Performance Analysis of a DF based Dual Hop Mixed RF-FSO System with a Direct RF Link

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December 13, 2015
1. Introduction
2. Problem Approached
3. Statistical Characteristics
4. Performance Metrics
5. Numerical Results
6. References
Motivation

- "Wireless" synonymous to RF technologies
- RF band is limited, licensed, and costly
- Free Space Optical Communications (FSO) / Optical Wireless Communication (OWC) → transmission in unguided propagation media through use of optical carriers, i.e., visible, IR, and UV band.

Figure: Electromagnetic spectrum
FSO: Advantages

- High rate communication over distances up to several kilometers (10 Gbps)
- FSO systems use very narrow laser beams ➞ inherent security and robustness to electromagnetic interference.
- Frequency used is above 300 GHz which is unlicensed worldwide.
- FSO systems are also easily deployable and can be reinstalled without the cost of dedicated fiber optic connections.
- Efficient solution for the "last mile" problem to bridge the gap between the end user and the backbone network.
- Enterprise/campus connectivity
- Video surveillance and monitoring
FSO : Applications I

FSO communication can be potentially employed in a diverse range of communication applications. Based on the transmission range, OWC can be studied in five categories -

- **Ultra-short range OWC** (chip-to-chip communications in stacked and closely-packed multi-chip packages)

- **Short range OWC** (wireless body area network (WBAN) and wireless personal area network (WPAN) applications, underwater communications)

  *Example*: Disaster Recovery, e.g., 9/11 Terrorist Attacks in NY City when financial corporations were left out with no landlines.
FSO: Applications II

- Medium range OWC (indoor IR and VLC for wireless local area networks (WLANs), inter-vehicular and vehicle-to-infrastructure communications)

- Long range OWC (inter-building connections)
  Example: Broadcasting of live events, e.g., during 2010 FIFA World Cup, BBC deployed FSO links for Ethernet-based transport of HD video between studio locations setup in South Africa.

- Ultra-long range OWC (inter-satellite links and deep space links)
FSO: Limitations

The performance of FSO systems is strongly AFFECTED by:

- Atmospheric Turbulence/Scintillations: variations in temperature and pressure of atmosphere $\implies$ variations in the refractive index along the transmission path $\implies$ channel fading.

- ATMOSPHERIC LOSS: Rain, snow, fog, pollution, dust, smoke, etc absorb laser light energy attenuating optical power of the signal and cause light scattering.

- MISALIGNMENT LOSS or POINTING ERRORS $\implies$ building sway phenomenon due to thermal expansion, earthquakes, etc.
FSO: Solutions

- Radio on FSO (RoFSO)
- MIMO-FSO
- Hybrid RF/FSO
- Asymmetric RF-FSO
- Serial FSO

Cooperation protocols
- Amplify-and-forward (AF)
- Decode-and-forward (DF)

Cooperative communication provides
- High reliability and fading mitigation
- Performance enhancement
- Broad and energy-efficient coverage area
In [S. Anees and M. R. Bhatnagar, IET Optoelectronics, 2015] Outage, BER, and capacity analysis for DF based asymmetric RF-FSO systems, where RF link \(\Rightarrow\) Nakagami distribution and FSO link \(\Rightarrow\) Gamma-Gamma turbulence & pointing errors.

In [I. S. Ansari, M. S. Alouini, and F. Yilmaz, IEEE VTC, 2013] BER analysis of fixed gain AF based mixed RF-FSO system, where RF link \(\Rightarrow\) Rayleigh distribution and FSO link \(\Rightarrow\) Gamma-Gamma turbulence & pointing errors and a direct RF link \(\Rightarrow\) Rayleigh distribution.

**Figure:** System Model of DF based dual-hop mixed RF-FSO system with a direct RF link.
Problem Statement

- Information theoretic analysis of DF based dual hop mixed RF-FSO communication system, where
  - S-R link is characterized by Nakagami-$m$ distributed fading
  - R-D link is characterized by Gamma-Gamma distributed turbulence and pointing error
  - S-D link is characterized by Nakagami-$m$ distributed fading
- The system uses SC at the receiver; it selects the link with maximum SNR
- The system uses SIM scheme and direct mode of detection
System model III

- Signal received by R and D from S:

\[ y_{s,q} = h_{s,q}x + e_{s,q} \]

* \( q \in \{r, d\} \)
* \( x \) denotes the signal transmitted by S
* \( h_{s,q} \) denotes the Nakagami-\( m \) distributed channel gain
* \( e_{s,q} \) denotes zero-mean AWGN noise with \( \sigma^2_{s,q} \) variance

- Signal received by D after optical-to-electrical conversion from S over the FSO link:

\[ y_{r,d} = \eta_{r,d}l_{r,d}\hat{x} + e_{r,d} \]

* \( l_{r,d} \) is the real-valued Gamma-Gamma distributed irradiance
* \( \eta_{r,d} \) is optical-to-electrical conversion coefficient
* \( e_{r,d} \) denotes zero-mean AWGN noise with \( \sigma^2_{r,d} \) variance
System model IV

- For a DF based mixed RF/FSO system without a direct link, the end-to-end signal-to-noise (SNR) \((\gamma_{s,r,d})\)
  
  \[ \gamma_{s,r,d} \approx \min(\gamma_{s,r}, \gamma_{r,d}) \]

- The instantaneous received SNR at D:
  
  \[ \gamma_z = \max(\gamma_{s,d}, \gamma_{s,r,d}) \]
Channel Model I : RF Link

Assuming the fading of RF link to be Nakagami-$m$ distributed, the PDF of $\gamma_{s,q}$ will be Gamma distributed

$$f_{\gamma_{s,q}}(\gamma) = \frac{m_{s,q}^{m_{s,q}} \gamma^{m_{s,q}-1}}{\Gamma(m_{s,q}) \bar{\gamma}_{s,q}^{m_{s,q}}} \exp \left( - \frac{m_{s,q} \gamma}{\bar{\gamma}_{s,q}} \right),$$

* $m \geq 1/2$ is the Nakagami parameter
PDF of $\gamma_{r,d}$ for direct detection

\[
f_{\gamma_{r,d}}(\gamma) = \frac{\xi^2}{2\gamma \Gamma(a) \Gamma(b)} G^{3,0}_{1,3} \left( \frac{\gamma}{\gamma_{r,d}} \left| \frac{\xi^2 + 1}{\xi^2} \right. \right)
\]

- $\gamma = \frac{\gamma_{r,d}}{I_0 A_0 \rho}$
- $f = \frac{\xi^2}{\xi^2 + 1}$
- $\xi = \frac{w_e}{2\sigma_s}$
- $w_e$ is the equivalent beamwaist
- $\sigma_s$ is the pointing error displacement standard deviation at the receiver
- $G(\cdot)$ is the Meijer-G function
CDF:

\[
F_{\gamma_{s,r,d}}(\gamma) = 1 - \left( 1 - \mathcal{K}_1 \gamma \left( m_{s,r}, \frac{m_{s,r} \gamma}{\bar{\gamma}_{s,r}} \right) \right) \\
\times \left( 1 - \mathcal{K}_2 G_{3,7}^6,1 \left( \mathcal{W}_{\gamma} \left| \begin{array}{c} 1, \mathcal{P}_1 \\ \mathcal{P}_2, 0 \end{array} \right. \right) \right)
\]

* \( \mathcal{K}_1 = \frac{1}{\Gamma(m_{s,r})} \)

* \( \mathcal{K}_2 = \frac{2^{z_1-2} \xi^2}{2\pi \Gamma(a) \Gamma(b)} \)

* \( \mathcal{W} = \frac{(fab)^2}{16 \bar{\gamma}_{r,d}} \)

* \( \mathcal{P}_1 = \frac{\xi^2 + 1}{2}, \frac{\xi^2 + 2}{2} \)

* \( \mathcal{P}_2 = \frac{\xi^2}{2}, \frac{\xi^2 + 1}{2}, \frac{a}{2}, \frac{a+1}{2}, \frac{b}{2}, \frac{b+1}{2} \)
PDF:

\[
f_{\gamma_{s,r,d}}(\gamma) = \left(\mathcal{K}_1 \left( \frac{m_{s,r}}{\gamma_{s,r}} \right)^{m_{s,r}} \right) \gamma^{m_{s,r} - 1} \exp \left( \frac{-m_{s,r}\gamma}{\gamma_{s,r}} \right)\]
\[
\times \left( 1 - \mathcal{K}_2 G^{6,1}_{3,7} \left( \mathcal{W}_\gamma \left| \frac{1}{\mathcal{P}_2}, 0 \right. \right) \right) - \mathcal{K}_2 \gamma^{-1}
\]
\[
\times \left( 1 - \mathcal{K}_1 \gamma \left( m_{s,r}, \frac{m_{s,r}\gamma}{\gamma_{s,r}} \right) \right) G^{6,0}_{2,6} \left( \mathcal{W}_\gamma \left| \frac{\mathcal{P}_1}{\mathcal{P}_2} \right. \right)
\]
Statistical Characteristics: Mixed RF-FSO Cooperative System with a Direct Link

CDF:

\[
F_{\gamma_z}(\gamma) = K_3 \gamma (m_{s,d}, \frac{m_{s,d} \gamma}{\gamma_{s,d}}) \left[ 1 - \left( 1 - K_1 \gamma (m_{s,r}, \frac{m_{s,r} \gamma}{\gamma_{s,r}}) \right) \right] \\
\times \left( 1 - K_2 G_{3,7}^{6,1} (\mathcal{W}_{\gamma | 1, P_1}) \right) \]

where \( K_3 = 1/\Gamma(m_{s,d}) \).

PDF:

\[
f_{\gamma_z}(\gamma) = F_{\gamma_{s,d}}(\gamma)f_{\gamma_{s,r,d}}(\gamma) + f_{\gamma_{s,d}}(\gamma)F_{\gamma_{s,r,d}}(\gamma).
\]
Outage Probability

- Without Direct Link:

\[ P_{\text{out}}(\gamma_{th}) = F_{\gamma_{s,r,d}}(\gamma_{th}) = 1 - \left( 1 - \mathcal{K}_1 \gamma \left( \frac{m_{s,r} \gamma_{th}}{\bar{\gamma}_{s,r}} \right) \right) \times \left( 1 - \mathcal{K}_2 G_{6,1}^{3,7} \left( \mathcal{W}_{\gamma_{th}} \left| \frac{1, P_1}{P_2, 0} \right) \right) \right). \]

- With Direct RF Link:

\[ P_{\text{out}}(\gamma_{th}) = \mathcal{K}_3 \gamma \left( \frac{m_{s,d} \gamma_{th}}{\bar{\gamma}_{s,d}} \right) \left[ 1 - \left( 1 - \mathcal{K}_1 \right) \right] \times \gamma \left( \frac{m_{s,r} \gamma_{th}}{\bar{\gamma}_{s,r}} \right) \left( 1 - \mathcal{K}_2 G_{6,1}^{3,7} \left( \mathcal{W}_{\gamma_{th}} \left| \frac{1, P_1}{P_2, 0} \right) \right) \right]. \]
**Table**: BER parameters for Various Modulation Techniques

<table>
<thead>
<tr>
<th>Modulation techniques</th>
<th>$\phi$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherent Binary Frequency Shift Keying (CBFSK)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Coherent Binary Phase Shift Keying (CBPSK)</td>
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<td>1</td>
</tr>
<tr>
<td>Non-Coherent Binary Frequency Shift Keying (NBFSK)</td>
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<td>0.5</td>
</tr>
<tr>
<td>Differential Binary Phase Shift Keying (DBPSK)</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Bit Error Rate II

Average BER Without Direct Link:

\[
P_e = \frac{K_1}{2\Gamma(\phi)} G_{1,2}^{1,2} \left( m_{s,r} \left| \frac{1 - \phi, 1}{m_{s,r}, 0} \right. \right) + \sum_{k=0}^{m_{s,r}-1} \frac{\psi^k K_2}{2k! \Gamma(\phi)} \left( \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^k \]

\[
\times \left( \psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^{-\phi-k} G_{6,2}^{4,7} \left( \frac{\mathcal{W}_{\bar{\gamma}_{s,r}}}{m_{s,r} + \psi \bar{\gamma}_{s,r}} \left| 1 - \phi - k, 1, \mathcal{P}_1 \right. \right) \left( \frac{W_{\bar{\gamma}_{s,r}}}{m_{s,r} + \psi \bar{\gamma}_{s,r}} \left| 1 - \phi - k, 1, \mathcal{P}_1 \right. \right).
\]
Bit Error Rate III

Average BER With Direct RF Link :

\[ P_e = \frac{\mathcal{K}_3}{2\Gamma(\phi)} G^{1,2}_{2,2} \left( \frac{m_{s,d}}{\psi \bar{\gamma}_{s,d}} \right| 1 - \phi, 1 \right) - \frac{\psi \phi \mathcal{K}_3}{2\Gamma(\phi)} \]

\[ \times \sum_{k=0}^{m_{s,r}-1} \left( \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^k \frac{1}{k!} \left( \psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^{-\phi-k} G^{1,2}_{2,2} \left( \frac{m_{s,d} \bar{\gamma}_{s,r}}{\bar{\gamma}_{s,d}(\psi \bar{\gamma}_{s,r} + m_{s,r})} \right| 1 - \phi - k, 1 \right) \]

\[ + \frac{\psi \phi \mathcal{K}_2}{2\Gamma(\phi)} \sum_{k=0}^{m_{s,r}-1} \left( \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^k \frac{1}{k!} \left( \psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^{-\phi-k} G^{6,2}_{4,7} \left( \frac{\mathcal{W} \bar{\gamma}_{s,r}}{(\psi \bar{\gamma}_{s,r} + m_{s,r})} \right| 1 - k - \phi, 1, \mathcal{P}_1 \right) \]

\[ - \frac{\psi \phi \mathcal{K}_2}{2\Gamma(\phi)} \sum_{k=0}^{m_{s,r}-1} \sum_{l=0}^{m_{s,d}-1} \left( \frac{m_{s,d}}{\bar{\gamma}_{s,d}} \right)^l \left( \frac{m_{s,r}}{\bar{\gamma}_{s,r}} \right)^k \frac{1}{l!k!} \left( \psi + \frac{m_{s,r}}{\bar{\gamma}_{s,r}} + \frac{m_{s,d}}{\bar{\gamma}_{s,d}} \right)^{-\phi-k-l} \]

\[ \times G^{6,2}_{4,7} \left( \frac{\mathcal{W}}{(\psi + \frac{m_{s,d}}{\bar{\gamma}_{s,d}} + \frac{m_{s,r}}{\bar{\gamma}_{s,r}})} \right| 1 - k - l - \phi, 1, \mathcal{P}_1 \right) \]
Result 1: Outage Probability

Figure: Outage Probability versus average SNR of the mixed RF-FSO system with direct link, for different values of fading parameters and $\xi=1.2$. 
Result II: BER for Different Modulation Schemes

Figure: Average BER versus average SNR of the dual hop mixed RF-FSO system with direct link, for different modulation techniques and fading parameters, $m_{s,d}=2$, $m_{s,r}=4$, $a=4.2$, $b=1.4$, and $\xi=1.2$. 
Result III: BER with and without Direct RF Link

**Figure:** Average BER versus average SNR of dual hop mixed RF-FSO system with and without direct link for CBFSK modulation technique and different values of fading parameters and $\xi$. 
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Thank You