PIPA: A New Proximal Interior Point Algorithm for Large-Scale Convex Optimization

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Interior Point Methods

Many problems in signal/image processing (image restoration, enhancement, denoising/deblurring, spectral unmixing) can be formulated as **constrained minimization problems** $\rightarrow$ need efficient methods for solving those.

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$$\mathcal{P}_0 : \quad \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f(x) \quad \text{s.t.} \quad (\forall i \in \{1, \ldots, p\}) \;\; c_i(x) \leq 0$$

where

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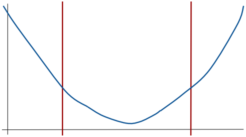
How to minimize f while ensuring that every iterate is feasible ?



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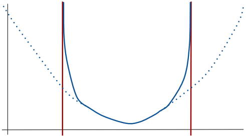
→ Add a barrier function



Logarithmic Barrier

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Unconstrained Subproblem

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Where $\mu > 0$ is the barrier parameter.

$\mathcal{P}_0$ is replaced by a sequence of subproblems $(\mathcal{P}_{\mu_j})_{j \in \mathbb{N}}$.

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- Main advantage : every iterate is feasible.
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✗ Require the inversion of an $n \times n$ matrix at each step : medium size applications.
✗ First or second order methods : limited to smooth functions. **[Armand *et al.*, 2000]**

Problem of Interest

Quality of the solution and robustness against noise can be improved by adding a **non-differentiable term** (ℓ_1, TV, ...).

Composite Constrained Problem

$$\mathcal{P}_0 : \quad \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad f(x) + g(x) \quad \text{s.t.} \quad (\forall i \in \{1, \dots, p\}) \;\; c_i(x) \leq 0$$

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- $f : \mathbb{R}^n \mapsto]-\infty, +\infty]$ convex, **non-differentiable**
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How to address the non-smooth term while ensuring that every iterate is feasible ?

→ Combine the logarithmic barrier method with proximal tools.

Notation and Definitions

- Let $\mathcal{S}^+(\mathbb{R}^N)$ be the set of symmetric positive definite matrices of $\mathbb{R}^{N\times N}$.
- The weighted norm induced by $U \in \mathcal{S}^+(\mathbb{R}^N)$ is $\|.\|_U = \sqrt{\langle . \mid U . \rangle}$.
- Let $\Gamma_0(\mathbb{R}^N)$ denote the set of proper lower semicontinuous convex functions from $\mathbb{R}^N$ to $]-\infty, +\infty]$.

Proximity Operator

The **proximal operator** [a] $\mathrm{prox}_f^U(x)$ of $f \in \Gamma_0(\mathbb{R}^N)$ at $x \in \mathbb{R}^N$ relative to the metric induced by $U \in \mathcal{S}^+(\mathbb{R}^N)$ is the unique vector $\widehat{y} \in \mathbb{R}^N$ such that

$$f(\widehat{y}) + \frac{1}{2}\|\widehat{y} - x\|_U^2 = \inf_{y\in\mathbb{R}^N} f(y) + \frac{1}{2}\langle y - x \mid U(y-x)\rangle.$$

a. http ://proximity-operator.net/

Example :

- Indicator function : projection.
- ℓ_1 norm : soft-thresholding.

Proposed Approach

$\mathcal{P}_0$ is replaced by a sequence of subproblems $(\mathcal{P}_{\mu_j})_{j\in\mathbb{N}}$.

Unconstrained Subproblems

$$\mathcal{P}_{\mu_j}: \quad \underset{x\in\mathbb{R}^n}{\text{minimize}} \ f(x) + \underbrace{g(x) - \boldsymbol{\mu_j} \sum_{\boldsymbol{i=1}}^{\boldsymbol{p}} \ln\left(-\boldsymbol{c_i}(\boldsymbol{x})\right)}_{\textit{smooth}}$$

Our algorithm comprises two interlocked loops.

- Given $\mu_j > 0$, $(x_{j,k})_k$ is produced via several forward-backward (proximal gradient) steps.
- Once $x_{j,k}$ is close enough to the solution of $\mathcal{P}_{\mu_j}$, the barrier parameter μ_j is updated.

Iteration Scheme

Forward-Backward Step

For j fixed,

$$x_{j,k+1} = \mathrm{prox}^{A_{j,k}}_{\gamma_{j,k} f}(x_{j,k} - \gamma_{j,k} A^{-1}_{j,k} \nabla \varphi_{\mu_j}(x_{j,k}))$$

where $\varphi_{\mu_j}(x) = g(x) - \mu_j \sum_{i=1}^{p} \ln(-c_i(x))$.

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- Gradient step on the smooth term ;
- Proximal step on the non-differentiable function f ;
- Preconditioner $A_{j,k}$ for acceleration **[Chouzenoux *et al.*, 2016]** ;
- Stepsize $\gamma_{j,k} > 0$ found using a backtracking strategy **[Salzo, 2017]** since φ_{μ_j} is not Lipschitz-differentiable.

Algorithm

Proximal Interior point Algorithm (PIPA)

Initialization

Let $\bar{\gamma} > 0$, $(\delta, \theta) \in]0,1[^2$, $\mu_0 > 0$;
Initialize $x_{0,0}$ such that $(\forall i \in \{1,\dots,p\})\ c_i(x_{0,0}) < 0$;

For $j = 0, 1, \dots$

- For $k = 0, 1, \dots$
  - **Choose $\boldsymbol{A_{j,k}}$** satisfying a boundedness condition ;
  - For $l = 0, 1, \dots$
    - $\tilde{x}^l_{j,k} = \operatorname{prox}^{A_{j,k}}_{\bar{\gamma}\theta^l f}(x_{j,k} - \bar{\gamma}\theta^l A^{-1}_{j,k} \nabla \varphi_{\mu_j}(x_{j,k}))$;
    - Stop if the **backtracking condition** is met ;
  - $x_{j,k+1} = \tilde{x}^l_{j,k}$;
  - $\gamma_{j,k} = \bar{\gamma}\theta^l$;
  - Stop if **precision conditions** are met ;
- $x_{j+1,0} = x_{j,k+1}$;
- **Update $\boldsymbol{\mu_j}$**;

Theoretical Results

Assumptions

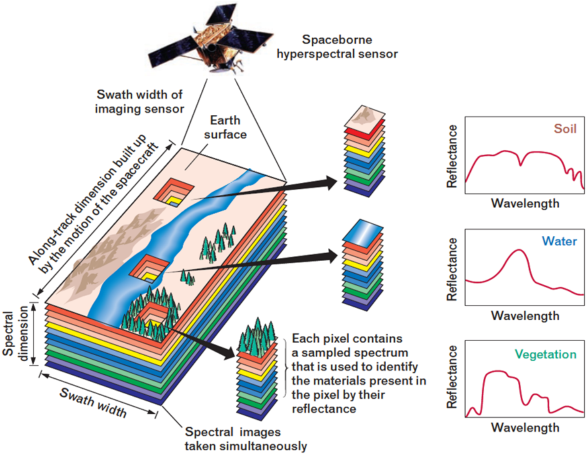
- The set of solutions to $\mathcal{P}_0$ is nonemtpy and bounded ;
- f, g and the constraints are convex, g is Lipschitz-differentiable and the constraints are continuously twice-differentiable ;
- The strict interior of the feasible set is nonempty ;
- $(\forall j \in \mathbb{N})$ $f + \varphi_{\mu_j}$ is a Kurdyka-Lojasiewicz (KL) function ;
- $(\forall j \in \mathbb{N})$ $(A_{j,k})_k$ are bounded from above and from below ;
- $\lim_{j\to\infty} \mu_j = 0$ and $(\forall i \in \{1,\ldots,4\})$ $\lim_{j\to\infty} \epsilon_{i,j}/\mu_j = 0$.

Convergence

Under some mild technical assumptions :

- for all $j \in \mathbb{N}$, $(x_{j,k})_{k\in\mathbb{N}}$ **converges to a solution to** $\mathcal{P}_{\mu_j}$;
- $(x_{j,0})_{j\in\mathbb{N}}$ **is bounded and every cluster point of it is a solution to** $\mathcal{P}_0$;
- if in addition strict complementarity holds, and if there exists $i \in \{1,\ldots,p\}$ such that c_i is strictly convex (or alternatively, for linear constraints, if some full rank property is satisfied) then $(x_{j,0})_{j\in\mathbb{N}}$ **converges to a solution to** $\mathcal{P}_0$.

Hyperspectral Unmixing Problem



Hyperspectral Unmixing Model

Optimization Problem

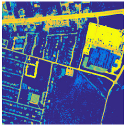
$$\begin{aligned}
\underset{X\in\mathbb{R}^{p\times n}}{\text{minimize}} \quad & \frac{1}{2}\|Y - SX\|_2^2 + \kappa \sum_{i=1}^{p} \|(WX_i)_d\|_1 \\
s.t. \quad & (\forall j \in \{1,\ldots,n\}) \ \sum_{i=1}^{p} X_{i,j} \leq 1 \\
& (\forall i \in \{1,\ldots,p\})(\forall j \in \{1,\ldots,n\}) \ X_{i,j} \geq 0
\end{aligned}$$

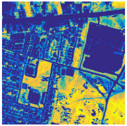
- p, n, s : number of endmembers, pixels, spectral bands
- $Y \in \mathbb{R}^{s\times n}$: observation
- $S \in \mathbb{R}^{s\times p}$: library
- $X \in \mathbb{R}^{p\times n}$: abundance matrix
- $W \in \mathbb{R}^{n\times n}$: orthogonal wavelet basis
- $\|(.)_d\|_1$: ℓ_1 norm of the detail wavelet coefficients

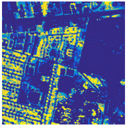
Experimental Setting

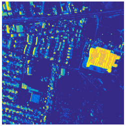
Realistic Data Simulation

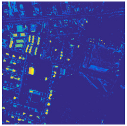
- Urban [1] data set : $p = 6$ endmembers (known spectral signatures), $s = 162$ spectral bands, $n = 256 \times 256$ pixels
- Gaussian noise : $\sigma^2 = 4.1 \times 10^{-3}$.

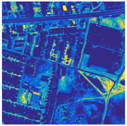

Asphalt Road


Grass


Tree


Roof


Metal


Dirt

1. http ://www.escience.cn/people/feiyunZHU/Dataset_GT.html

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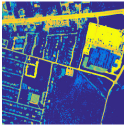
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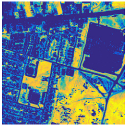
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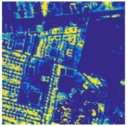
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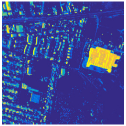
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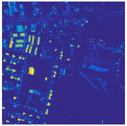
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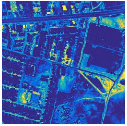
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Reconstruction Model

- Regularization weight : $\kappa = 10^{-2}$.
- W : orthogonal Daubechies 4 wavelet decomposition over 2 resolution levels.

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Implementation

- Matlab R2016b, Intel Xeon 3.2 GHz processor and 16 GB of RAM.
- Code will be available soon on `https://github.com/mccorbineau`.

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Comparison

State-of-the-Art Algorithms

No reg : interior point least squares algorithm without regularization **[Chouzenoux *et al.*, 2014]**

ADMM : alternating direction of multipliers method **[Setzer *et al.*, 2010]**

PDS : primal-dual splitting algorithm **[Combettes *et al.*, 2014]**

GFBS : generalized forward-backward splitting algorithm **[Raguet *et al.*, 2013]**

Evaluation Metric

Signal-to-Noise Ratio

$$\mathrm{SNR} = -10\log_{10}\left(\sum_{i=1}^{p}\frac{\|X_i-\bar{X}_i\|_2^2}{\|\bar{X}_i\|_2^2}\right) \quad ; \quad \mathrm{SNR}_i = -10\log_{10}\left(\frac{\|X_i-\bar{X}_i\|_2^2}{\|\bar{X}_i\|_2^2}\right)$$

where $\bar{X}_i$ is the true abundance map of the i^{th} endmember.

Distance from the Iterates to the Solution

$$\frac{\|x_{j,k}-x_\infty\|_2}{\|x_\infty\|_2}$$

where x is the vectorization of X and x_∞ is obtained after a very large number of iterations.

Quantitative Results

- No reg : SNR = 1.96 dB / With regularization : SNR = 3.65 dB

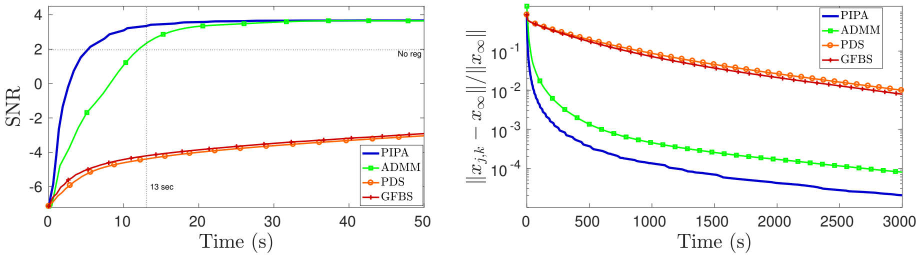

FIGURE – Left : global SNR versus time. Right : distance from the iterates to the solution versus time.

	Asphalt Road	Grass	Tree	Roof	Metal	Dirt
No reg	10.12	11.21	11.86	14.91	4.90	13.68
ADMM	6.75	11.47	12.56	14.66	**7.57**	11.47
PDS	2.06	3.33	4.73	6.63	-0.08	10.27
GFBS	2.17	3.57	4.76	7.66	0.05	10.31
PIPA	**10.98**	**11.70**	**12.73**	**15.19**	7.06	**14.57**

TABLE – SNR (dB) for all endmembers after 13 sec.

Asphalt Road

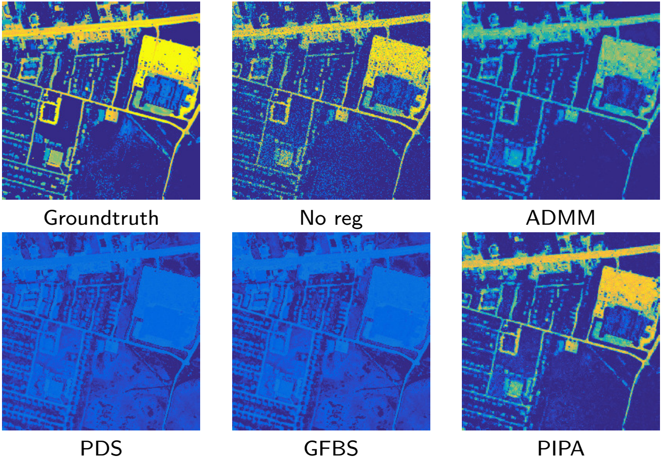

FIGURE – Abundance map of asphalt road after 13 sec.

No reg	10.12
ADMM	6.75
PDS	2.06
GFBS	2.17
PIPA	**10.98**

TABLE – SNR (dB)

Grass

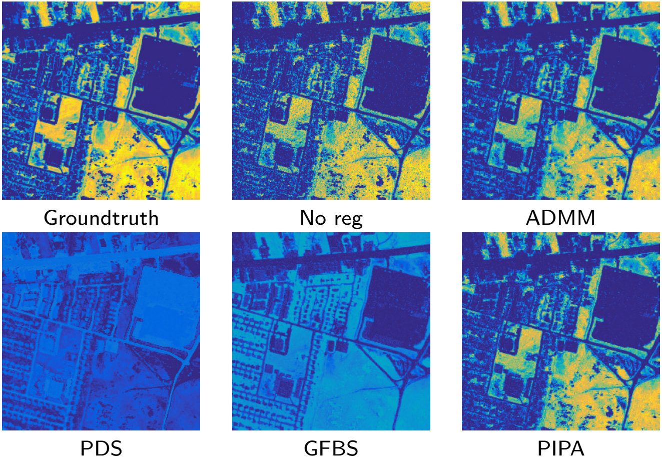

FIGURE – Abundance map of grass after 13 sec.

No reg	11.21
ADMM	11.47
PDS	3.33
GFBS	3.57
PIPA	**11.70**

TABLE – SNR (dB)

Dirt

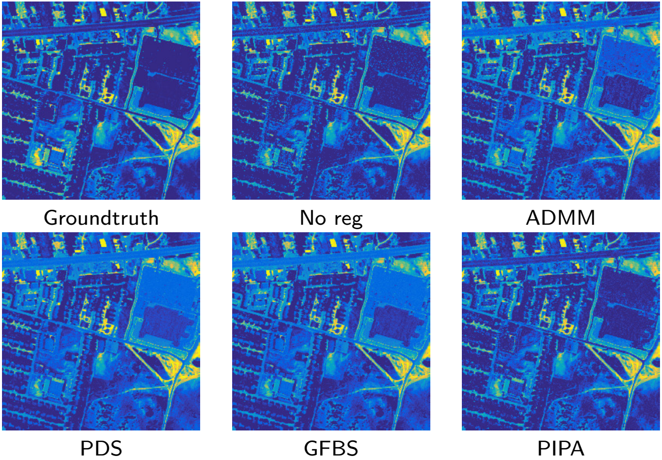

FIGURE – Abundance map of dirt after 13 sec.

No reg	13.68
ADMM	11.47
PDS	10.27
GFBS	10.31
PIPA	**14.57**

TABLE – SNR (dB)

Conclusion

Application of a new proximal interior point algorithm to hyperspectral unmixing with a non-differentiable regularization.

- Convergence guaranteed under mild assumptions.
- Possibility to include an arbitrary preconditioner.
- Good performance in the context of a large-scale image recovery application.

→ Extension of the convergence proof to inexact proximity operator.

→ Other applications.

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Thank you !

Stopping Criteria

Backtracking [Salzo, 2017]

For j and k fixed, the backtracking procedure stops if :

$$\varphi_{\mu_j}(\tilde{x}^l_{j,k}) - \varphi_{\mu_j}(x_{j,k}) - \left\langle \tilde{x}^l_{j,k} - x_{j,k} \mid \nabla \varphi_{\mu_j}(x_{j,k}) \right\rangle \leq \frac{\delta}{\bar{\gamma}\theta^l} \|\tilde{x}^l_{j,k} - x_{j,k}\|^2_{A_{j,k}}$$

If $f := 0$, Armijo linesearch along the steepest direction.

Stopping Criteria

Backtracking **[Salzo, 2017]**

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If $f := 0$, Armijo linesearch along the steepest direction.

Accuracy for Solving $\mathcal{P}_{\mu_j}$

The barrier parameter is decreased as soon as the following criteria are met :

$$\|x_{j,k} - x_{j,k+1}\| \leq \epsilon_{1,j} \qquad \frac{1}{\gamma_{j,k}}\|A_{j,k}(x_{j,k} - x_{j,k+1})\| \leq \epsilon_{2,j}$$

$$\sum_{i=1}^{p}\left|\frac{c_i(x_{j,k+1})}{c_i(x_{j,k})} - 1\right| \leq \epsilon_{3,j} \qquad \mu_j\left\|\sum_{i=1}^{p}\frac{\nabla c_i(x_{j,k}) - \nabla c_i(x_{j,k+1})}{c_i(x_{j,k})}\right\| \leq \epsilon_{4,j}$$

where $\{(\epsilon_{i,j})_{j\in\mathbb{N}}\}_{i\in\{1,\ldots,4\}}$ and $(\mu_j)_{j\in\mathbb{N}}$ are strictly positive sequences converging to 0 such that $(\forall i \in \{1,\ldots,4\})\ \lim_{j\to\infty} \epsilon_{i,j}/\mu_j = 0$.

Tree

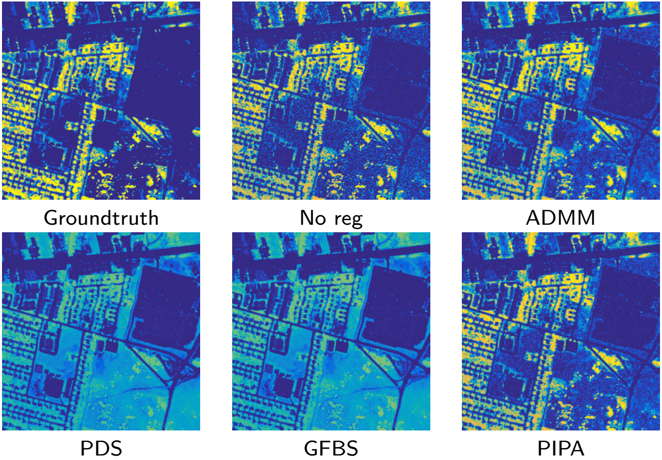
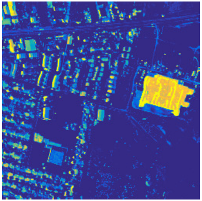


FIGURE – Abundance map of tree after 13 sec.

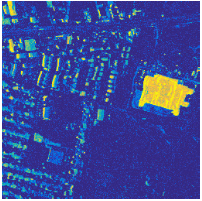
No reg	11.86
ADMM	12.56
PDS	4.73
GFBS	4.76
PIPA	**12.73**

TABLE – SNR (dB)

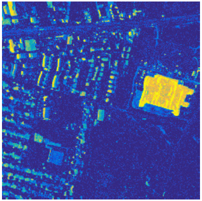
Roof



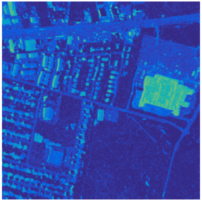
Groundtruth



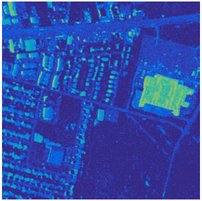
No reg



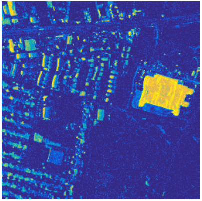
ADMM



PDS



GFBS



PIPA

FIGURE – Abundance map of roof after 13 sec.

No reg	14.91
ADMM	14.66
PDS	6.63
GFBS	7.66
PIPA	**15.19**

TABLE – SNR (dB)

Metal

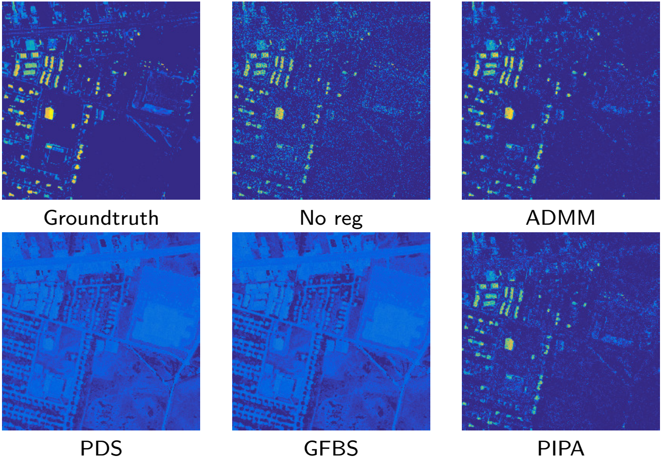


FIGURE – Abundance map of metal after 13 sec.

No reg	4.90
ADMM	**7.57**
PDS	-0.08
GFBS	0.05
PIPA	7.06

TABLE – SNR (dB)