Exemplar-Embed Complex Matrix Factorization for Facial Expression Recognition

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1. Major Contribution

- ✓ This paper presents an image representation approach, which is called exemplar-embed complex matrix factorization (EE-CMF) and is based on matrix factorization in the complex domain.
- ✓ The proposed EE-CMF approach effectively improve the performance of facial expression recognition.
- ✓ Wirtinger's calculus was employed to determine derivatives and the gradient descent method was utilized to solve the complex optimization problems

2. Existing Representation Methods

- Principal component analysis (PCA) based method
- Linear discriminant analysis (LDA)
- Nonnegative matrix factorization (NMF)
- Semi-NMF, Convex-NMF, Cluster NMF

Nonnegative Matrix Factorization

Given an $N\times M$ input data matrix $\mathbf{X}=(\mathbf{x}_1,\mathbf{x}_2,\dots,\mathbf{x}_M)$

M is the number of facial images and each column $\mathbf{x}_m$ corresponds to an image with size $N=a\times b$.

The NMF problem is to find $\mathbf{W}$ and $\mathbf{V}$ that can minimize the follow objective function:

$$\min_{\mathbf{W}\geq 0,\mathbf{V}\geq 0} \mathrm{O}_{NMF}(\mathbf{W},\mathbf{V})=\frac{1}{2}\|\mathbf{X}-\mathbf{WV}\|_F^2$$

To relax the constraint of nonnegative data, EE-NMF imposes a constraint that the column vectors of **W** must lie within the column space of **X**, i.e. **W** = **XA**

$$\min_{\mathbf{A}\geq 0,\mathbf{V}\geq 0} O_{conNMF}(\mathbf{A},\mathbf{V})=\frac{1}{2}\|\mathbf{X}-\mathbf{XAV}\|_F^2$$

The factors $\mathbf{V}$ and $\mathbf{A}$ are updated as follows:

$$\mathbf{A}\leftarrow\mathbf{A}\sqrt{\frac{(\mathbf{X}^T\mathbf{X})^+\mathbf{V}^T+(\mathbf{X}^T\mathbf{X})^-\mathbf{AVV}^T}{(\mathbf{X}^T\mathbf{X})^-\mathbf{V}^T+(\mathbf{X}^T\mathbf{X})^+\mathbf{AVV}^T}}$$

$$\mathbf{V}\leftarrow\mathbf{V}\sqrt{\frac{\mathbf{A}^T(\mathbf{X}^T\mathbf{X})^++\mathbf{A}^T(\mathbf{X}^T\mathbf{X})^-\mathbf{AV}}{\mathbf{A}^T(\mathbf{X}^T\mathbf{X})^-+\mathbf{A}^T(\mathbf{X}^T\mathbf{X})^+\mathbf{AV}}}$$

where $\mathbf{X}^T\mathbf{X}=(\mathbf{X}^T\mathbf{X})^+-(\mathbf{X}^T\mathbf{X})^-$

3. Wirtinger's Calculus and Complex Optimization

✓ If $g(z,z^*)=f(x,y)$ where $z=x+jy$

$$\frac{\partial g}{\partial z}=\frac{1}{2}\left(\frac{\partial f}{\partial x}-i\frac{\partial f}{\partial y}\right)\qquad \frac{\partial g}{\partial z^*}=\frac{1}{2}\left(\frac{\partial f}{\partial x}+i\frac{\partial f}{\partial y}\right)$$

$$\nabla f(\mathbf{Z})=2\frac{df(\mathbf{Z})}{d\mathbf{Z}^*}=\frac{\partial f(\mathbf{Z})}{\partial\,\mathrm{Re}\,\mathbf{Z}}+i\frac{\partial f(\mathbf{Z})}{\partial\,\mathrm{Im}\,\mathbf{Z}}$$

For the real-valued function $f(\mathbf{Z},\mathbf{Z}^*)$, we have

$$\Delta f(\mathbf{Z},\mathbf{Z}^*)\approx\langle\nabla_{\mathbf{Z}}f,\Delta\mathbf{Z}^*\rangle+\langle\nabla_{\mathbf{Z}^*}f,\Delta\mathbf{Z}\rangle$$

$$\Delta f(\mathbf{Z},\mathbf{Z}^*)\approx\langle\nabla_{\mathbf{Z}}f,\Delta\mathbf{Z}^*\rangle+\langle\nabla_{\mathbf{Z}^*}f,\Delta\mathbf{Z}\rangle=2\mathrm{Re}\{\langle\nabla_{\mathbf{Z}^*}f,\Delta\mathbf{Z}\rangle\}$$

4. Proposed Method

mapping $f:\mathbb{R}^N\to\mathbb{C}^N$

$$f(\mathbf{x}_t)=\mathbf{z}_t=\frac{1}{\sqrt{2}}e^{i\alpha\pi\mathbf{x}_t}=\frac{1}{\sqrt{2}}\begin{bmatrix}e^{i\alpha\pi\mathbf{x}_t(1)}\\ \vdots\\ e^{i\alpha\pi\mathbf{x}_t(N)}\end{bmatrix}$$

EE-CMF aims at factorizing **Z** into two matrices, $\mathbf{W}\in\mathbb{C}^{M\times K}$ and $\mathbf{V}\in\mathbb{C}^{K\times M}$ to satisfy the follow objective function:

$$\min_{\mathbf{W},\mathbf{V}}\mathrm{O}_{conCMF}(\mathbf{W},\mathbf{V})=\min_{\mathbf{W},\mathbf{V}}\frac{1}{2}\|\mathbf{Z}-\mathbf{ZWV}\|_F^2$$

where $\|\mathbf{Z}-\mathbf{ZWV}\|_F^2=Trace(\mathbf{Z}-\mathbf{ZWV})^H(\mathbf{Z}-\mathbf{ZWV})$

$$=Trace(\mathbf{Z}^H\mathbf{Z}-\mathbf{V}^H\mathbf{W}^H\mathbf{Z}^H\mathbf{Z}-\mathbf{Z}^H\mathbf{ZWV}+\mathbf{V}^H\mathbf{W}^H\mathbf{Z}^H\mathbf{ZV})$$

Optimal Solution: Wirtinger's calculus

First, fix **W** and solve **V** by

$$\min_{\mathbf{V}} f(\mathbf{V})\ \text{where}\ f(\mathbf{V})=\frac{1}{2}\|\mathbf{Z}-\mathbf{ZWV}\|_F^2$$

where **V** be can solved iteratively

$$\mathbf{V}_{t+1}=\mathbf{V}_t-\beta\nabla_{\mathbf{V}^*}f(\mathbf{V}_t)$$

$$\nabla_{\mathbf{V}^*}f(\mathbf{V})=2\frac{\partial f(\mathbf{V})}{\partial\mathbf{V}^*}=\frac{\partial f(\mathbf{V})}{\partial(\mathrm{Re}\,\mathbf{V})}+i\frac{\partial f(\mathbf{V})}{\partial(\mathrm{Im}\,\mathbf{V})}$$

$$\nabla_{\mathbf{V}^*}f(\mathbf{V})=-\mathbf{W}^H\mathbf{Z}^H\mathbf{Z}+\mathbf{W}^H\mathbf{Z}^H\mathbf{ZWV}$$

Then, **W** is updated based on the Moore–Penrose pseudoinverse, †, and $\mathbf{W}=\mathbf{Z}(\mathbf{VZ})^\dagger$ with fixed $\mathbf{V}$

5. Simulation Results

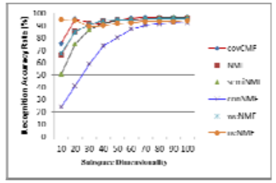
Cohn–Kanade (CK) database

surprise sadness anger fear disgust happiness

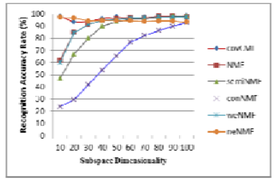
JFFA database

surprise sadness anger fear disgust happiness

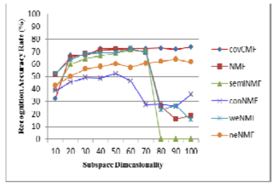
CK database training: set = 1 : 4



CK database training: set = 2 : 3



JFFA database training: set = 1 : 4



JFFA database training: set = 2 : 3