Robust Energy-Efficient Transmit Design for MISOME Wiretap Channels

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Background

- **Physical-layer (PHY) security**
  - **PHY security can overcome the inherent difficulties of cryptographic methods.**
  - **Most research activities focussed on the issue of secrecy rate maximization.**
  - **It is also an imporatnt problem to characterize the tradeoff between the secrecy performance and the energy consumption.**
- **PHY security and energy efficiency (EE)**
  - **Secrecy EE (SEE): the ratio of the achievable secrecy rate to the total power consumption.**
  - **considered only in a few existing works, with simple system settings (e.g., perfect CSI, one Eve).**
- Main contributions
  - **More general channel setting: MISOME wiretap channel with imperfect CSI on all links**
  - **Problem formulation: maximization of the worst-case SEE (WC-SEE) with secrecy rate constraint**
  - **The optimal covariance matrix is obtained by applying the fractional programming and rank relaxation methods.**
  - **The rank relaxation is proved to be tight.**

System Model

- **A multi-antenna transmitter (Alice) intends to send confidential information to a single-antenna legitimate receiver (Bob), in the presence of K multi-antenna eavesdroppers (Eves).**

$$y_b = \mathbf{h}_b \mathbf{x} + z_b,$$
$$\mathbf{y}_{e,k} = \mathbf{G}_{e,k}\mathbf{x} + \mathbf{z}_{e,k}, \forall k \in \mathcal{K}$$

where $\mathcal{K} \triangleq \{1,2,\ldots,K\}$, $\mathbf{h}_b \in \mathbb{C}^{1\times N_b}$ is the channel response between Alice and Bob, N_t is the number of transmit antennas employed by the transmitter, $\mathbf{G}_{e,k} \in \mathbb{C}^{N_e \times N_b}$ is the channel response between Alice and Eve k, $N_{e,k}$ is the number of transmit antennas employed by Eve, z_b and $\mathbf{z}_{e,k}$ are AWGN, and $\mathbf{x}$ is the coded confidential information following $\mathbf{x} \sim CN(0,\mathbf{Q}_c)$.

- **Deterministically bounded CSI error model**

$$\mathbf{h}_b = \tilde{\mathbf{h}}_b + \Delta\mathbf{h}_b, \|\Delta\mathbf{h}_b\|_F \le \varepsilon_b$$
$$\mathbf{G}_{e,k} = \tilde{\mathbf{G}}_{e,k} + \Delta\mathbf{G}_{e,k}, \|\Delta\mathbf{G}_{e,k}\|_F \le \varepsilon_{e,k}, \forall k \in \mathcal{K},$$

- **The worst-case secrecy rate**

$$R_s^{\text{worst}}(\mathbf{Q}_c) = \min_{\mathbf{h}_b \in B_b} \log(1+\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H) - \max_{k=1,\ldots,K} \max_{\mathbf{G}_{e,k}\in B_{e,k}} \log\det(\mathbf{I}+\mathbf{G}_{e,k}\mathbf{Q}_c\mathbf{G}_{e,k}^H)$$

where B_b and $B_{e,k}$ are the sets of all admissible CSI associated with $\mathbf{h}_b$ and $\mathbf{G}_{e,k}$, respectively.

- **Power consumption model [Xu et al. ’13]**

$$P_t = \mathrm{Tr}(\mathbf{Q}_c) + P_c$$

where P_c is a constant transmit independent power.

- **Problem Formulation**

Our work focuses on the design of $\mathbf{Q}_c$, to maximize the WC-SEE with QoS and total power constraints.

$$\max_{\mathbf{Q}_c} \frac{R_s^{\text{worst}}(\mathbf{Q}_c)}{\mathrm{Tr}(\mathbf{Q}_c)+P_c} \quad \text{s.t. } R_s^{\text{worst}}(\mathbf{Q}_c) \ge \tau_s, \mathrm{Tr}(\mathbf{Q}_c) \le P, \mathbf{Q}_c \succeq \mathbf{0}. \qquad (1)$$

where τ_s is preset requirement of the worst-case secrecy rate.

Problem Solving

- **A Dinkelbach Method-based Reformulation**

We first define a parametric problem with respect to λ as follow.

$$F(\lambda) = \max_{\mathbf{Q}_c \in \mathcal{F}} R_s^{\text{worst}}(\mathbf{Q}_c) - \lambda(\mathrm{Tr}(\mathbf{Q}_c)+P_c) \qquad (2)$$

where $\mathcal{F}$ is the feasible set of problem (1).

Lemma 1 $F(\lambda)$ is a strictly decreasing and continuous function w.r.t. λ, and it has a unique zero solution.
Lemma 2 Assume that λ^* is the unique zero solution to F(λ), then F(λ*) and (1) have the same optimal solution, and the optimal objective function value of (1) is λ*.

Our strategy is to optimize (2) and obtain $F(\lambda)$ with a given λ, and employ the ***Dinkelbach method*** to seek the optimal λ.

Introducing a slack variable β, one can check that (2) is equivalent to the following problem (3).

$$F(\lambda) = \max_{\mathbf{Q}_c,\beta} \left\{\log(\beta^{-1}+\beta^{-1}\min_{\mathbf{h}_b\in B_b}\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H) - \lambda(\mathrm{Tr}(\mathbf{Q}_c)+P_c)\right\}$$
$$\text{s.t. } \log\det(\mathbf{I}+\mathbf{G}_{e,k}\mathbf{Q}_c\mathbf{G}_{e,k}^H) \le \log\beta, \forall k \in \mathcal{K}, \mathbf{G}_{e,k}\in B_{e,k}$$
$$\log(1+\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H) - \log\beta \ge \tau_{cs}, \forall \mathbf{h}_b \in B_b,$$
$$\mathrm{Tr}(\mathbf{Q}_c) \le P, \mathbf{Q}_c \succeq \mathbf{0} \qquad (3)$$

- **A tight rank relaxation**

Lemma 3 The following inequality holds, det(**I** + **A**) ≥ 1 + Tr(**A**) for any PSD matrix **A**. Moreover, the equality holds if and only if rank(**A**) ≤ 1.

By applying Lemma 3, problem (3) can be relaxed as

$$F_{relax}(\lambda) = \max_{\mathbf{Q}_c,\beta} \left\{\log(\beta^{-1}+\beta^{-1}\min_{\mathbf{h}_b\in B_b}\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H) - \lambda(\mathrm{Tr}(\mathbf{Q}_c)+P_c)\right\}$$
$$\text{s.t. } 1+\mathrm{Tr}(\mathbf{G}_{e,k}\mathbf{Q}_c\mathbf{G}_{e,k}^H) \le \beta, \forall k \in \mathcal{K}, \mathbf{G}_{e,k}\in B_{e,k}$$
$$\log(1+\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H) - \log\beta \ge \tau_{cs}, \forall \mathbf{h}_b \in B_b,$$
$$\mathrm{Tr}(\mathbf{Q}_c) \le P, \mathbf{Q}_c \succeq \mathbf{0} \qquad (4)$$

with $F_{\text{relax}}(\lambda) \ge F(\lambda)$.

slack variable η introduced

- **A two-stage reformulation of (4)**

Outer problem

$$F_{\text{relax}}(\lambda) = \max_{\eta} \{\log\gamma(\lambda,\eta) - \eta\}$$
$$\text{s.t. } \lambda(\eta_{\min}+P_c) \le \eta \le \lambda(P+P_c), \qquad (5)$$

The inner problem

$$\gamma(\lambda,\eta)=\max_{\mathbf{Q}_c,\beta}\ \beta^{-1}(1+\min_{\mathbf{h}_b\in B_b}\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H)$$
$$\text{s.t.}\quad \lambda(\mathrm{Tr}(\mathbf{Q}_c)+P_c)\le\eta, \tag{6}$$
$$\text{constraints in (4) satisfied.}$$

❑SDP-based Reformulation of the Inner Problem

Step 1: variable transformation

Introduce the transformation $\alpha=1/\beta,\mathbf{Z}=\mathbf{Q}_c/\beta$ to rewrite (6) as

$$\gamma(\lambda,\eta)=\max_{\mathbf{Z},\alpha}\min_{\mathbf{h}_b\in B_b}\alpha+\mathbf{h}_b\mathbf{Z}\mathbf{h}_b^H$$
$$\text{s.t.}\quad \lambda(\mathrm{Tr}(\mathbf{Z})+\alpha P_c)\le\alpha\eta,$$
$$\alpha+\mathrm{Tr}(\mathbf{G}_{e,k}\mathbf{Z}\mathbf{G}_{e,k}^H)\le 1,\forall k\in\mathcal{K},\mathbf{G}_{e,k}\in B_{e,k} \tag{7}$$
$$\alpha+\mathbf{h}_b\mathbf{Z}\mathbf{h}_b^H\ge 2^{\tau_{cs}},\forall\mathbf{h}_b\in B_b,$$
$$\mathrm{Tr}(\mathbf{Z})\le P\alpha,\mathbf{Z}\succeq\mathbf{0}.$$

Step 2: S-procedure

$$\alpha+\mathrm{Tr}(\mathbf{G}_{e,k}\mathbf{Z}\mathbf{G}_{e,k}^H)\le 1,\forall k\in\mathcal{K},\mathbf{G}_{e,k}\in B_{e,k}\Leftrightarrow$$

$$\mathbf{X}_k(\rho_k,\alpha,\mathbf{Z})=$$
$$\begin{bmatrix}\rho_k\mathbf{I}_{N_tN_{e,k}}-(\mathbf{Z}^T\otimes\mathbf{I}_{N_{e,k}}) & -(\mathbf{Z}^T\otimes\mathbf{I}_{N_{e,k}})\tilde{\mathbf{g}}_{e,k}\\ -\tilde{\mathbf{g}}_{e,k}^H(\mathbf{Z}^T\otimes\mathbf{I}_{N_{e,k}})^H & -\rho_k\varepsilon_{e,k}^2-\tilde{\mathbf{g}}_{e,k}(\mathbf{Z}^T\otimes\mathbf{I}_{N_{e,k}})\tilde{\mathbf{g}}_{e,k}^H+1-\alpha\end{bmatrix}\succeq\mathbf{0}$$

in which $\tilde{\mathbf{g}}_{e,k}=\mathrm{vec}(\widetilde{\mathbf{G}}_{e,k})$, please refer to our paper for details.

$$\alpha+\mathbf{h}_b\mathbf{Z}\mathbf{h}_b^H\ge 2^{\tau_{cs}},\forall\mathbf{h}_b\in B_b,\Leftrightarrow$$
$$\mathbf{U}(t,\alpha,\mu,\mathbf{Z})=\begin{bmatrix}t\mathbf{I}+\mathbf{Z} & \mathbf{Z}\tilde{\mathbf{h}}_b^H\\ \tilde{\mathbf{h}}_b\mathbf{Z} & -\mu\varepsilon_b^2+\tilde{\mathbf{h}}_b\mathbf{Z}\tilde{\mathbf{h}}_b^H-v+\alpha\end{bmatrix}\succeq\mathbf{0}$$

in which $v\triangleq\max\{2^{\tau_{cs}},\mu\}$.

Then problem (7) can be transformed into an SDP problem given in (8), which can be efficiently solved via CVX. The outer problem can be handled via a one-dimensional search.

$$\gamma(\lambda,\eta)=\max_{\mathbf{Z},\alpha,\mu,t,\{\rho_k\}_{k\in K}}\ \mu$$
$$\text{s.t.}\quad \lambda(\mathrm{Tr}(\mathbf{Z})+\alpha P_c)\le\alpha\eta,$$
$$\mathbf{X}_k(\rho_k,\alpha,\mathbf{Z})\succeq\mathbf{0},\rho_k\ge 0,\forall k\in\mathcal{K} \tag{8}$$
$$\mathbf{U}(t,\alpha,\mu,\mathbf{Z})\succeq\mathbf{0},t\ge 0,$$
$$\mathrm{Tr}(\mathbf{Z})\le P\alpha,\mathbf{Z}\succeq\mathbf{0},$$

❑Overall Algorithm

Algorithm 1 Dinkelbach Algorithm for Solving the WC-SEE Maximization Problem (3)

1: Initiate $n=0$, $\epsilon>0$ and λ_n such that $F(\lambda_n)\ge 0$.
2: **Repeat**
3: Perform a one-dimensional line search over η to get $(\eta^*,\gamma(\lambda_n,\eta^*))$, wherein each $\gamma(\lambda_n,\eta)$ is returned by solving the problem (8) ;
4: Retrieve the corresponding $\mathbf{Q}_c^*$ via the variable transformation;
5: Compute $F(\lambda_n)=R_s^{\text{worst}}(\mathbf{Q}_c^*)-\lambda_n(\mathrm{Tr}(\mathbf{Q}_c^*)+P_c)$;
6: Update $\lambda_{n+1}=\frac{R_s^{\text{worst}}(\mathbf{Q}_c^*)}{\mathrm{Tr}(\mathbf{Q}_c^*)+P_c}$;
7: Update $n=n+1$;
8: **until** $|F(\lambda_{n-1})|\le\epsilon$
9: Output λ_n as the maximum WC-SEE.

Dinkelbach method

❑Tightness proof of the relaxation

Proposition 1 There exist an optimal solution ($\mathbf{Q}_c^*$,η^*) of problem (4), for which rank($\mathbf{Q}_c^*$)≤1, and the optimal $\mathbf{Q}_c^*$ can be constructed by solving a power minimization problem (9).

Sketch of proof: Suppose that we have solved (6) with the optimal value E_η. Then, we study the power minimization problem below.

$$\min_{\mathbf{Q}_c\succeq\mathbf{0}}\ \mathrm{Tr}(\mathbf{Q}_c)$$
$$\text{s.t.}\quad \min_{\mathbf{h}_b\in B_b}1+\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H\ge\beta E_\eta,$$
$$\lambda(\mathrm{Tr}(\mathbf{Q}_c)+P_c)\le\eta, \tag{9}$$
$$1+\mathrm{Tr}(\mathbf{G}_{e,k}\mathbf{Q}_c\mathbf{G}_{e,k}^H)\le\beta,\forall k\in\mathcal{K},\mathbf{G}_{e,k}\in B_{e,k}$$
$$\log(1+\mathbf{h}_b\mathbf{Q}_c\mathbf{h}_b^H)-\log\beta\ge\tau_{cs},\forall\mathbf{h}_b\in B_b.$$

Problem (9) can be converted into an SDP problem via *S*-procedure. The remaining proof consists of two steps:
Step 1: It can be proved, by contradiction, that the optimal solution of (9) is bound to be optimal to (6). Hence, it suffices to prove that the optimal $\mathbf{Q}_c$ of (9) is of rank one.
Step 2: By checking the Karush-Kuhn-Tucker (KKT) conditions of (9), one can verify that the optimal $\mathbf{Q}_c$ of (9) is of rank one. The details are omitted here. Similar proof can be found in [Li et al.'13].

Numerical Results

❑ Benchmark scheme (worst-case secrecy rate maximization, WC-SRM)

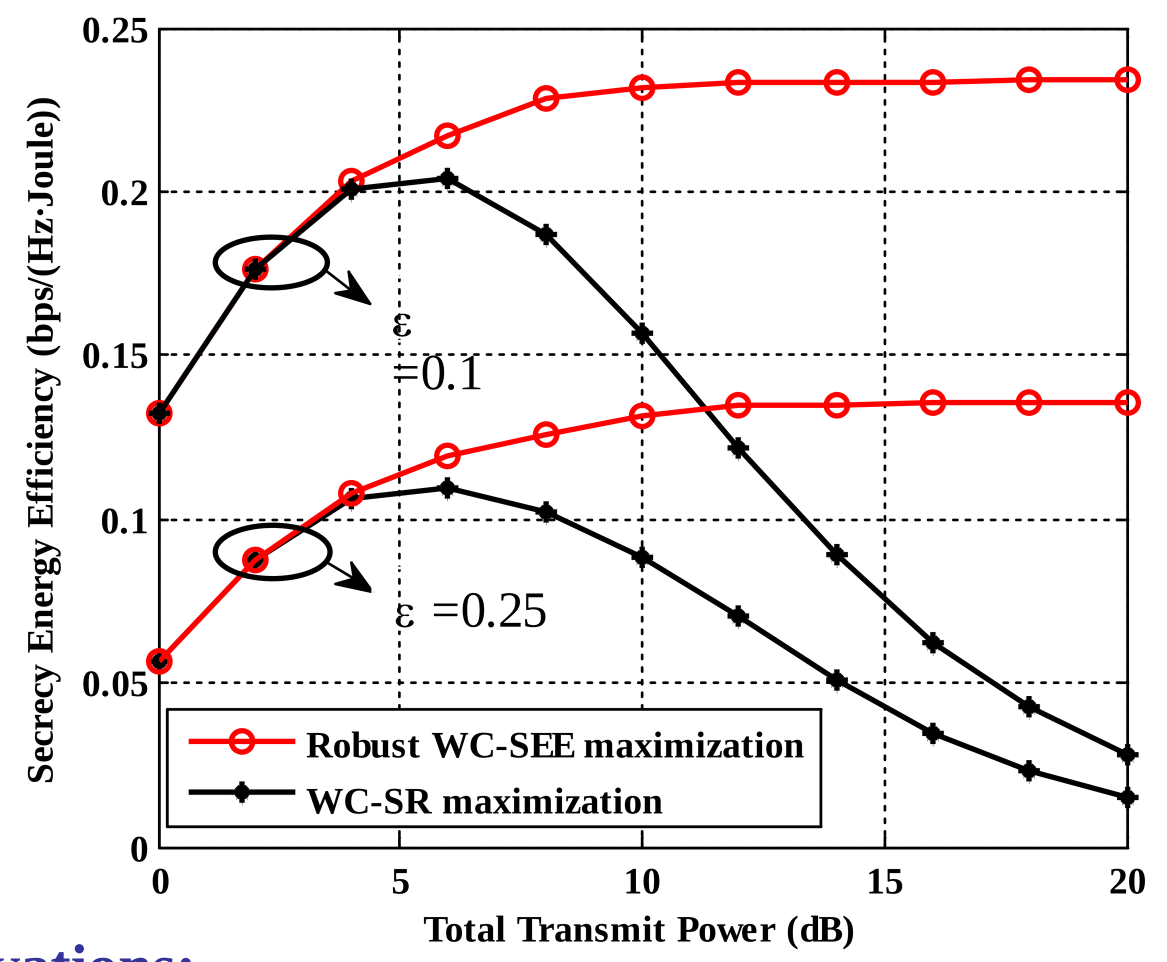
$$\hat{\mathbf{Q}}_c=\arg\max_{\mathbf{Q}_c\succeq\mathbf{0},\mathrm{Tr}(\mathbf{Q}_c)\le P}R_s^{\text{worst}}(\mathbf{Q}_c)$$
$$\text{s.t.}\quad R_s^{\text{worst}}(\mathbf{Q}_c)\ge\tau_s. \tag{10}$$

the resultant SEE

$$SEE_{\text{SRM}}=\frac{R_s^{\text{worst}}(\hat{\mathbf{Q}}_c)}{\mathrm{Tr}(\hat{\mathbf{Q}}_c)+P_c} \tag{11}$$

❑Simulation setting

- Transmit antenna#: N_t=6
- Eves#: K=2
- Eves' antennas#: $N_{e,k}$=3 for all k
- Transmit independent power P_c=7dB
- Required secrecy rate τ_s=1.5bps/Hz
- Channel uncertainty $\varepsilon_b=\varepsilon_{e,k}=\varepsilon$
- Average of 100 channel trials



Observations:
✓When P≤4dB, both schemes increase with the transmit power and achieve identical SEE performance.
✓After that, the performance achieved by the WC-SRM scheme degrades significantly, since it has used up all power budget.

Concluding Remarks

❑ The input transmit covariance was optimized to maximize the WC-SEE with constrained QoS.

❑ By resorting to the fractional programming theory and introducing a tight convex relaxation, we manage to recast the primal fractional optimization problem as a sequence of SDP problems.

❑ We proved that our obtained method can admit a rank-one solution, which guarantees the tightness of the convex relaxation.

❑Numerical results showed that our proposed WC-SEE maximization optimal strategy achieves SEE no less than that achieved by the WC-SR maximization optimal strategy.