A LOW-RANK APPROACH FOR INTERFERENCE MANAGEMENT IN DENSE WIRELESS NETWORKS

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Wireless Big Data and Network Densification

Wireless big data era: overwhelming data traffic brought by the explosive growth of mobile devices and various application scenarios

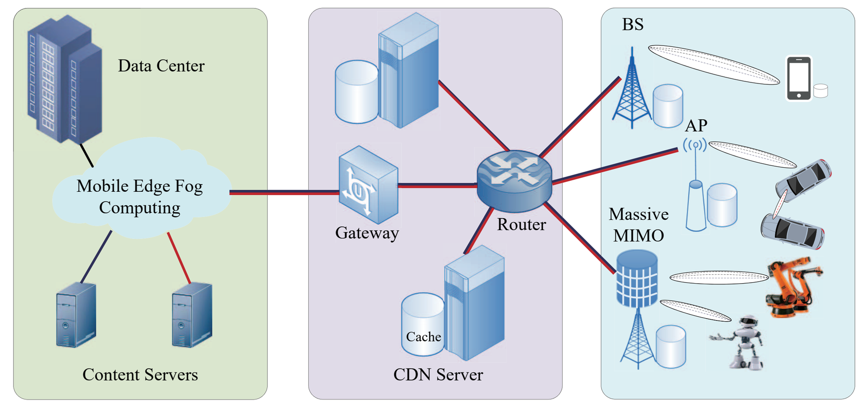


Figure: Massive devices and applications in Fog-RAN.

Key requirements: low-latency, ultra-high data rate, ultra reliability.

Various applications: Internet of Things, mobile virtual reality, autonomous driving etc.

Solution: Network densification.

Challenge

High-dimensional channel state information at the transmitters (CSIT) acquisition in dense wireless networks for interferences management

Prior Works on CSIT reduction

Prior works: Abundant CSIT → Relaxed CSIT

- Perfect CSIT: [Cadambe and Jafar, TIT 08] (IT paper award '09)
- Delayed CSIT: [Maddah-Ali and Tse, TIT 12] (IT & ComSoc paper award '15)
- Alternating CSIT: [Tandon, et al., TIT 13], partial and imperfect CSIT [Shi, et al., TSP 14]

Massive overhead for CSIT acquisition still exists!

Start here ⟶ Applicable? ⟵ Prior works

No CSIT — CSIT ⟶ — Perfect CSIT

Topological Interference Management: No CSIT

Partially connected interference channel:

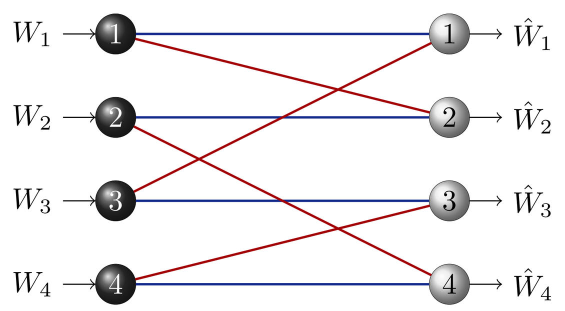


Figure: Blue lines: desired message. Red lines: interference links.

Interference alignment condition without CSIT [1, 2]

$$\begin{cases} \boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_j = 0,\ \forall j \in \mathcal{V}_i, j \neq i & \triangleright \text{ interference cancellation} \\ \boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_i \neq 0,\ i = 1, \cdots, K. & \triangleright \text{ desired message preservation} \end{cases}$$

Assumption: Mixed Network Connectivity Information

Channel links for each user: **strong links** (red lines) and **weak links** (red dashed lines) due to path loss and shadowing, e.g.

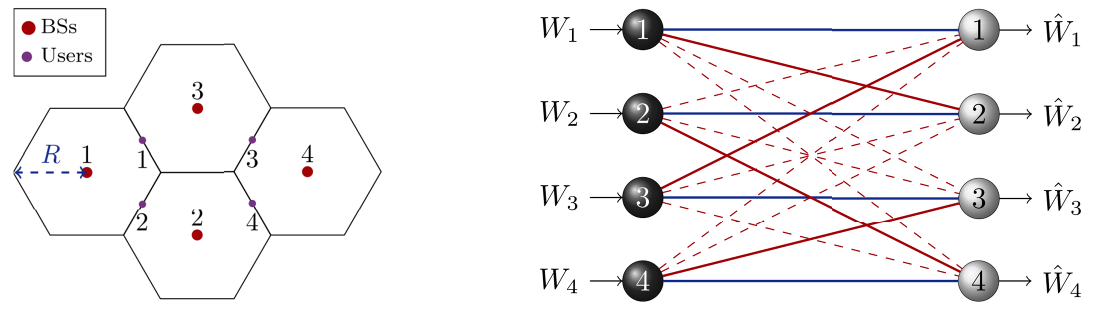


Figure: a) An example of 4 cells case. b) Heterogeneity of channel links.

Mixed Network Connectivity Information: Connectivity information for strong links ($\mathcal{V}_i$), statistical information for weak links ($\bar{\mathcal{V}}_i$).

Proposed: Interference Leakage Minimization

Goal: Achieve high sum rate per channel use with mixed network connectivity information

$$C_{\text{sum}} = \frac{1}{r}\sum_{i=1}^{K}\log(1+\text{SINR}_i), \quad \text{SINR}_i = \frac{\left|\boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_i\right|^2 |h_{ii}|^2}{\sigma^2\|\boldsymbol{u}_i\|^2 + \sum_{j\neq i}\left|\boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_j\right|^2 |h_{ij}|^2}$$

Proposed Approach: tradeoff of interference leakage and channel use

$$\begin{cases} \min\ I = \sum_{j\in\bar{\mathcal{V}}_i}\mathbb{E}(|h_{ij}|^2)\left|\boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_j\right|^2 & \triangleright \text{ Interference leakage minimization} \\ \min\ \text{channel use } r & \triangleright \text{ DoF maximization} \\ \boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_j = 0, \forall j \in \mathcal{V}_i, j \neq i & \triangleright \text{ Strong interferences cancellation} \\ \boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_i \neq 0,\ i = 1, \cdots, K & \triangleright \text{ Desired message preservation} \end{cases}$$

Low Rank Approach: $\boldsymbol{X} = [\boldsymbol{u}_i^{\mathsf{H}}\boldsymbol{v}_j]$

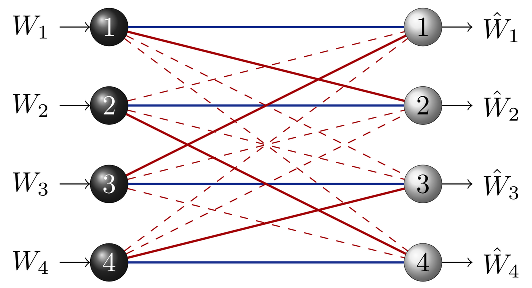
$$\begin{aligned} \underset{\boldsymbol{X}}{\text{minimize}}\ \ & \text{rank}(\boldsymbol{X}) \\ \text{subject to}\ \ & \mathcal{P}_\Omega(\boldsymbol{X}) = \boldsymbol{I}, \\ & \sum_{j\in\bar{\mathcal{V}}_i}\mathbb{E}(|h_{ij}|^2)|X_{ij}|^2 \leq \tau_0 \end{aligned}$$

Unique Challenges: non-convex, nuclear norm relaxation always yields identity matrix!

Example: 4 hexagonal cells

For above mentioned cellular network, assuming:

- Strong links: the distance from BS is less than $3R/2$.
- Weak links: the distance from BS is greater than or equal to $3R/2$.



(a) Mixed network connectivity information

$$\begin{bmatrix} 1 & \star & 0 & \star \\ 0 & 1 & \star & \star \\ \star & \star & 1 & 0 \\ \star & 0 & \star & 1 \end{bmatrix}$$

(b) Corresponding low rank matrix recovery problem

Two-Stage Algorithm by Riemannian Trust Region

$f(\boldsymbol{X};\lambda) = \|\mathcal{P}_\Omega(\boldsymbol{X}) - \boldsymbol{I}\|_F^2 + \lambda\sum_{j\in\bar{\mathcal{V}}_i}\mathbb{E}(|h_{ij}|^2)|X_{ij}|^2.$

1) Find the minimal rank r when strong interferences can be canceled

$$\begin{aligned} \text{minimize}\ \ & \text{rank}(\boldsymbol{X}) \\ \text{subject to}\ \ & f(\boldsymbol{X};0) = 0 \end{aligned}$$

2) Minimize interference leakage under fixed channel use r

$$\begin{aligned} \underset{\boldsymbol{X}}{\text{minimize}}\ \ & f(\boldsymbol{X};\lambda) \\ \text{subject to}\ \ & \text{rank}(\boldsymbol{X}) = r, \end{aligned}$$

Allows for a **unified form** of minimizing a smooth function $f(\boldsymbol{X};\lambda_0)$ under fixed rank constraint

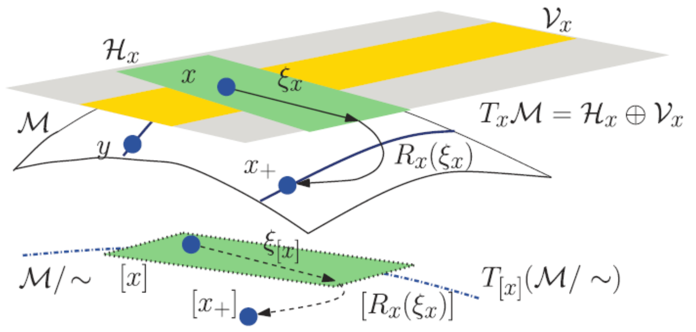
Algorithm 1 Proposed Unified Framework

```
1:  Inputs:
        K, Ω, E(|h_ij|^2) for V̄_i, accuracy ε1, ε2
2:  Initialize:
        r ← 1
3:  while r ≤ K do
4:      Minimize f(X; 0) under fixed rank r
5:      if ||f(X; 0)||_F^2 / 2 ≤ ε1 then
6:          break;
7:      end if
8:      r ← r + 1
9:  end while
10: Minimize f(X; λ) under fixed rank r
11: Outputs:
        r, X
```

Numerical Algorithm

Riemannian trust region [3] algorithm for fixed-rank problem:

$$\mathscr{P}_{\text{LIM}}:\ \text{minimize}\ \ f(\boldsymbol{X};\lambda_0)\quad \text{subject to rank}(\boldsymbol{X}) = r.$$



- Generalize Euclidean gradient and Hessian to Riemannian gradient and Hessian.
- Finding search direction by solving trust region subproblem

$$\begin{aligned} \underset{\boldsymbol{\xi}_{\boldsymbol{X}}\in\mathcal{H}_{\boldsymbol{X}}\mathcal{M}_r}{\text{minimize}}\ \ & m(\boldsymbol{\xi}_{\boldsymbol{X}}) \\ \text{subject to}\ \ & g_{\boldsymbol{X}}(\boldsymbol{\xi}_{\boldsymbol{X}},\boldsymbol{\xi}_{\boldsymbol{X}}) \leq \delta^2, \end{aligned}$$

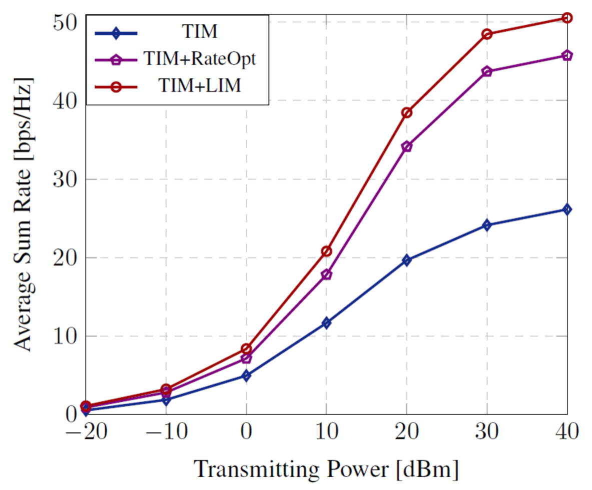
- Exploit the non-uniqueness of matrix factorization on equivalent class

$$[\boldsymbol{X}] = \{(\boldsymbol{U}\boldsymbol{M}^{-1}, \boldsymbol{V}\boldsymbol{M}^T) : \boldsymbol{M} \in \mathcal{GL}(r)\}$$

Simulation

DoF versus transmitting power constraint per channel:

- "TIM": Setting weak links as zeros [2].
- "TIM+RateOpt": RateOpt is an alternating minimization method [4] with mixed network connectivity information.
- "TIM+LIM": The proposed algorithm.



Conclusions & Possible Extensions

- Only some statistical information, big progress!
- Algorithmic advantages on interference management.

Extensions

Although we focus on the single antenna transmitters and users in this work, it can be readily extended to more general cases according to our recent works:

- when users are cache-enabled [5].
- both transmitters and users are cache-enabled [6] in MISO channel.

References

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