A Spectral Graph Wiener Filter in Graph Fourier Domain for Improved Image Denoising

Ali Can YAĞAN, Mehmet Tankut ÖZGEN

Anadolu University, Faculty of Engineering, Department of Electrical and Electronics Engineering, 26555 Eskişehir, Turkey,

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MOTIVATION

-Improving denoising performances achieved by various spectral graph based denoising methods.

-Proposing a post-processing step that yields consistent accuracy improvement for different choices of weighted adjacency and graph Laplacien matrices used in computing the graph Fourier transform and for different processing methods used to denoise ontained transform coefficients.

-Attaining a post-processing Wiener scheme directly in graph Fourier domain that can be estimated from and applied to the entire image, or can be used patchwise in a locally adaptive manner.

BACKGROUND

-Graph weights are computed in three different ways with two different formulas

$$K_{i,j} = \exp\left[-\left\|\tilde{\mathbf{x}}_\mathbf{i}-\tilde{\mathbf{x}}_\mathbf{j}\right\|^2/h_x^2\right]\exp\left[-\left\|\mathbf{p_i}-\mathbf{p_j}\right\|^2/h_p^2\right]$$

$$K^{\mathbf{MS}}_{i,j} = exp\left[-\left(\left\|\tilde{\mathbf{x}}_\mathbf{i}-\tilde{\mathbf{x}}_\mathbf{j}\right\| + \beta\left\|\mathbf{p_i}-\mathbf{p_j}\right\|\right)^2/\delta^2\right]$$

-Graph Fourier Transform (GFT) of the noisy image and the inverse GFT of the denoised image is computed as in below, respectively.

$$\mathbf{y}_f = V^T\mathbf{y}$$

$$\hat{\mathbf{x}} = V\hat{\mathbf{x}}_f = \sum_{l=0}^{N-1}\hat{x}_f(\lambda_l)\,\mathbf{v_l}$$

-Denoising process is performed by lowpass filtering or hard thresholding in GFT domain.

$$\hat{x}_f(\lambda_l) = \frac{1}{1+\alpha\lambda_l}y_f(\lambda_l)$$

$$\hat{x}_f(\lambda_l) = y_f(\lambda_l)\mathbf{1}(|y_f(\lambda_l)| \geq t)$$

Set Graph Weights (K) → Derive unnormalized Laplacian and perform its eigendecomposition → Compute GFT of the observed noisy image → Perform denoising process by lowpass filtering in GFT domain → Compute the denoised image by the inverse GFT

Flow Chart 1. SGM1 and SGM2 algorithms.

Set Graph Weights (K) → Derive normalized Laplacian and perform its eigendecomposition → Compute the GFT of the observed noisy image → Perform denoising process by hard thresholding in GFT domain → Compute the denoised image $\hat{x}_1$ by the inverse GFT → Set the new graph weights from the cleaner output image of the first step $\hat{x}_1$ → Derive normalized Laplacian and perform its eigendecomposition → Compute the GFT of the observed noisy image → Perform denoising process by hard thresholding in GFT domain → Compute the denoised image $\hat{x}_f$ by the inverse GFT

Flow Chart 2. SGM3 algorithm.

METHOD

-Wiener filtering scheme, integrated to these spectral graph based denoising algorithms, implemented directly in GFT domain without requiring power spectral density or correlation function estimation.

$$\widehat{W}(\lambda_l) = max\left\{0, \frac{\hat{x}_f^2(\lambda_l)-\hat{\sigma}^2}{\hat{x}_f^2(\lambda_l)}\right\}$$

-The proposed Wiener filter can be applied either to the observed, noisy image $\mathbf{y}$ or to the denoised image $\hat{\mathbf{x}}$..

$$\hat{x}_{W,f}(\lambda_l) = \widehat{W}(\lambda_l)y_f(\lambda_l)$$

$$\hat{x}_{W,f}(\lambda_l) = \widehat{W}(\lambda_l)\,\hat{x}_f(\lambda_l)$$

EXPERIMENTAL RESULTS

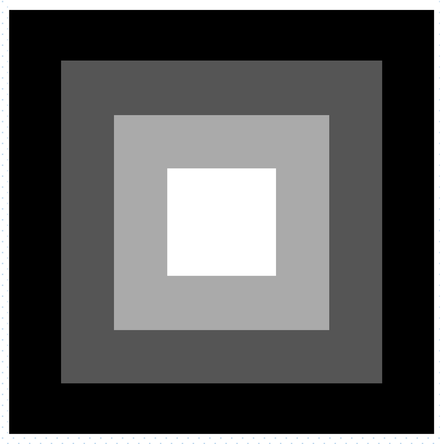

Figure 1. Original box image

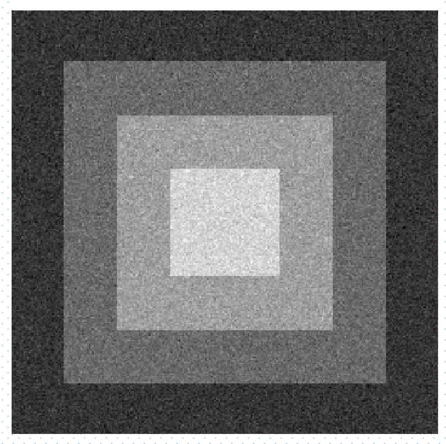

Figure 2. Noisy box image

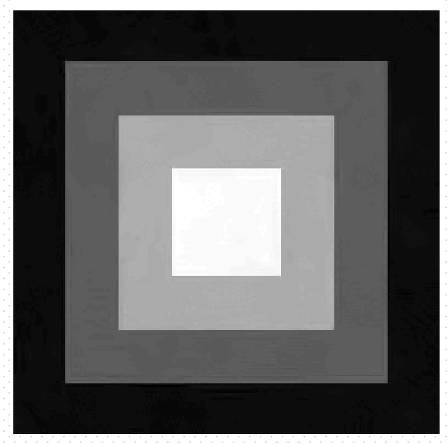

Figure 3. Denoised box image by BM3D

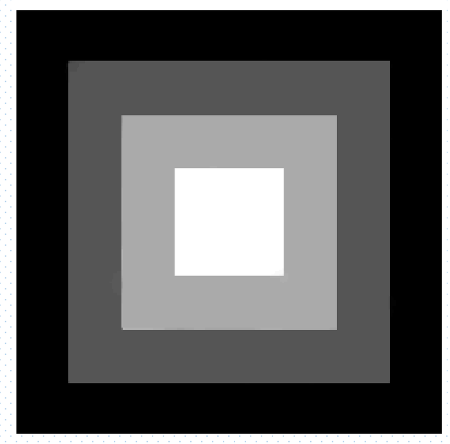

Figure 4. Denoised box image by SGM3W

Figure 5. Original montage image

Figure 6. Noisy montage image

Figure 7. Denoised montage image by BM3D

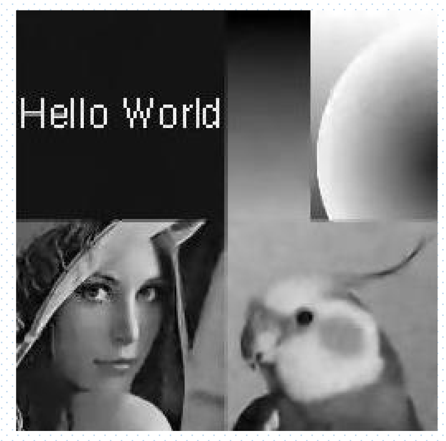

Figure 8. Denoised montage image by SGM1W

	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$
SGM1	65.081	59.749	55.279	53.206	41.214
SGM1W	**67.577**	**62.484**	**58.017**	**54.859**	**43.013**
BM3D	56.141	50.123	46.587	44.102	42.144

Table 1. PSNR values (in dB) for the stripes for SGM1, SGM1W and BM3D algorithms for different noise standard deviation values..

	Box	Stripes	House	Montage	Peppers	Cameraman
SGM1	47.409	55.279	31.744	32.641	30.403	29.897
SGM1W	**53.105**	**58.017**	32.658	33.834	31.232	30.281
SGM2	48.225	45.683	32.099	32.05	30.754	29.626
SGM2W	**53.051**	45.772	32.884	33.564	31.293	30.216
SGM3	57.249	45.733	30.022	29.729	27.193	27.672
SGM3W	**57.269**	45.734	30.152	30.257	27.447	28.324
BM3D	45.859	46.587	34.932	35.097	32.687	31.585

Table 2. PSNR values (in dB) for SGM1, SGM1W, SGM2, SGM2W, SGM3, SGM3W and BM3D algorithms.

CONCLUSION

-This Wiener filter applied as a post-processing step yields consistent accuracy improvement for different choices of graph weights and graph Laplacian matrices (normalized and unnormalized).

-It can be estimated from and applied to the entire image, or can be used patchwise in a locally adaptive manner.

-With this improvement, combined spectral graph methods have attained significantly higher PSNR values than the BM3D methods for box and stripes images, and for upper two subimages of montage image.

ACKNOWLEDGEMENT

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