JOINT CHANNEL AND CARRIER FREQUENCY ESTIMATION FOR M-ARY CPM OVER FREQUENCY-SELECTIVE CHANNEL USING PAM DECOMPOSITION

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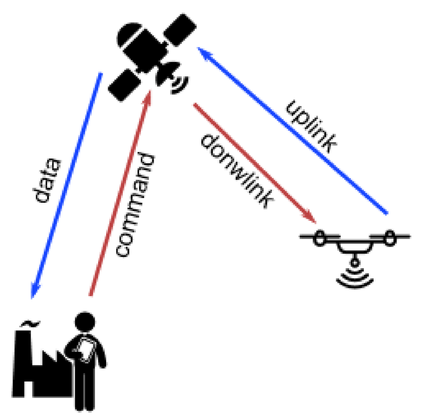
1. Continuous Phase Modulation

1.1. Motivation

- UAV (Unmanned Aerial Vehicle) control-link by SatCom



- Robustness to non-linearity (due to its constant complex envelop)
- Usually used over AWGN channels
- No discrete equivalent channel in case of frequency-selective channels

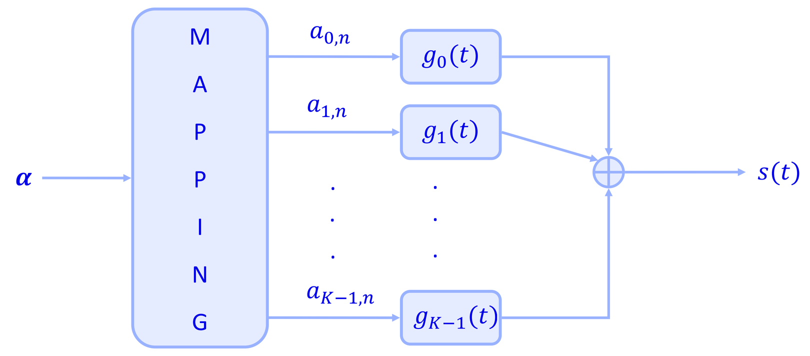
1.2. CPM signals

$\boldsymbol{\alpha} = \{\alpha_i\}$ sequence of N symbols taken in the M-ASK alphabet
Transmitted signal (complex envelop)

$$s_b(t) = \exp^{j\phi(t,\boldsymbol{\alpha})} \quad (1)$$

where $\phi(t, \boldsymbol{\alpha})$ is the information phase.

PAM Decomposition
Laurent representation **[1]** (extended by Mengali and Morelli to the M-ary case **[2]**): **sum of modulated PAM** with pseudo-symbols $a_{k,n}$.



1.3. Received Signal

Frequency-selective channel $h(t) = \sum_l h_l \delta(t - \tau_l) \quad (2)$

Received signal with **carrier-frequency offset**

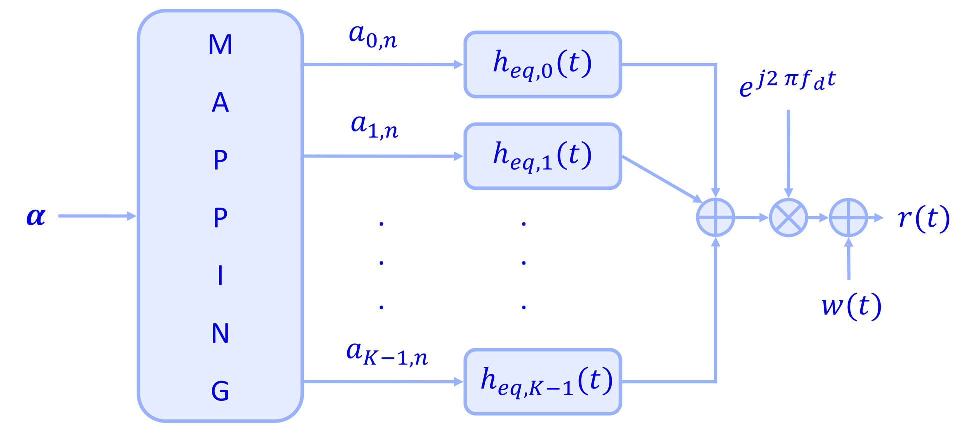
$$r(t) = e^{j2\pi f_d t}\{h * s\}(t) + w(t) \quad (3)$$

with $w(t)$ complex Gaussian white noise with spectral density N_0

2. A linear representation

2.1. Equivalent channels

Received signal



Sampling at R_s:

$$r(mT_s) = \sum_{n=0}^{N-1}\sum_{k=0}^{K-1} a_{k,n} h_{eq,k}((m-n)T_s) + w(mT_s) \quad (4)$$

$$\text{with } h_{eq,k}(t) = (h * g_k)(t) \quad (5)$$

2.2. Matrix-wise representation

Let define L_c the equivalent channel length and for $k = 0$ to $K - 1$:

$$\boldsymbol{A}_k = \begin{pmatrix} a_{k,0} & 0 & \dots & 0 \\ a_{k,1} & a_{k,0} & \ddots & \vdots \\ \vdots & \dots & \ddots & 0 \\ a_{k,L_c-1} & \dots & a_{k,1} & a_{k,0} \\ \vdots & \vdots & \vdots & \vdots \\ a_{k,N-1} & a_{k,N-2} & \dots & a_{k,N-L_c} \end{pmatrix} \quad (6)$$

$$\boldsymbol{\Gamma}(f_d) = \text{diag}\{e^0, e^{j2\pi f_d T}, \dots, e^{j2\pi f_d (N-1)T}\} \quad (7)$$

$$\text{and } \boldsymbol{h}_{eq,k} = [h_{eq,k}[0], h_{eq,k}[1], \dots, h_{eq,k}[L_c - 1]] \quad (8)$$

$$\boldsymbol{A} = [\boldsymbol{A}_0, \boldsymbol{A}_1, \dots, \boldsymbol{A}_{K-1}] \quad (9)$$

$$\boldsymbol{h}_{eq} = [\boldsymbol{h}_{eq,0} | \boldsymbol{h}_{eq,1} | \dots | \boldsymbol{h}_{eq,K-1}]^T \quad (10)$$

Matrix model of the received signal

$$\boldsymbol{r} = \boldsymbol{\Gamma}(f_d)\boldsymbol{A}\boldsymbol{h}_{eq} + \boldsymbol{w} \quad (11)$$

3. Joint ML Estimation

Likelihood function (similar to **[3]** for linear modulations and binary GMSK)

$$\Delta(\boldsymbol{r}; \widetilde{\boldsymbol{h}}_{eq}; \widetilde{f}_d) = \frac{1}{(\pi\sigma_n^2)^N}\exp\left\{\frac{-1}{N}[\boldsymbol{r} - \boldsymbol{\Gamma}(\widetilde{f}_d)\boldsymbol{A}\widetilde{\boldsymbol{h}}_{eq}][\boldsymbol{r} - \boldsymbol{\Gamma}(\widetilde{f}_d)\boldsymbol{A}\widetilde{\boldsymbol{h}}_{eq}]^H\right\} \quad (12)$$

Least Squares Estimation of the equivalent channels for a fixed $\widetilde{f}_d$

$$\widehat{\boldsymbol{h}}_{eq}(\widetilde{f}_d) = (\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H\boldsymbol{\Gamma}^H(\widetilde{f}_d)\boldsymbol{r} \quad (13)$$

Likelihood function (to maximize) for **Carrier Frequency Estimation** using Eq (13) in Eq (12)

$$g(\widetilde{f}_d) = \boldsymbol{r}^H\boldsymbol{\Gamma}(\widetilde{f}_d)\boldsymbol{B}\boldsymbol{\Gamma}^H(\widetilde{f}_d)\boldsymbol{r} \quad (14)$$

$$\text{with } \boldsymbol{B} = \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H \quad (15)$$

This can be done using a **FFT** as:

$$g(\widetilde{f}_d) = -\rho(0) + 2\Re\{\sum_{m=0}^{N-1}\rho(m)e^{-j2\pi m\widetilde{f}_d}\} \quad (16)$$

$$\text{with } \rho(m) = \sum_{k=m}^{N-1}[\boldsymbol{B}]_{k-m,k} r(k) r^*(k-m) \quad (17)$$

Hence,

$$\widehat{f}_d = \text{argmax } g(\widetilde{f}_d) \quad (18)$$

Procedure

- Compute $\widehat{f}_d$ using Eq (16)
- Counter-rotate the received signal $\boldsymbol{r}$ according to $\widehat{f}_d$
- Compute $\widehat{h}_{eq}$ using Eq (13)

References

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[2] Mengali, U., & Morelli, M. (1995). Decomposition of M-ary CPM signals into PAM waveforms. *IEEE transactions on information theory*, 41(5), 1265-1275.

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4. Results

Preamble of 128 symbols modulated by a GMSK with $M = 4$, the modulation index $h = 1/4$, the CPM memory $L_{\text{cpm}} = 3$ and $BT = 0.3$, and a FFT of size 2048.
Uniformly random CFO between $-0.5R_s$ and $0.5R_s$.
We consider the urban GSM channel and that all equivalent discrete channels have a length $L_c = 8$.

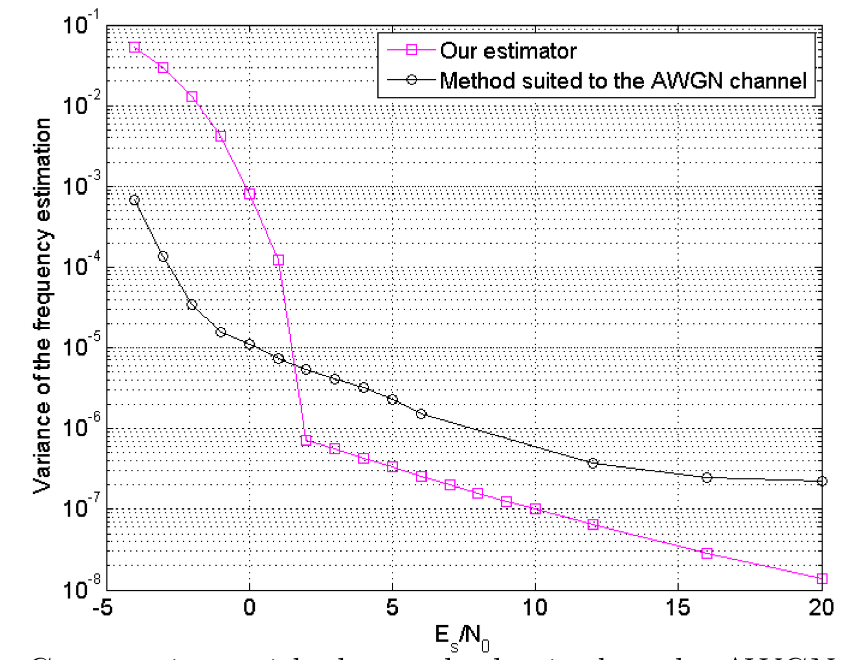

Figure 1: Comparaison with the method suited to the AWGN channel **[4]**

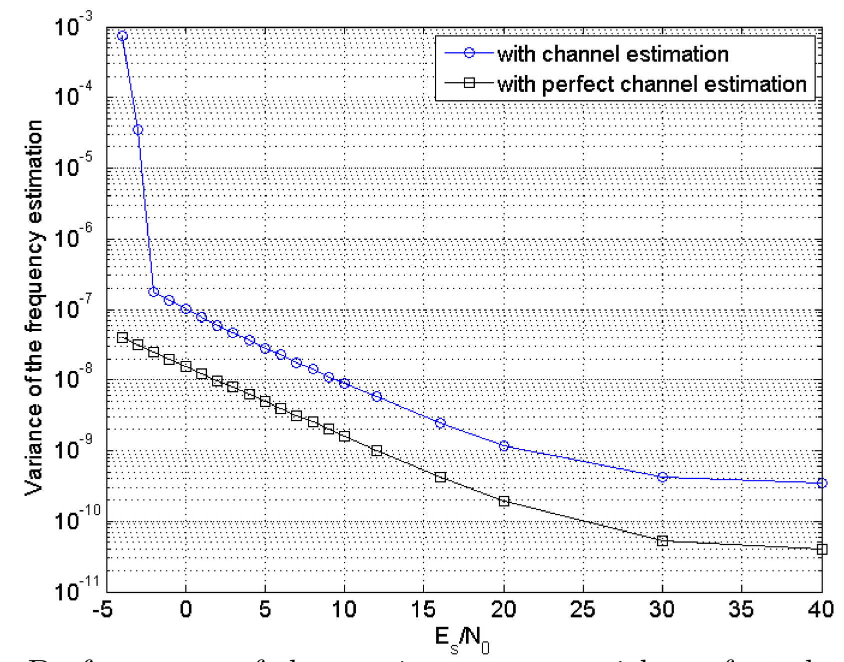

Figure 2: Performance of the carrier recovery with perfect channel knowledge over the urban GSM channel

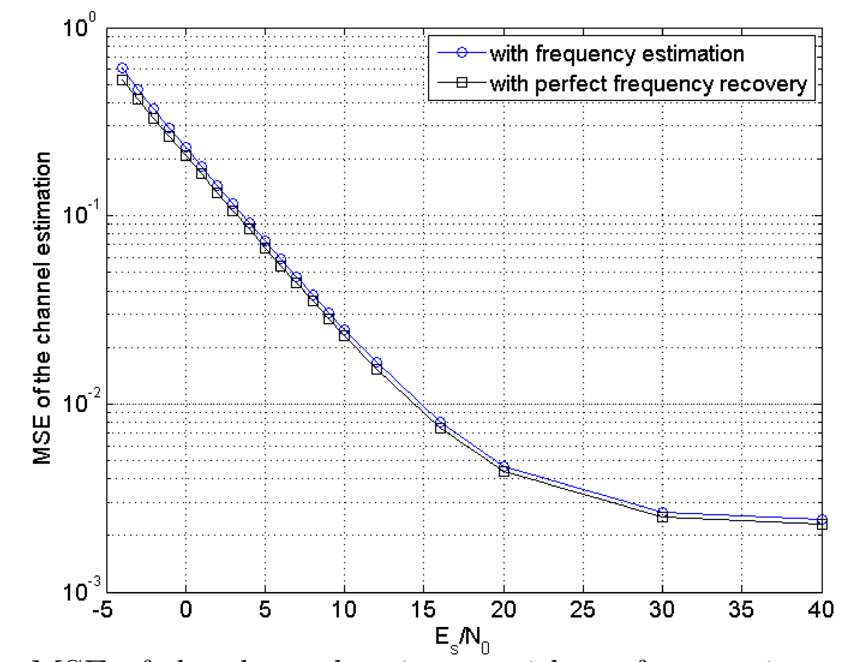

Figure 3: MSE of the channel estimate with perfect carrier recovery over the urban GSM channel

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