Supplementary Materials

: Robust Estimation of Bump Height for Wafer-Level Packaging Using Optical Triangulation

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This supplementary document provides a detailed derivation of Equation (1) from the main manuscript, which maps the pixel coordinates (u, v) of the t^{th} raw images to 3D space (x, y, h) using optical triangulation geometry.

A. Notation and Parameters

Throughout this document, we follow the notation defined in the main manuscript:

- u, v, t: column index, row index, and acquisition order of the raw image
- C: total number of rows in the raw image (i.e., $v \in [0, C-1]$)
- d: size of a camera pixel (μm)
- α: glancing angle of the beam (angle between the beam and the surface plane)
- I_t: the raw image acquired at acquisition order t
- l: scanning resolution (μm)

B. Height Mapping: $h(u, v, t)$

We acquired raw images by projecting a line beam onto the bump at a glancing angle α, where the beam extends along the x-axis. During acquisition, both the beam and the camera, which were aligned at the same angle, moved together along the y-axis with a scan resolution l. Fig. 1 illustrates how the reflected beam from the bump surface is captured by the camera, shown in the y-h plane. The row index v of the raw image is counted from the top ($v = 0$) to the bottom ($v = C - 1$), so reflections from higher points on the bump appear at smaller v on the camera. Using scanning parameters such as d and C, we can express the height $h(u, v, t)$ as

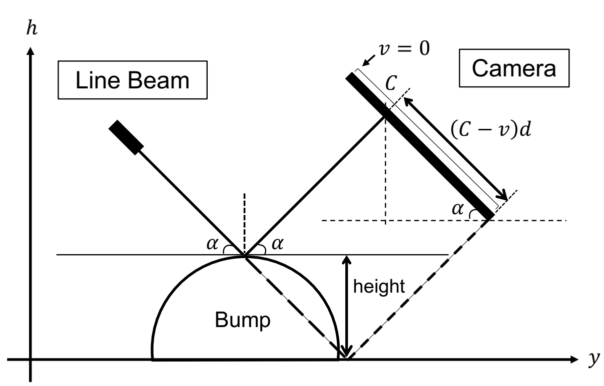

Fig. 1. Optical triangulation geometry in the y–h plane

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$$h(u, v, t) = d(C - v)\sin\alpha, \tag{1}$$

which is independent of the column index u and the acquisition order t. The height is determined only by the row index v.

C. Location Mapping: $x(u, v, t)$ and $y(u, v, t)$

We now derive the mapping from coordinates (u, v) in the t^{th} raw image to the 2D world coordinates (x, y) on the scan plane. Since we are using a line beam, the world coordinate $x(u, v, t)$ is determined solely by the column index u, and is independent of v and t, as in

$$x(u, v, t) = du. \tag{2}$$

As shown in Fig. 2, the raw images are sequentially acquired along the y-axis with a scan resolution of l. Each raw image I_t corresponds to the acquisition order t. We define the starting position of the t^{th} raw image along the y-axis at the base row ($v = C - 1$), as

$$y(u, C - 1, t) = lt. \tag{3}$$

The y-coordinate corresponding to each row index v in the t^{th} raw image can be computed by subtracting an offset from the starting position. The offset is determined by applying optical triangulation. Finally, $y(u, v, t)$ is formulated as

$$y(u, v, t) = \underbrace{lt}_{\text{Starting position}} - \underbrace{d(C - v - 1)\cos\alpha}_{\text{Offset}}, \tag{4}$$

which is independent of the column index u.

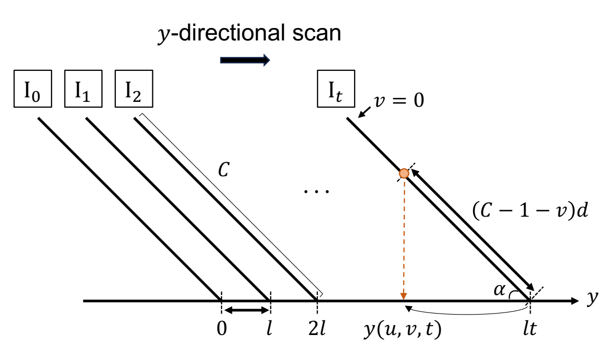

Fig. 2. Multi-frame acquisition process along the y-direction.

D. Matrix Form of Coordinate Mapping

We express the coordinate mapping in matrix form by combining the individual expressions for $x(u,v,t)$, $y(u,v,t)$, and $h(u,v,t)$. This transformation is a one-to-one mapping from the coordinates (u,v) in the t^{th} raw image to the corresponding 3D spatial coordinates (x,y,h), as in

$$\begin{bmatrix} x \\ y \\ h \end{bmatrix} = \begin{bmatrix} d & 0 & 0 \\ 0 & d\cos\alpha & l \\ 0 & -d\sin\alpha & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ t \end{bmatrix} + \begin{bmatrix} 0 \\ d(1-C)\cos\alpha \\ dC\sin\alpha \end{bmatrix}. \quad (5)$$