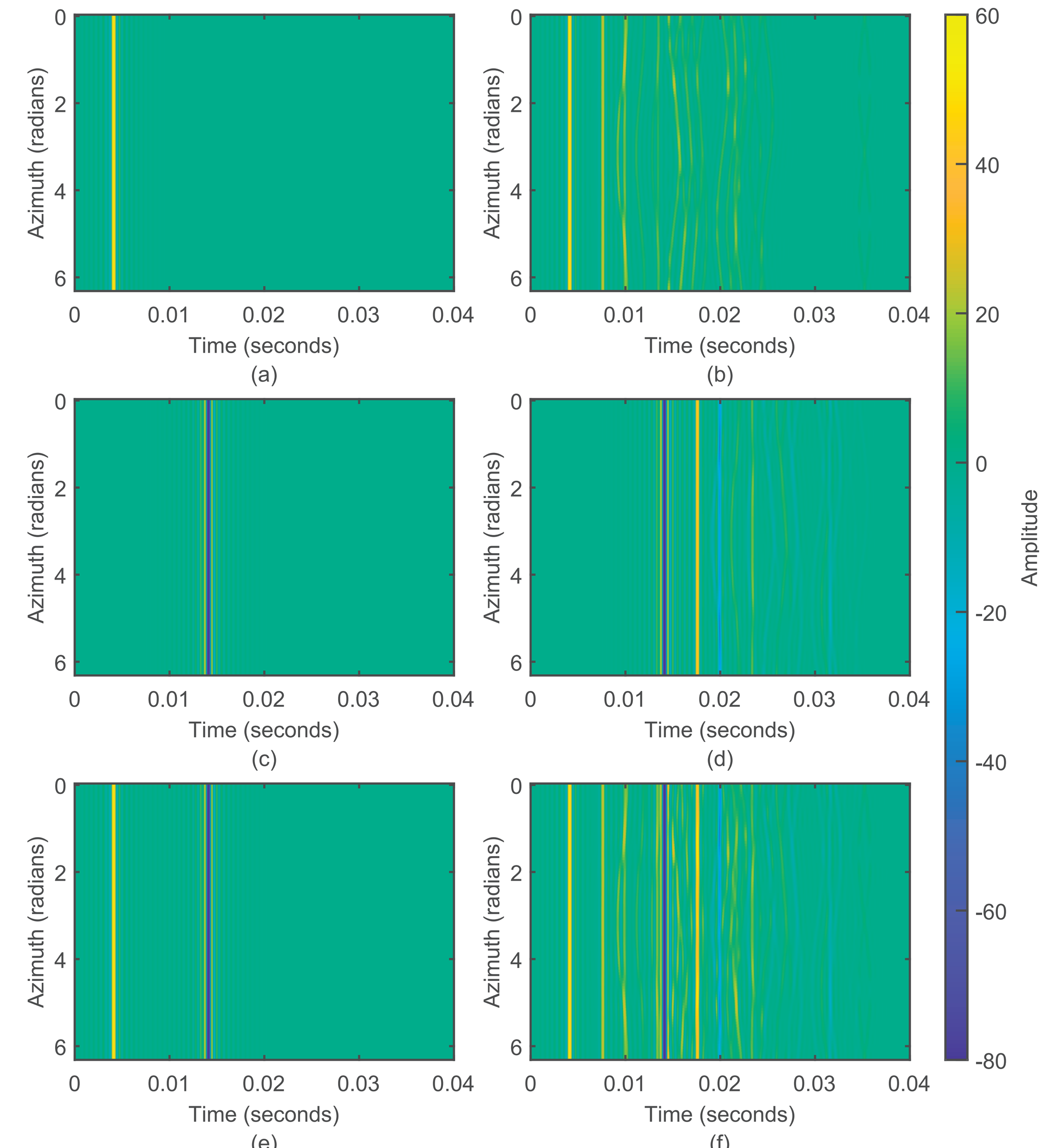

Figure 5: Observed signals on the equator of the observation sphere.
In (a) and (b), $d(t, \theta^{(s')}, \phi^{(s')}) = Y_0^0(\theta^{(s')}, \phi^{(s')})\delta(t)$.
In (c) and (d), $d(t, \theta^{(s')}, \phi^{(s')}) = Y_1^0(\theta^{(s')}, \phi^{(s')})\delta(t-0.01)$.
In (e) and (f), $d(t, \theta^{(s')}, \phi^{(s')}) = Y_0^0(\theta^{(s')}, \phi^{(s')})\delta(t) + Y_1^0(\theta^{(s')}, \phi^{(s')})\delta(t-0.01)$.
Moreover, (a), (c) and (e) are in anechoic condition; while (b), (d), and (f) are in reverberant condition.

References

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