Distributed Cooperative Charging for Plug-in Electric Vehicles:

A Consensus+Innovations Approach

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Outline

- **Motivation and Background**
- Approach and Methodology
- Problem Formulation
  - Plug-in Electric Vehicles' Cooperative Charging (PEV-CC)
- Performance analysis
- Summary

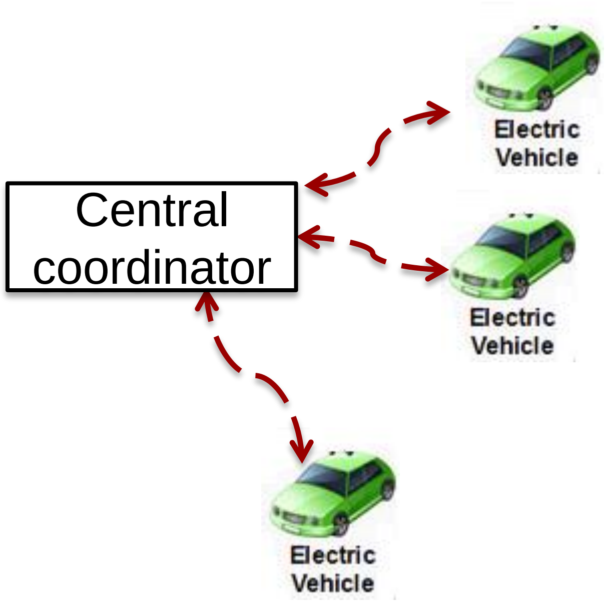
Motivation

- Towards a smarter grid
  - Abundance of local computation/communication
  - Emergence of affordable small scale energy resources
  - Increased integration of power electronics

Source: http://solutions.3m.com/wps/portal/3M/en_EU/SmartGrid/EU-Smart-Grid/

Motivation

- Traditional/Centralized
  - Requires complete set of information



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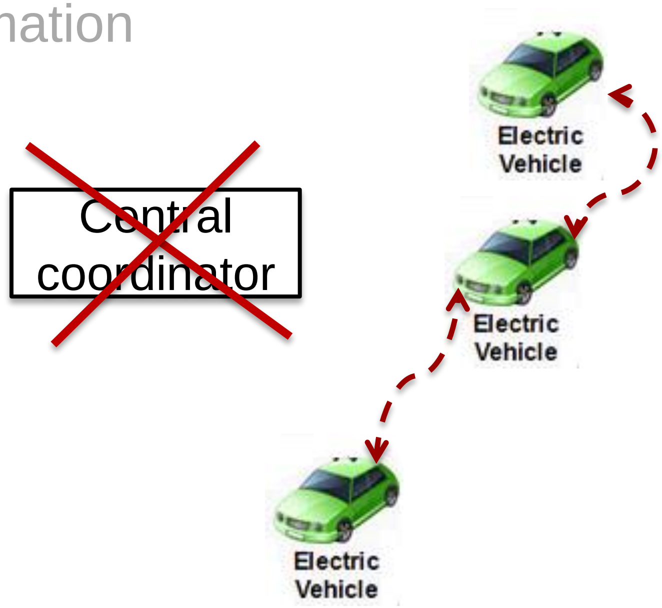

Distributed approach

- Distributed control approaches:
  - No need for central coordinator
  - Distributes the computation among entities.
  - Each entity exchanges limited information with a few other entities

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- Advantages:
  - Suitable for smart grid
    - No need for complete set of information
    - A promising solution approach for coordinating geographically scattered resources like PEVs.
    - Preserves data confidentiality

Existing work

- Non-cooperative agents:
  - Game theory approach
- Cooperative agents:
  - Decomposition based methods, e.g., ADMM
    - Requires information exchange with a central entity
  - Consensus based methods:
    - Enforcing agreement on a common variable
    - Requires a leader node to have access to total load information

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Our assumption: Cooperative agents

Our approach: Fully distributed consensus-based approach

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- Optimal dispatch problem

$$\begin{aligned} \min_{\mathbf{Y}} \quad & \mathbf{F}(\mathbf{Y}) \\ \text{s.t.} \quad & \mathbf{M}(\mathbf{Y}) = 0 \\ & \mathbf{N}(\mathbf{Y}) \leq 0 \end{aligned}$$

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- Lagrangian Function

$$L = \mathbf{F}(\mathbf{Y}) + \lambda^T \mathbf{M}(\mathbf{Y}) + \mu^T \cdot \mathbf{N}(\mathbf{Y})$$

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Dual variables

  - Optimality conditions

$$\frac{\partial L}{\partial x} = g(x) \qquad \text{and Complementary Slackness Condition}$$

Vector of primal and dual variables

$$\text{where} \quad x^T = [\mathbf{Y}\ , \lambda\ , \mu]^T$$

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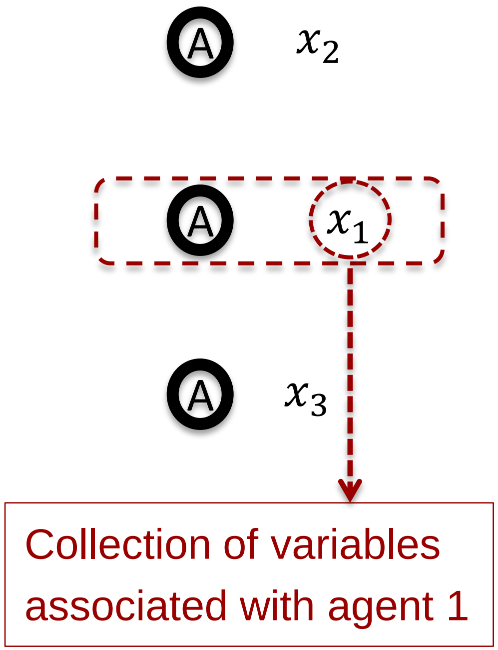

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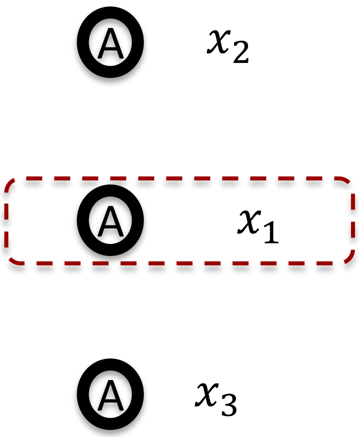

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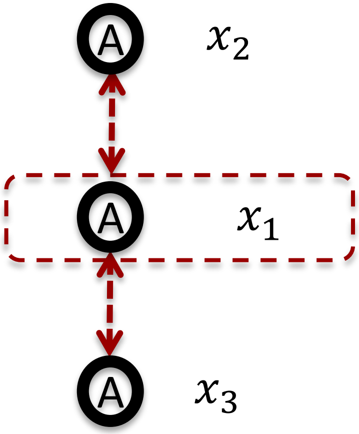

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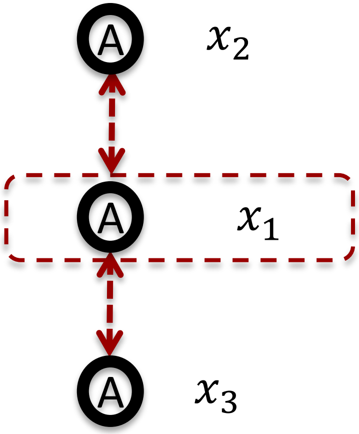

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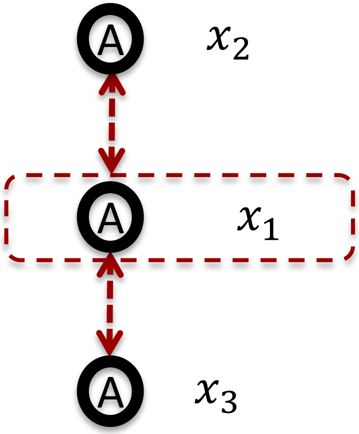

- Update local variables

$$x_i(k+1) = x_i(k) + \mathrm{C} \sum_{j \in \Omega_i} x_i(k) - x_j(k) + \Phi^T \cdot g_i\big(x_j(k)\big)$$

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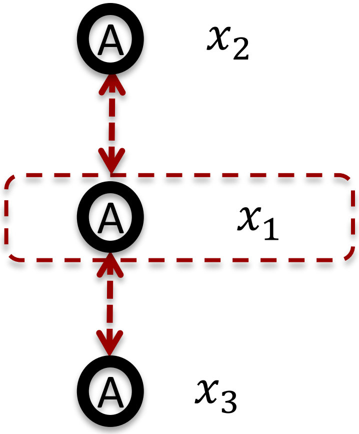

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Consensus term

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Energy constraint ⇠ $AX_v \le b_v$

PEV-CC formulation

$$\text{minimize}_{\mathbf{x}_v,\mathbf{L}} \quad \mathbf{L}^\top \cdot c_1 \cdot \mathbf{L} + c_2^\top \cdot \mathbf{L}$$

s.t.

$$\mathbf{L} = \sum_{v \in V} \mathbf{x}_v$$

$$A \cdot \mathbf{x}_v \leq b_v, \quad \forall v \in \{1, \cdots, V\}$$
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μ_v

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→ Local Constraints

Optimality conditions

$$
\begin{aligned}
\frac{\partial \mathfrak{L}}{\partial \mathbf{L}} &= 2c_1 \cdot \mathbf{L} + c_2 - \lambda = 0 \\
\frac{\partial \mathfrak{L}}{\partial \mathbf{x}_v} &= \lambda + A^\top \cdot \mu_v + \mu_{v,+} - \mu_{v,-} = 0 \\
\frac{\partial \mathfrak{L}}{\partial \lambda} &= -\mathbf{L} + \sum_{v \in V} \mathbf{x}_v = 0 \\
\frac{\partial \mathfrak{L}}{\partial \mu_v} &= A \cdot \mathbf{x}_v - b_v \leq 0 \\
\frac{\partial \mathfrak{L}}{\partial \mu_{v,+}} &= \mathbf{x}_v - \overline{x}_v \leq 0 \\
\frac{\partial \mathfrak{L}}{\partial \mu_{v,-}} &= -\mathbf{x}_v + \underline{x}_v \leq 0
\end{aligned}
$$

These equations involve both local and global information

Distributed variables

$\mathbf{x}_v$ $\mathbf{L}$ λ μ_v

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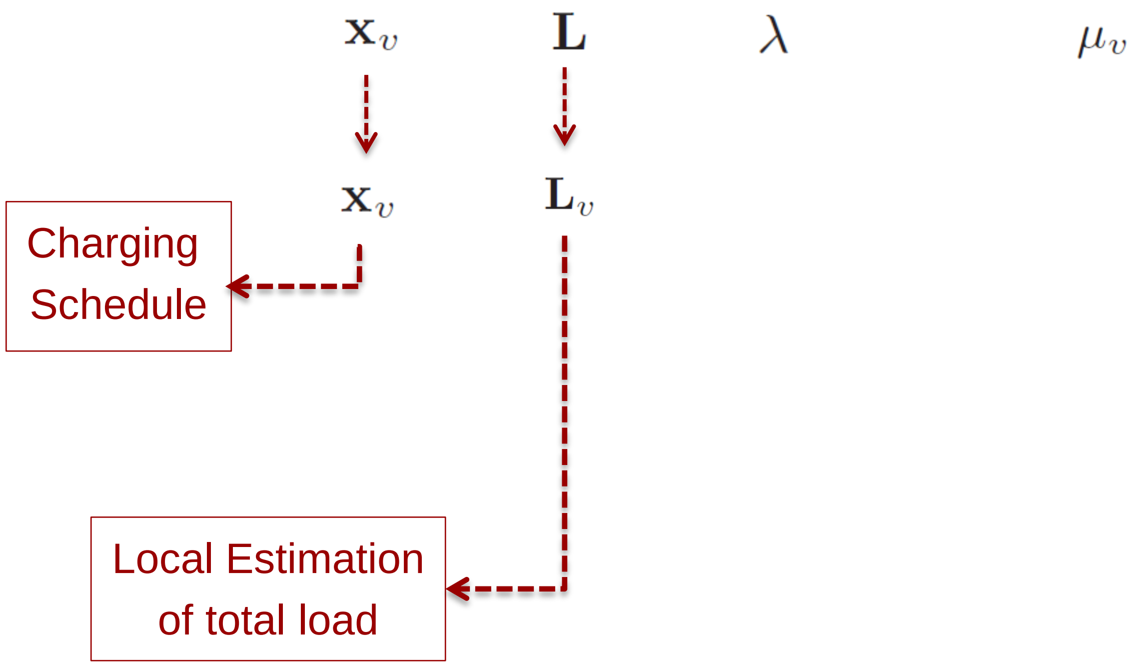
$\mathbf{x}_v$

Charging Schedule

Distributed variables



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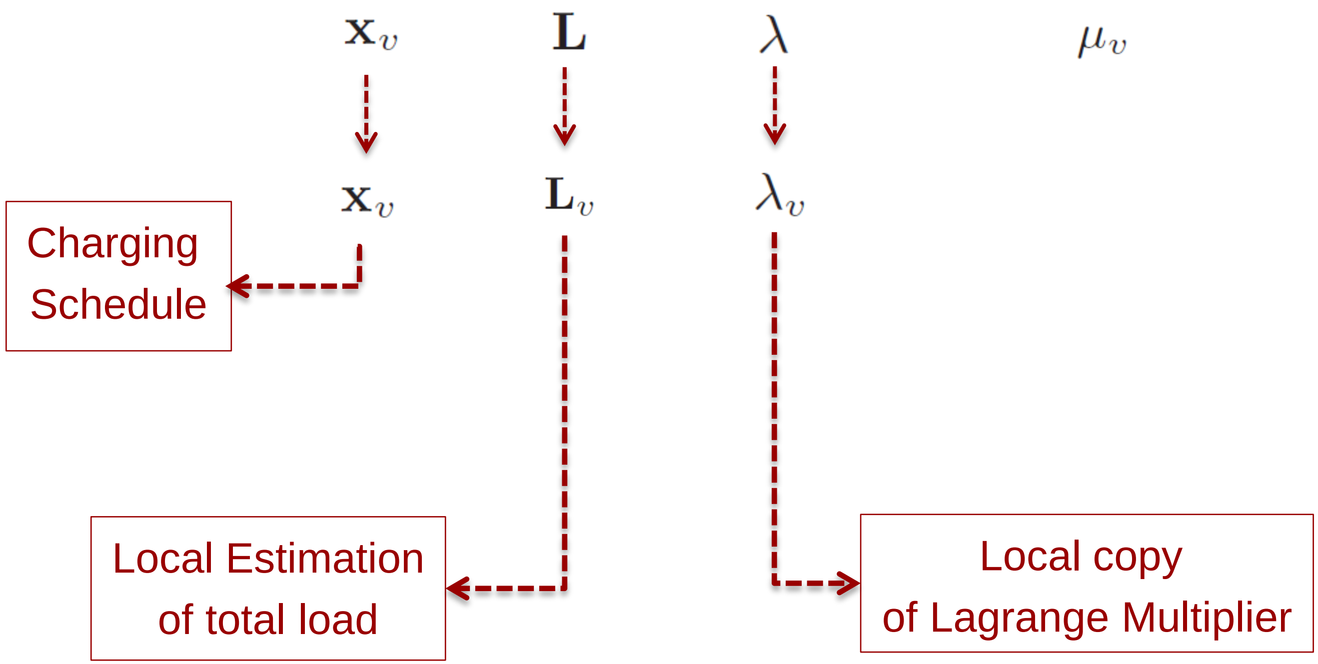

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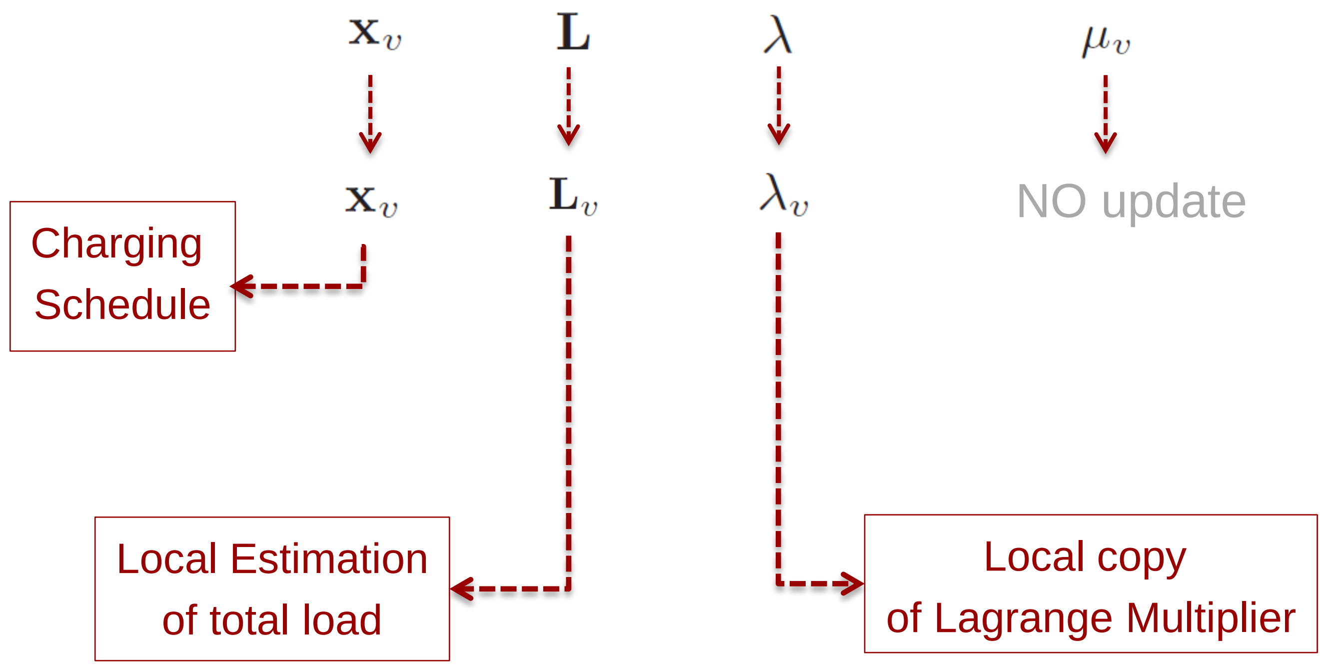

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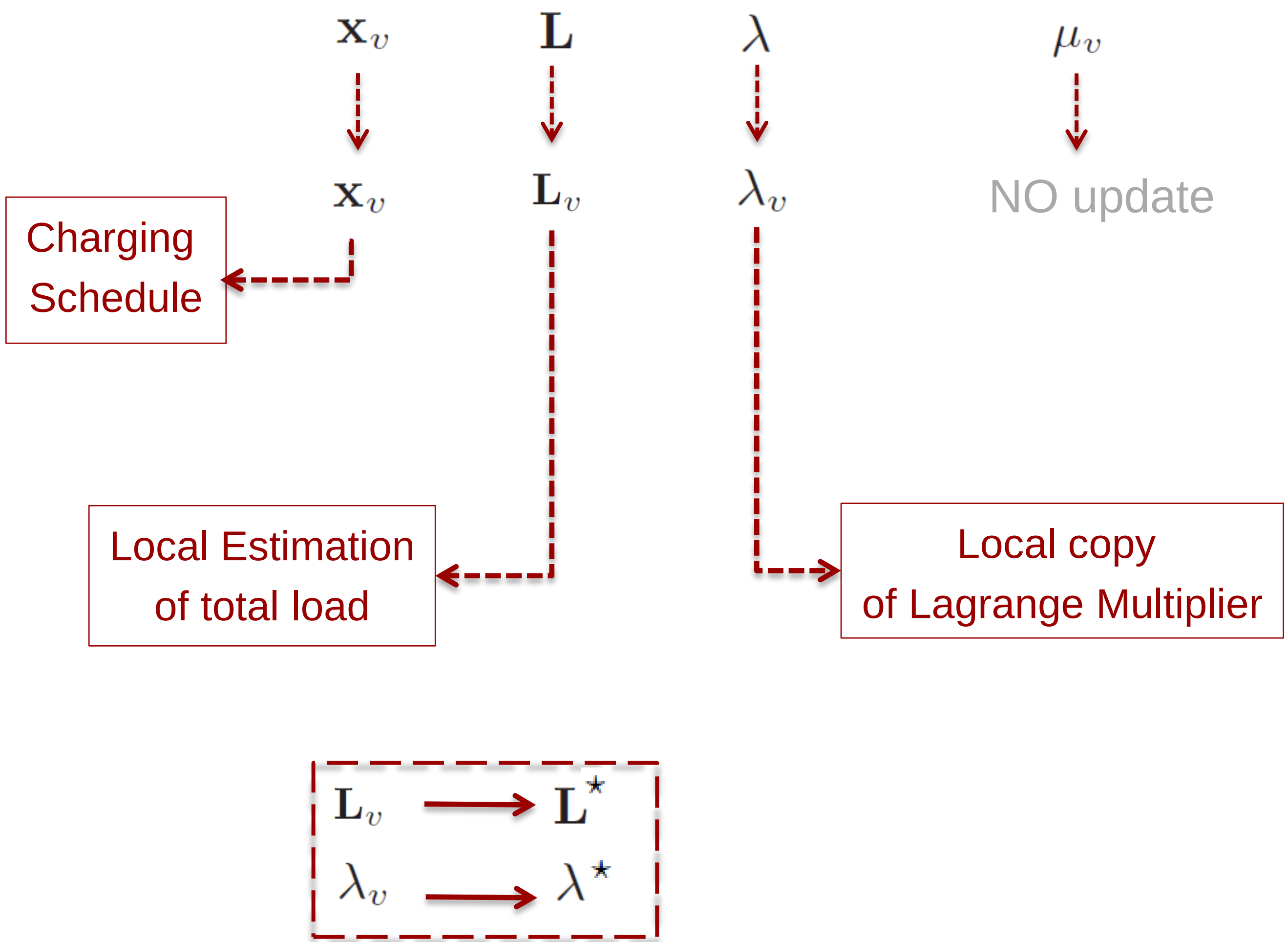

Distributed updates

The proposed update for λ_v :

$$\lambda_v(k+1) = \mathbb{P}\left[\lambda_v(k) - \beta_k \overbrace{\left(\sum_{w \in \Omega_v} (\lambda_v(k) - \lambda_w(k))\right)}^{\text{neighborhood consensus}} - \alpha_k \underbrace{\left(\frac{\mathbf{L}_v(k)}{V} - \mathbf{x}_v(k)\right)}_{\text{local innovation}}\right]_{[c_2,\infty)} .$$

The proposed update for $\mathbf{L}_v$:

$$\mathbf{L}_v(k+1) = \frac{\lambda_v(k) - c_2}{2c_1}$$

Distributed updates

The proposed update for $\mathbf{x}_v$:

Step1: Variable update

$$\mathbf{x}_v(k+1) = \mathbb{P}\left[\mathbf{x}_v(k) + \delta_k \left(\frac{\mathbf{L}_v(k)}{V} - \mathbf{x}_v(k)\right) - \eta_k \left(\lambda_v(k)\right)\right]_{\mathcal{F}}$$

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Step2: Feasibility

$$A \cdot \mathbf{x}_v \leq b_v$$
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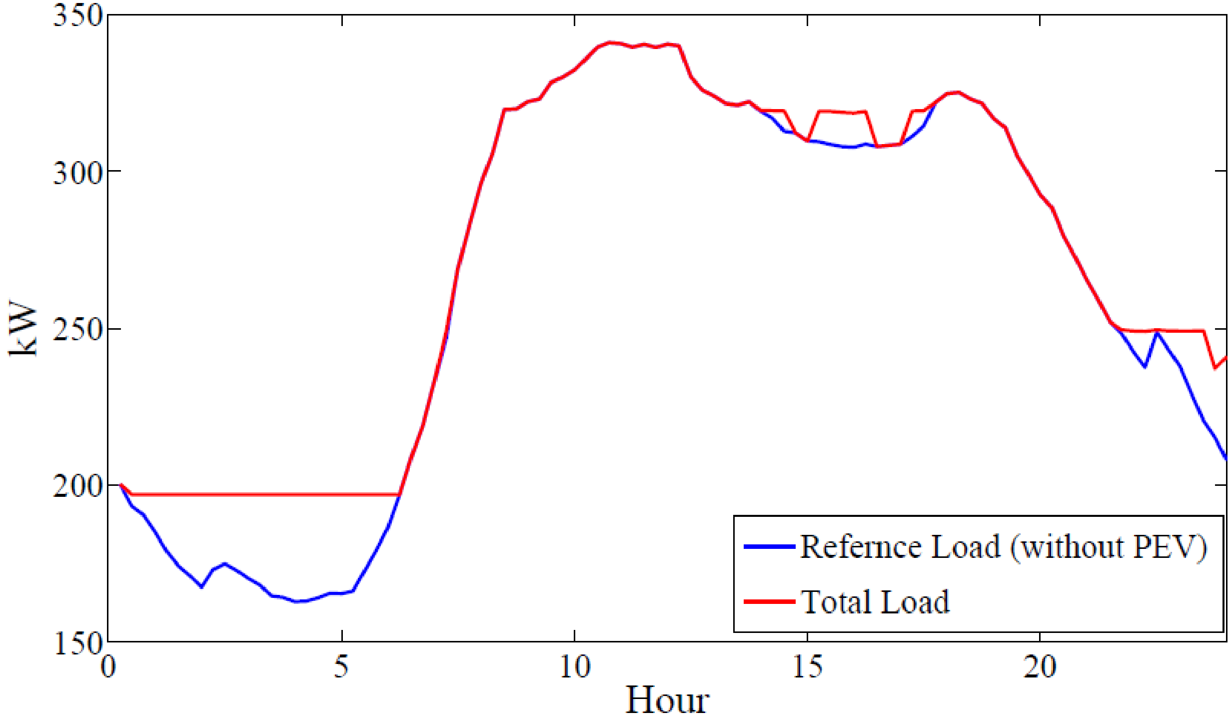
Test system

- Fleet of 25 EVs
  - Communication topology: ring graph

- Convergence Measures

$$rel_{obj} = \frac{|f - f^*|}{f^*},$$

Valley filling



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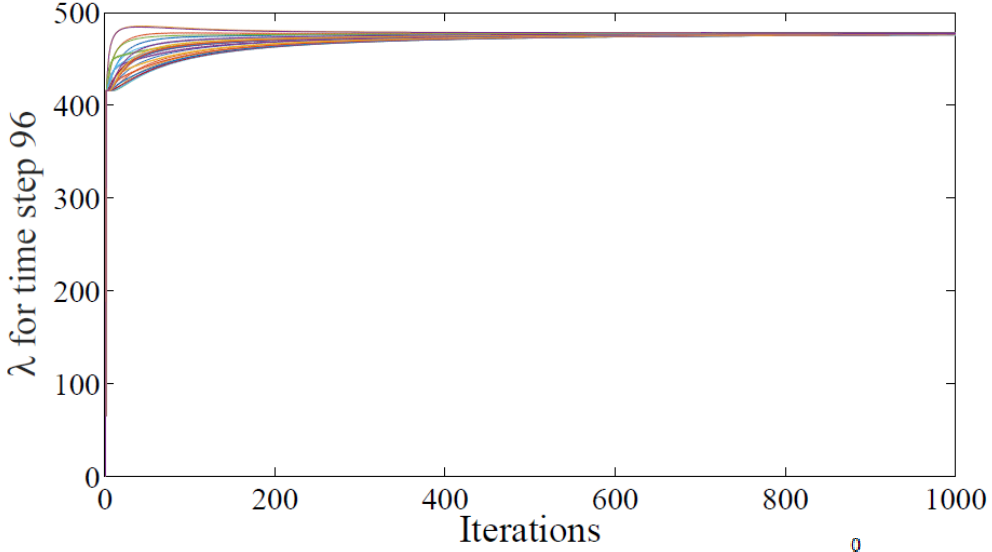

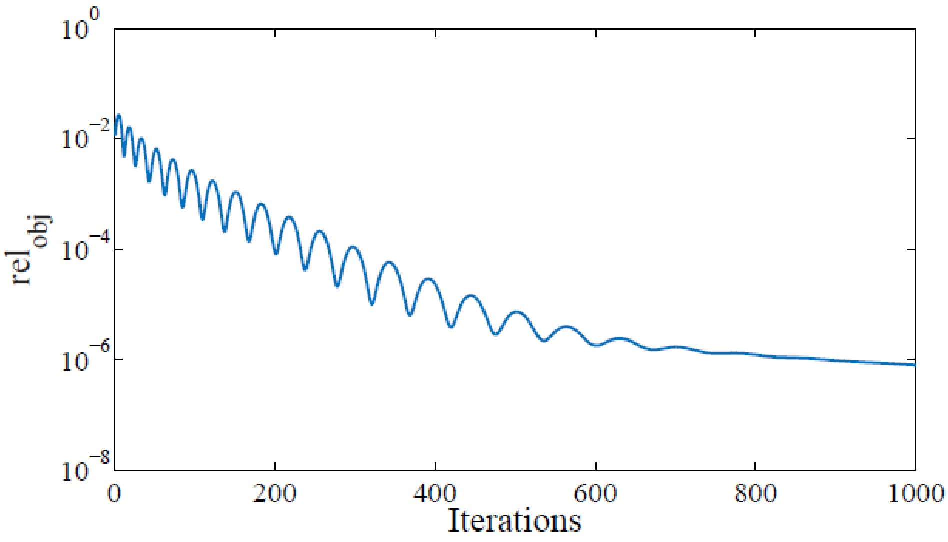

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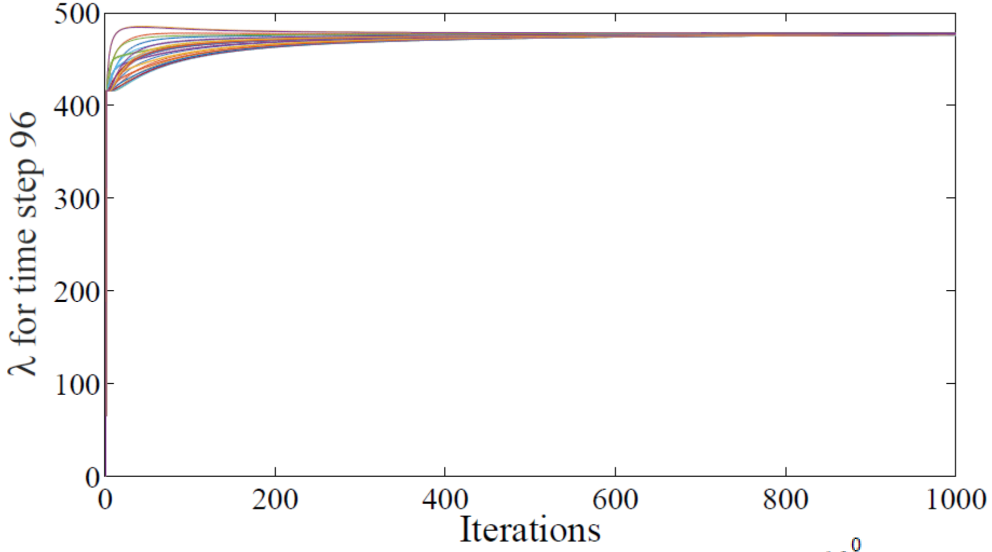
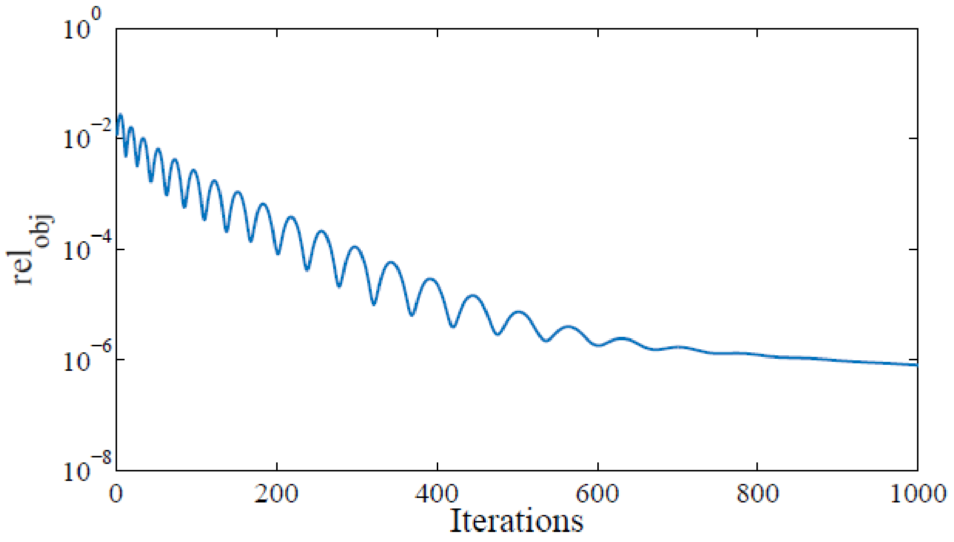


Feasible solution at each iteration



Summary

- Proposed distributed solution:
  - Distributes the computation among distributed entities (PEVs).
  - Each PEV exchanges limited information with a few other PEVs.
  - Achieves feasible solution at each iteration.

Questions