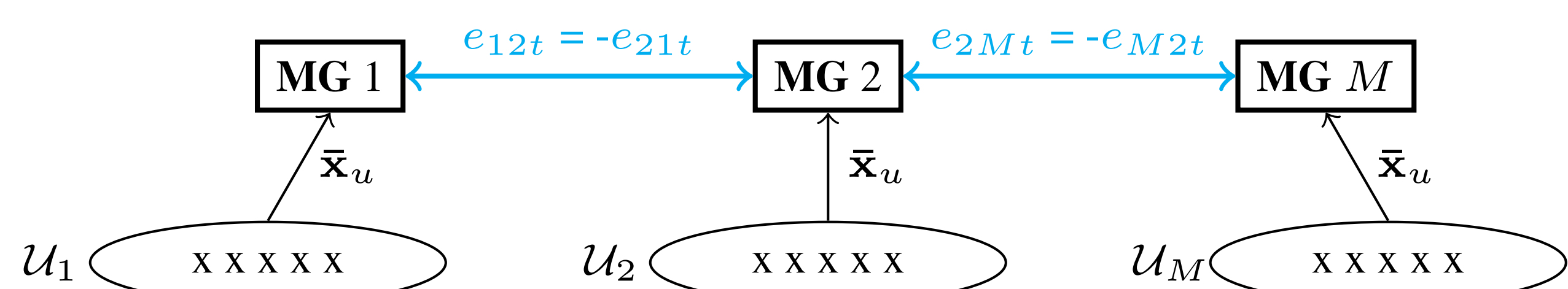


Motivation

- The smart grid will be composed of smaller grids, known as MicroGrids (MGs), in which energy is locally generated and consumed.
 - MGs can operate in islanded mode, i.e., without connection to the main grid.
 - MGs improve reliability in power delivery and efficiency of energy usage.
- Demand Response (DR) programs are another alternative for reducing costs at the MG.
 - Low energy consumers have been traditionally ignored in DR programs due to their reduced impact.
 - DR Aggregators (AGs) have recently appeared as new market agents, capable of controlling and managing compounds of small energy consumers, granting omnipresent access to DR programs.

Energy trading without demand response



Problem formulation:

Each MG designs its energy generation and trading strategy, $\mathbf{y}_m = [\mathbf{g}_m, \mathbf{e}_m]$, to maximize its own benefit, i.e., the difference between incomes and costs (generation + energy transfer costs):

$$q_m \left(\sum_{u \in \mathcal{U}_m} \mathbf{1}_{LT}^T \bar{\mathbf{x}}_u \right) - \underbrace{\left(\mathbf{1}_T^T \mathbf{c}_m(\mathbf{g}_m) + \mathbf{1}_{T|\mathcal{M}_m}^T \gamma_m(\mathbf{e}_m) \right)}_{f_m(\mathbf{y}_m)}. \quad (1)$$

Design constraints:

- The energy generated by the m -th MG must satisfy $\mathbf{g}_m \in \mathcal{G}_m \triangleq \{\mathbf{g}_m \in \mathbb{R}^T : g_{mt} \in [0, \hat{G}_m], \forall t\}$.
- The traded energy must satisfy $\mathbf{e}_m \in \mathcal{E}_m \triangleq \{\mathbf{e}_m \in \mathbb{R}^{T|\mathcal{M}_m}| : e_{mm't} \in [\hat{E}_{mm't}^-, \hat{E}_{mm't}^+], \forall t\}$, where $\hat{E}_{mm't}^- \leq 0$ and $\hat{E}_{mm't}^+ \geq 0$ limit the maximum energy sold and bought by MG m to/from MG m' , respectively.
- MG pairs $(m, m') \in \mathcal{M}$ must reach consensus on the traded energy, i.e.,

$$e_{mm't} = -e_{m'mt}, \quad \forall (m, m') \in \mathcal{M}, \forall t \in \mathcal{T}.$$
- Load balancing: $g_{mt} = \sum_{u \in \mathcal{U}_m} \mathbf{1}_L^T \bar{\mathbf{x}}_u - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall m, t$.

M-player Generalized Nash Equilibrium Problem (GNEP):

The m -th MG revenue maximization problem can be written as follows:

$$\min_{\mathbf{y}_m \in \mathcal{Y}_m(\mathbf{e}_{-m})} f_m(\mathbf{y}_m), \quad (2)$$

where the feasible set of MG m , \mathcal{Y}_m , is coupled with the strategy of the other MGs, $\mathbf{e}_{-m} \triangleq (\mathbf{e}_{m'})_{m' \in \mathcal{M}_m}$:

$$\mathcal{Y}_m(\mathbf{e}_{-m}) \triangleq \left\{ \mathbf{g}_m \in \mathcal{G}_m, \mathbf{e}_m \in \mathcal{E}_m : g_{mt} = \sum_{u \in \mathcal{U}_m} \mathbf{1}_L^T \bar{\mathbf{x}}_u - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall t, e_{mm't} + e_{m'mt} = 0, \forall m' \in \mathcal{M}_m, \forall t \right\}.$$

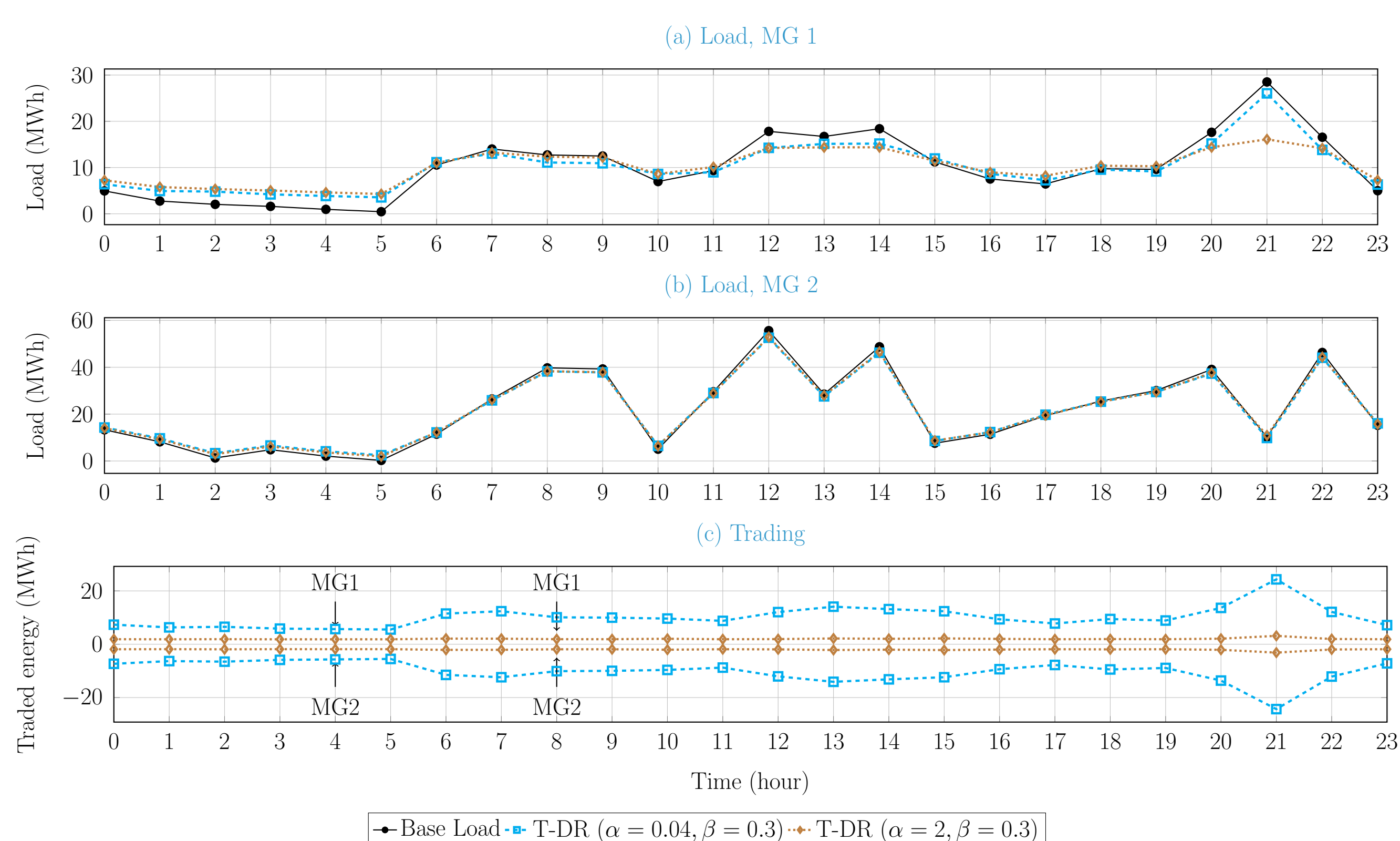
Variational solutions to the GNEP:

Proposition 1. The variational solutions of the GNEP defined by (2), $\forall m = 1, \dots, M$, are solutions of the following Network Utility Maximization (NUM) problem:

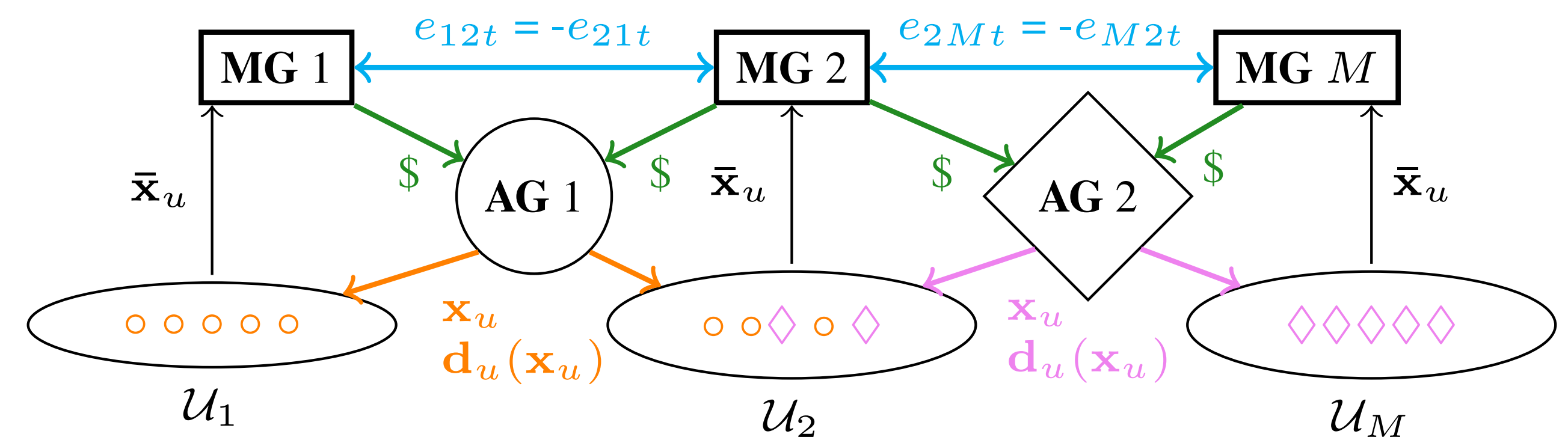
$$\min_{\mathbf{y} \in \mathcal{Y}} \sum_{m=1}^M f_m(\mathbf{y}_m), \quad (3)$$

where \mathbf{y} contains the strategy of the different MGs, $\mathbf{y} \triangleq [\mathbf{g}, \mathbf{e}]$ with $\mathbf{g} = (\mathbf{g}_m)_{m=1}^M$ and $\mathbf{e} = (\mathbf{e}_m)_{m=1}^M$, and where its associated feasible set is $\mathcal{Y} \triangleq \left\{ \mathbf{e} \in \prod_{i=1}^M \mathcal{E}_i, \mathbf{g} \in \prod_{i=1}^M \mathcal{G}_i : g_{mt} = \sum_{u \in \mathcal{U}_m} \mathbf{1}_L^T \bar{\mathbf{x}}_u - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall m, t, e_{mm't} + e_{m'mt} = 0, \forall (m, m') \in \mathcal{M}, \forall t \right\}$. Additionally, the converse implication holds true as well.

- The variational solutions of the GNEP are convenient equilibrium points as they achieve the same performance than a cooperative strategy aimed at minimizing the total cost.
- We obtain the variational solutions of the GNEP by solving (3) in a distributed way by means of dual decomposition.



Energy trading with DR aggregators



Problem formulation:

- AGs can shift user loads from $\bar{\mathbf{x}}_u$ to \mathbf{x}_u to reduce MGs' costs.
- m -th MG offers a fraction $\Gamma_m \in [0, 1]$ of its savings to the AGs. In particular, AG a receives $\Gamma_{am} \Delta f_m(\mathbf{y}_m)$.

Design constraints:

- Design constraints 1, 2, and, 3 in the left column.
- Load balancing: $g_{mt} = \left(\sum_{a=1}^A \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_L^T \mathbf{x}_{ut} \right) - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall m, t$.
- All requested loads must be scheduled within the time horizon \mathcal{T} , i.e.,

$$\mathcal{X}_u \triangleq \left\{ \mathbf{x}_u \in \mathbb{R}_+^{LT} : \sum_{t \in \mathcal{T}} x_{lut} = \sum_{t \in \mathcal{T}} \bar{x}_{lut}, \forall l \right\}.$$

(M + A)-player GNEP:

- The m -th MG problem reads

$$\min_{\mathbf{y}_m} f_m(\mathbf{y}_m)(1 - \Gamma_m) \quad (4a)$$

$$\text{s. t. } \mathbf{y}_m \in \mathcal{Y}_m(\mathbf{e}_{-m}, (\bar{\mathbf{x}}_a)_{a=1}^A), \quad (4b)$$

where $\mathcal{Y}_m(\mathbf{e}_{-m}, (\bar{\mathbf{x}}_a)_{a=1}^A) = \left\{ \mathbf{g}_m \in \mathcal{G}_m, \mathbf{e}_m \in \mathcal{E}_m : e_{mm't} + e_{m'mt} = 0, \forall m' \in \mathcal{M}_m, \forall t, g_{mt} = \left(\sum_{a=1}^A \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_L^T \mathbf{x}_{ut} \right) - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall t \right\}$.

- The problem of the a -th AG problem, whose actions are $\bar{\mathbf{x}}_a \triangleq (\mathbf{x}_u)_{u \in \cup_{m=1}^M \mathcal{U}_{am}}$, is:

$$\min_{\bar{\mathbf{x}}_a} \sum_{m=1}^M \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_{LT}^T \mathbf{d}_u(\mathbf{x}_u) \quad (5a)$$

$$\text{s. t. } \bar{\mathbf{x}}_a \in \mathcal{Y}_a(\mathbf{e}, \bar{\mathbf{x}}_{-a}), \quad \text{where} \quad (5b)$$

$$\mathcal{Y}_a(\mathbf{y}, \bar{\mathbf{x}}_{-a}) = \left\{ \mathbf{x}_u \in \mathcal{X}_u, u \in \cup_{m=1}^M \mathcal{U}_{am} : g_{mt} = \left(\sum_{a=1}^A \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_L^T \mathbf{x}_{ut} \right) - \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt}, \forall m, t \right\}.$$

Variational solutions:

Proposition 2. The variational solutions of the GNEP in (4) and (5) are equivalent to the solutions of the following optimization problem

$$\min_{\mathbf{z} \in \mathcal{Z}} \sum_{m=1}^M f_m(\mathbf{y}_m)(1 - \Gamma_m) + \sum_{a=1}^A \sum_{m=1}^M \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_{LT}^T \mathbf{d}_u(\mathbf{x}_u) \quad (6a)$$

$$\text{s. t. } e_{mm't} + e_{m'mt} = 0, \quad \forall (m, m') \in \mathcal{M}, \forall t \quad (6b)$$

$$g_{mt} + \mathbf{1}_{|\mathcal{M}_m|}^T \mathbf{e}_{mt} - \sum_{a=1}^A \sum_{u \in \mathcal{U}_{am}} \mathbf{1}_L^T \mathbf{x}_{ut} = 0, \forall m, t. \quad (6c)$$

where \mathbf{z} stacks the vectors $\mathbf{y}_m, \forall m$, and $\bar{\mathbf{x}}_a, \forall a$, with feasible set

$$\mathcal{Z} \triangleq \left\{ \mathbf{z} : \mathbf{g}_m \in \mathcal{G}_m, \mathbf{e}_m \in \mathcal{E}_m, \mathbf{x}_u \in \mathcal{X}_u, \forall m, u \right\}.$$

Simulation Setup

- $M = 2$ MGs and one AG, slot duration is set to one hour.
- Oil generators with: $\mathcal{G}_1 = [0, 12]$ MWh and $\mathcal{G}_2 = [0, 100]$ MWh, $c_{1t}(x) = 86.39 + 56.56x + 0.33x^2$ and $c_{2t}(x) = 781.52 + 43.66x + 0.05x^2$.
- $\mathcal{E}_m = [-100, 100]$ MWh.
- $q_m = 200$ \$/MWh.
- $\gamma_{mm't}(x) = \alpha x^2, \forall m, m', t$, where α is a constant in \$/MWh².
- $d_{lut}(x_{lut}) = \beta(x_{lut} - \bar{x}_{lut})^2, \forall l, u, t$, where β is a constant in \$/(MWh)².

