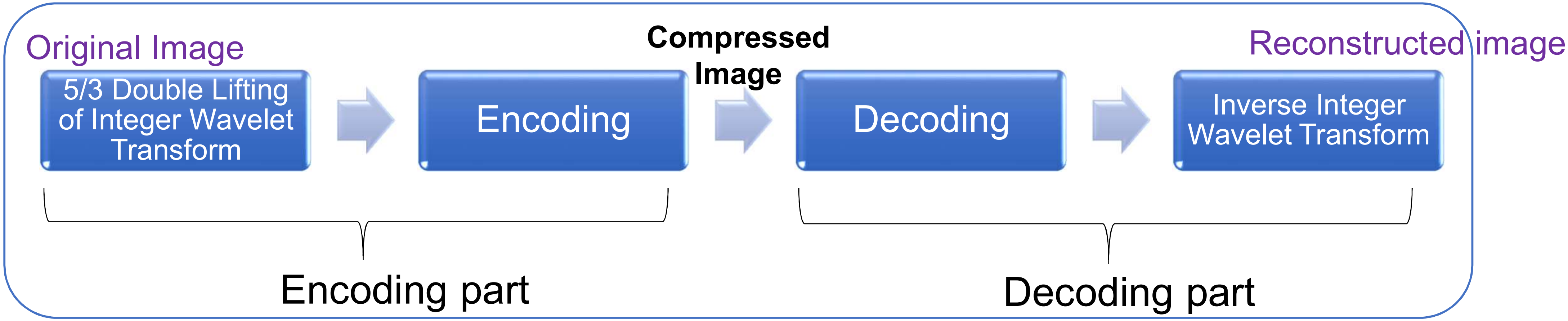
NON-SEPARABLE QUADRUPLE LIFTING STRUCTURE FOR FOUR-DIMENSIONAL INTEGER WAVELET TRANSFORM WITH REDUCED ROUNDING NOISE

Fairoza Amira Binti Hamzah*,
Taichi Yoshida*,
Masahiro Iwahashi*
Nagaoka University of Technology, Japan

Two types of data compression

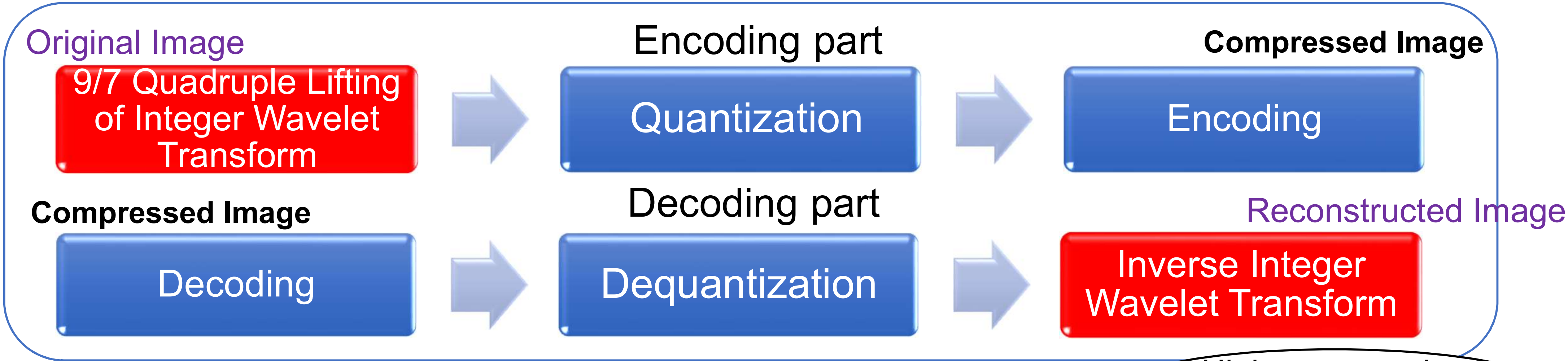
Lossless Data Compression in JPEG 2000



Reconstructed image = Original Image

Low compression ratio

Lossy Data Compression in JPEG 2000

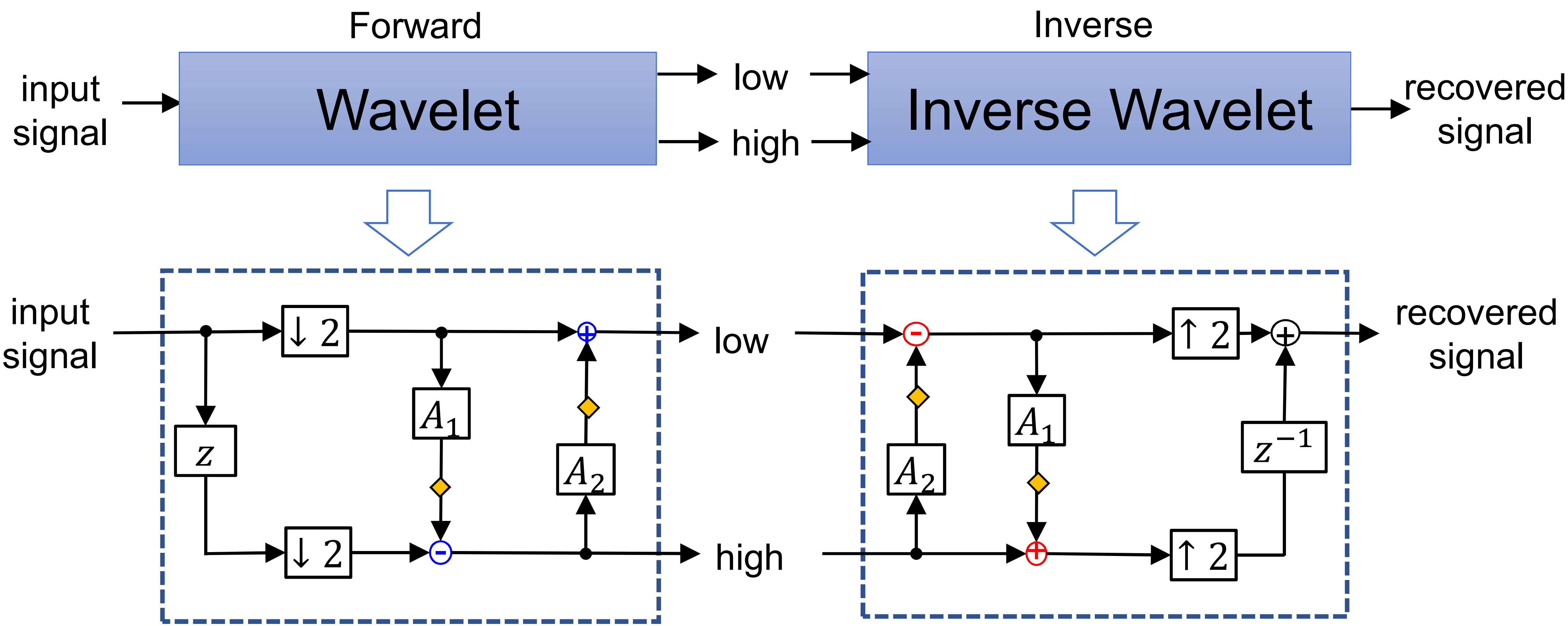


Reconstructed image ≠ Original Image

High compression ratio

Rounding Noise in Lifting Structure of Integer Wavelet Transform

(example in double type)



"◇" denotes "rounding" operator

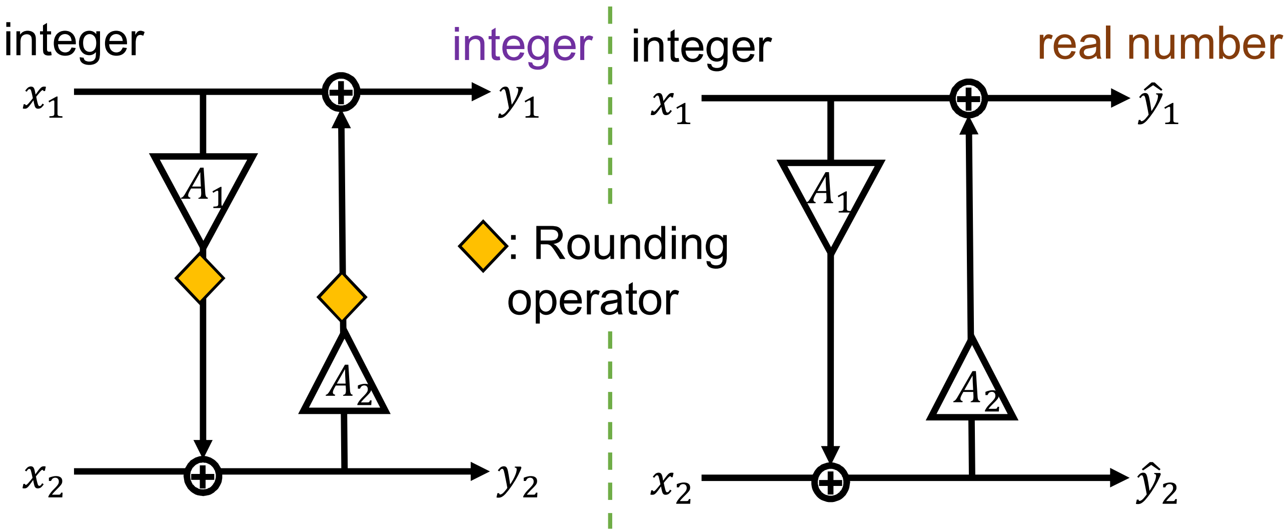
z : delay

$\downarrow 2$: downsampling by 2

A_1, A_2: coefficients of filter bank

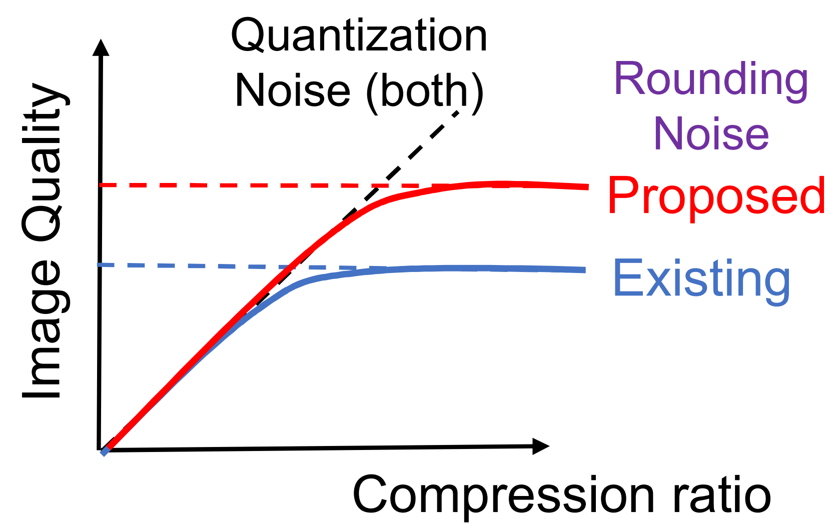
Rounding operators are reduced in this research

Rounding Noise



Rounding noise $= y - \hat{y}$

Evaluation of Coding Performance



Effect

- The lower the rounding noise, the higher the compression performance

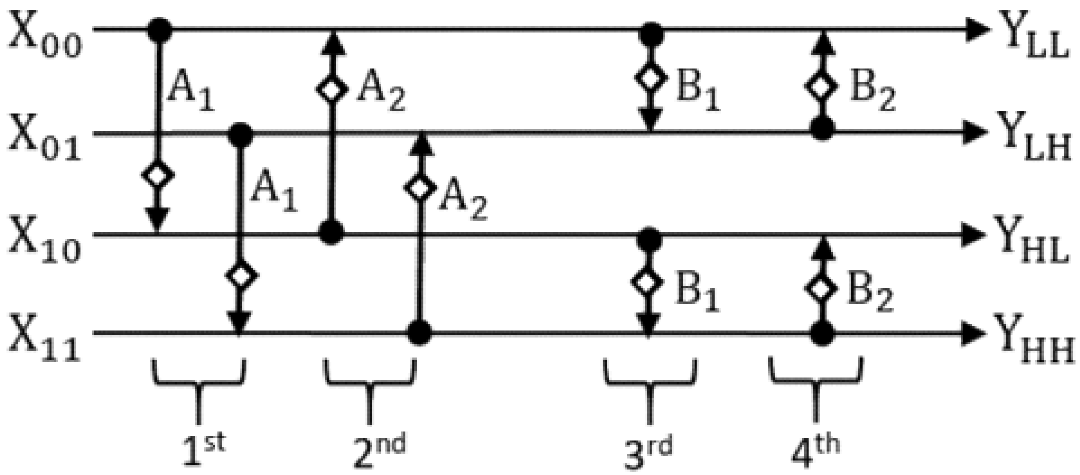
How?

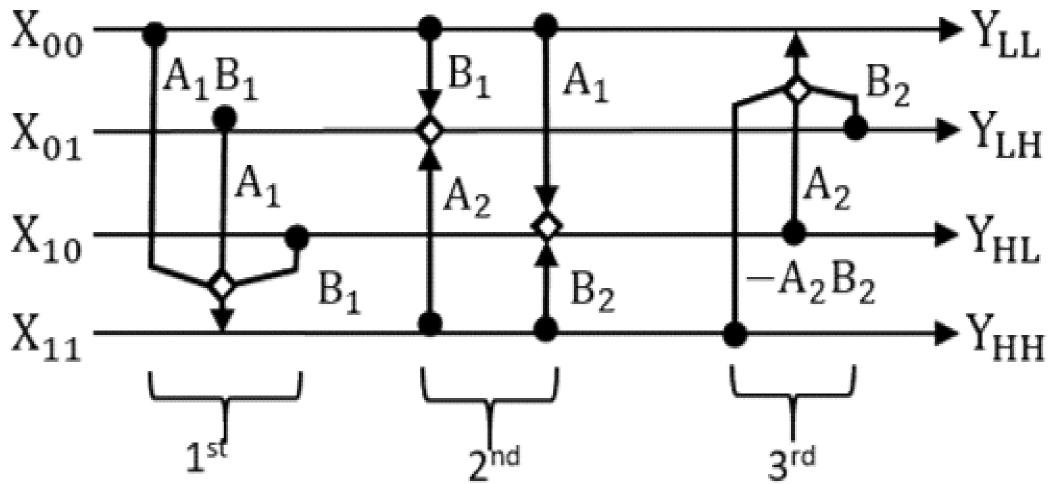
- By introducing the '**non-separable**' structure

Two Lifting Structure

(example in 2D for double type)

Separable	**Non-separable**





◇ : rounding operator

$$A_1 A_2 B_1 B_2$$

x-dimension | *y-dimension*

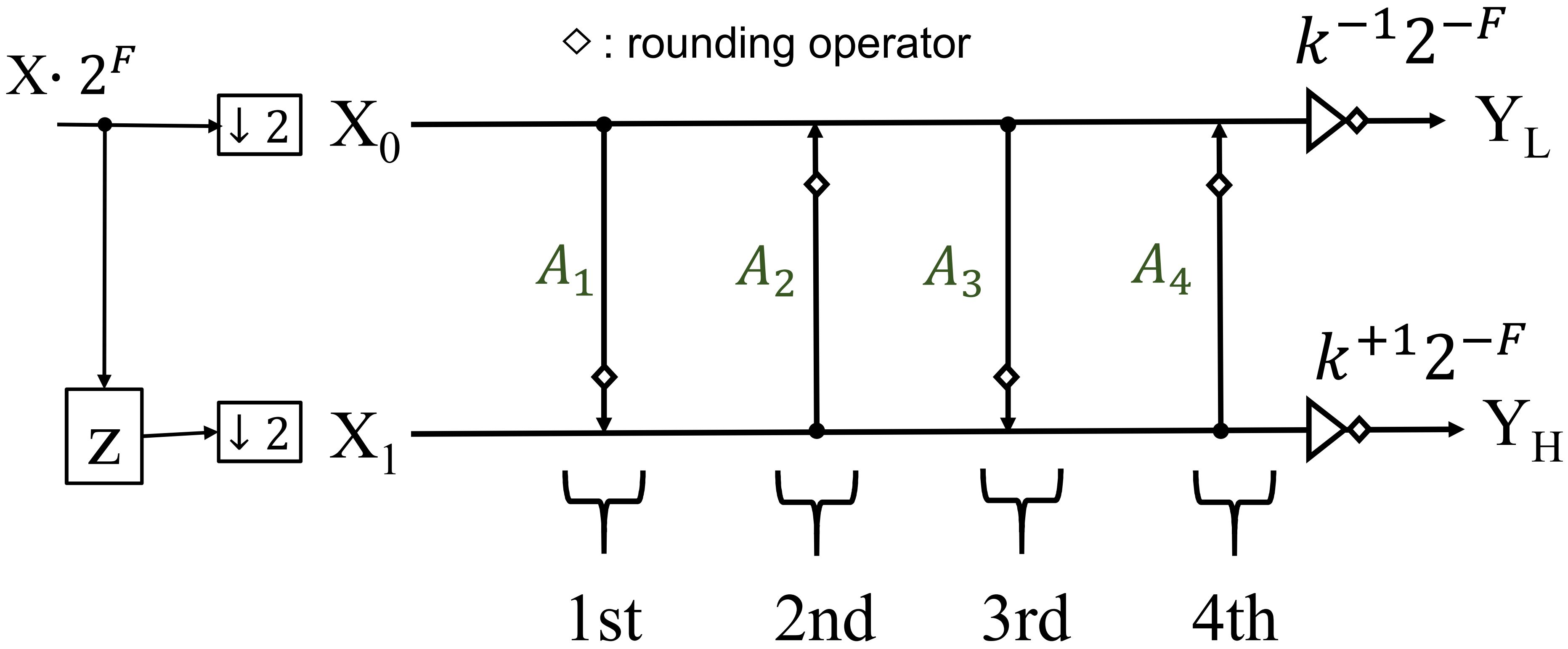
$$(A_1 A_2 B_1 B_2)_{2D}$$

x & y-dimension (simultaneously)

	Separable	Non-separable
Rounding Operators	8	4

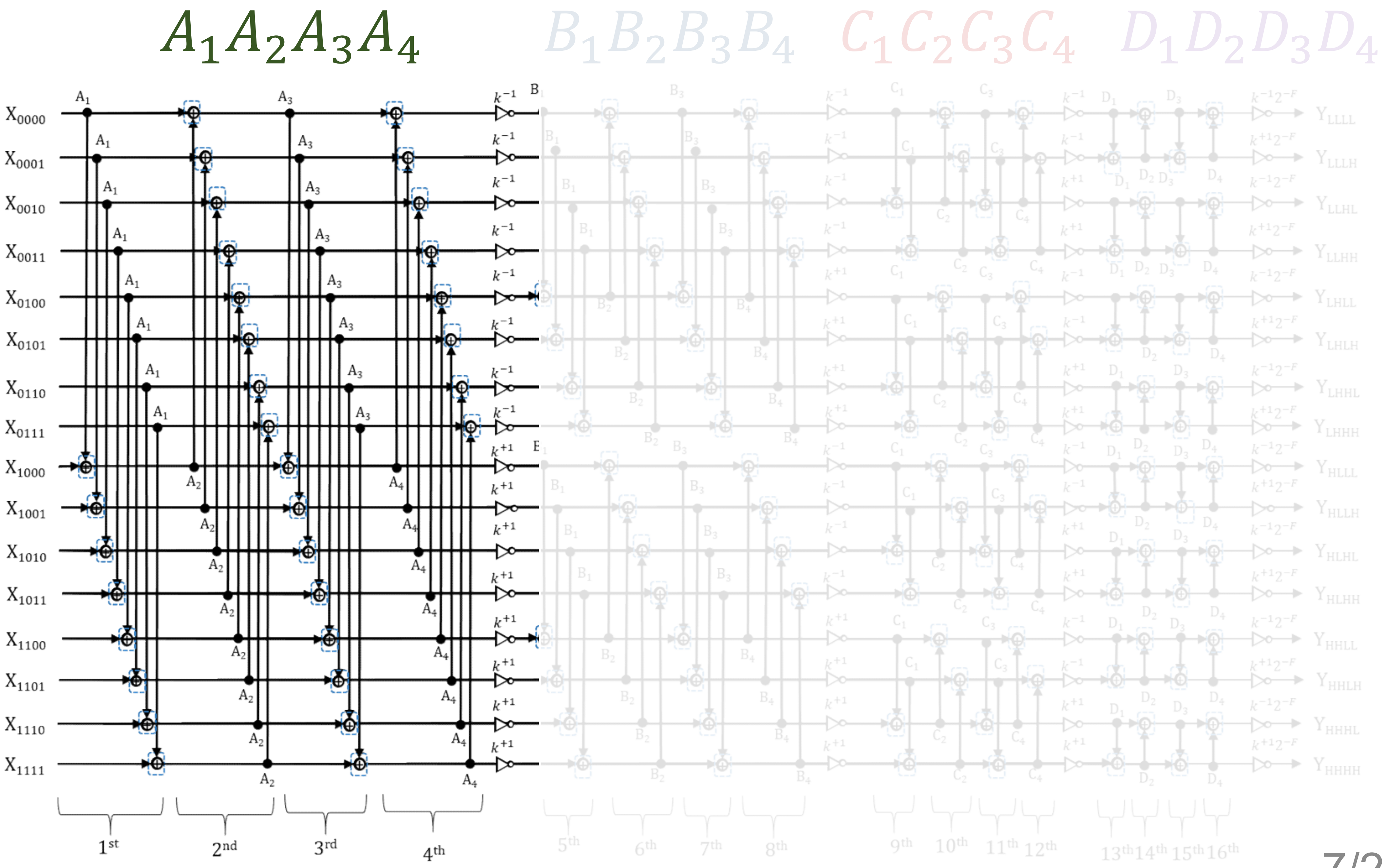
Reduced!

Quadruple 1D IWT $A_1 A_2 A_3 A_4$



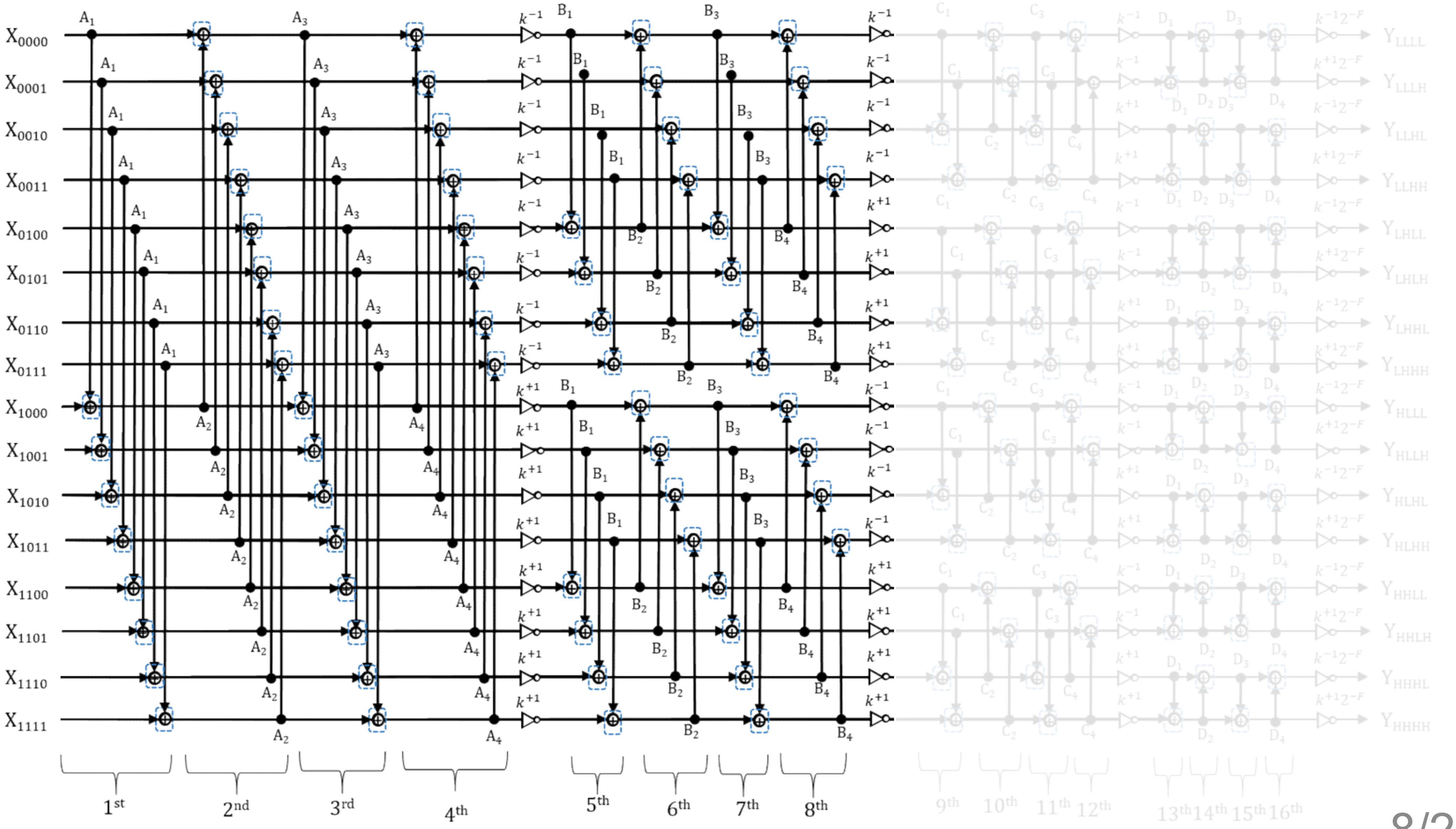
z : delay
↓ 2 : downsampling by 2
A_1, A_2, A_3, A_4: coefficients of filter bank

Quadruple Separable 4D



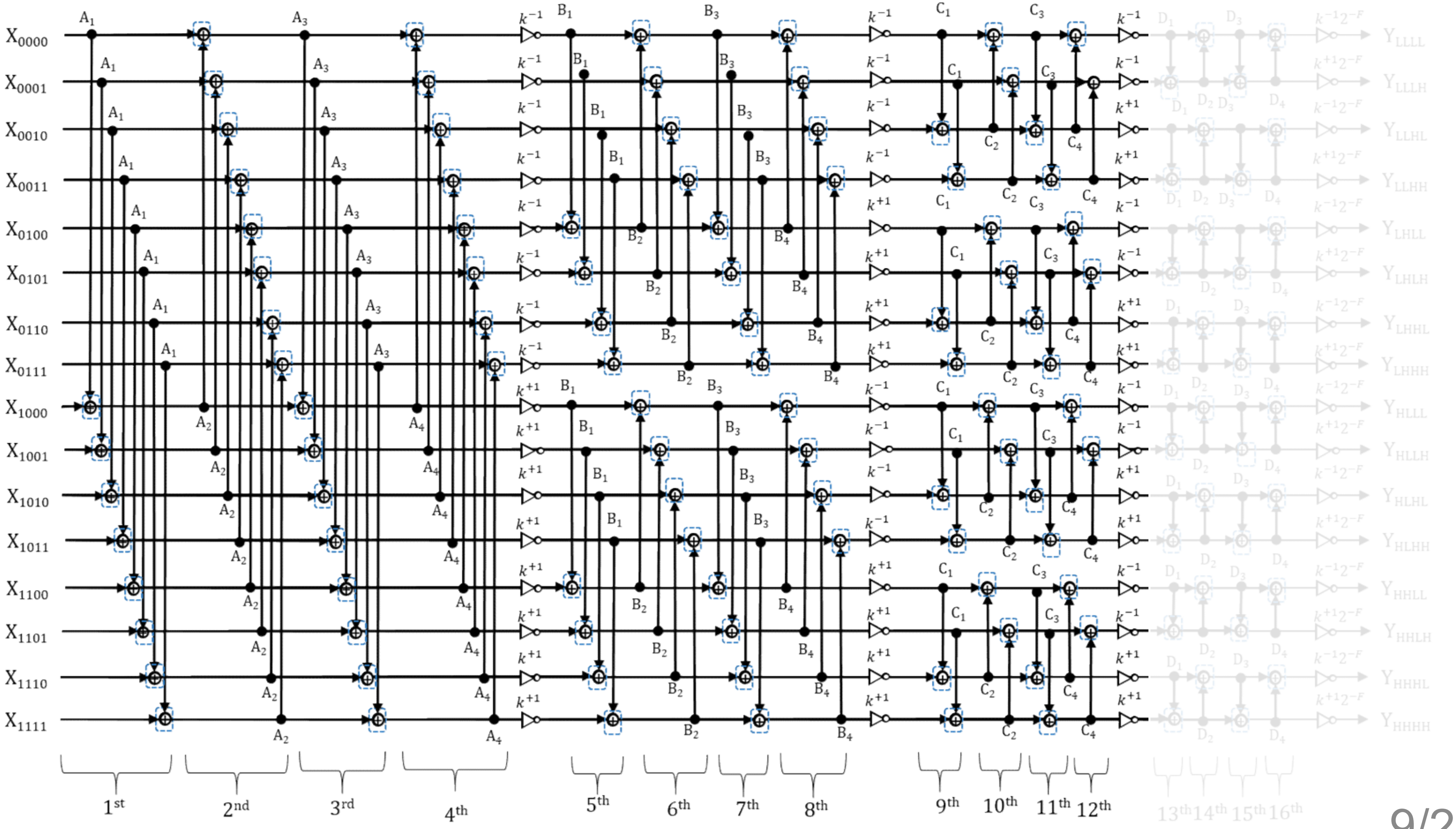
Quadruple Separable 4D

$A_1A_2A_3A_4$ $B_1B_2B_3B_4$ $C_1C_2C_3C_4$ $D_1D_2D_3D_4$



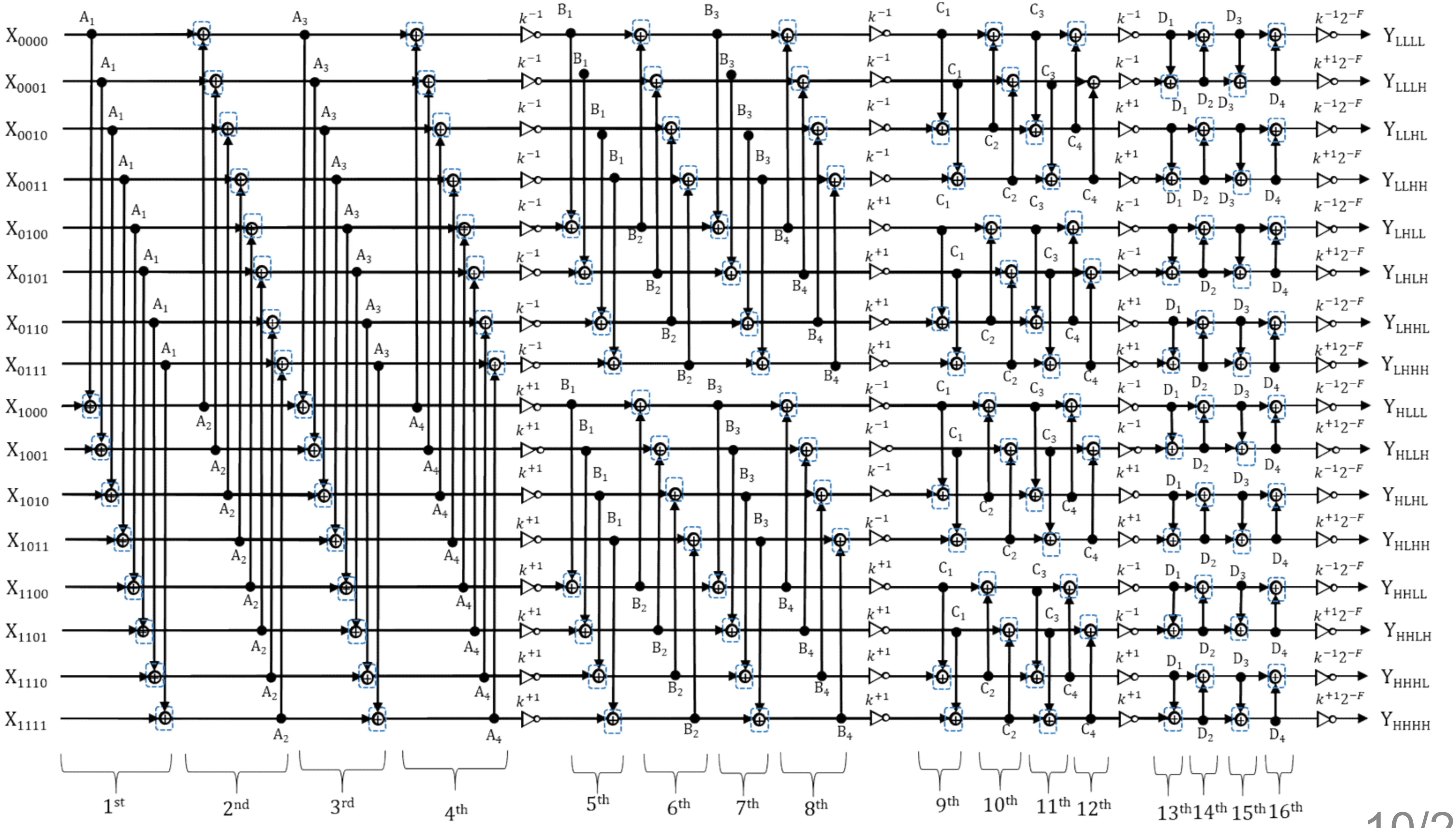
Quadruple Separable 4D

$A_1 A_2 A_3 A_4$ $B_1 B_2 B_3 B_4$ $C_1 C_2 C_3 C_4$ $D_1 D_2 D_3 D_4$



Quadruple Separable 4D

$A_1 A_2 A_3 A_4$ $B_1 B_2 B_3 B_4$ $C_1 C_2 C_3 C_4$ $D_1 D_2 D_3 D_4$



Possible combinations of the structures

x-dimension	y-dimension	z-dimension	t-dimension

2D

$A_1A_2A_3A_4$ $B_1B_2B_3B_4$

$A_1A_2(A_3A_4B_1B_2)_{2D}B_3B_4$

$(A_1A_2B_1B_2)_{2D}$ $(A_3A_4B_3B_4)_{2D}$

⋮

24 Possible Combinations

3D

$A_1A_2A_3A_4$ $B_1B_2B_3B_4$ $C_1C_2C_3C_4$

$(A_1A_2B_1B_2C_1C_2)_{3D}(A_3A_4B_3B_4C_3C_4)_{3D}$

$A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}C_3C_4$

⋮

40320 Possible Combinations

4D

$A_1A_2A_3A_4$ $B_1B_2B_3B_4$ $C_1C_2C_3C_4$ $D_1D_2D_3D_4$

$A_1A_2A_3A_4(B_1B_2C_1C_2D_1D_2)_{3D}(B_3B_4C_3C_4D_3D_4)_{3D}$

$A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$

⋮

2.092×10^{13} Possible Combinations

How to find the best structure from the 2.092×10^{13} structures?

By maintaining the original lifting structure, which is

$$A_1 A_2 A_3 A_4 B_1 B_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$$

and turn it into the non-separable structure

$$A_1 A_2 (A_3 A_4 B_1 B_2)_{2D} (B_3 B_4 C_1 C_2)_{2D} (C_3 C_4 D_1 D_2)_{2D} D_3 D_4$$

Comparison of the structures

4D DWT (9,7) Quadruple Lifting Structure:

Separable 4D (Existing I):

$$A_1A_2A_3A_4B_1B_2B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4$$

Non-separable 1D and 3D for 4D (Existing II):

$$A_1A_2A_3A_4(B_1B_2C_1C_2D_1D_2)_{3D}(B_3B_4C_3C_4D_3D_4)_{3D}$$

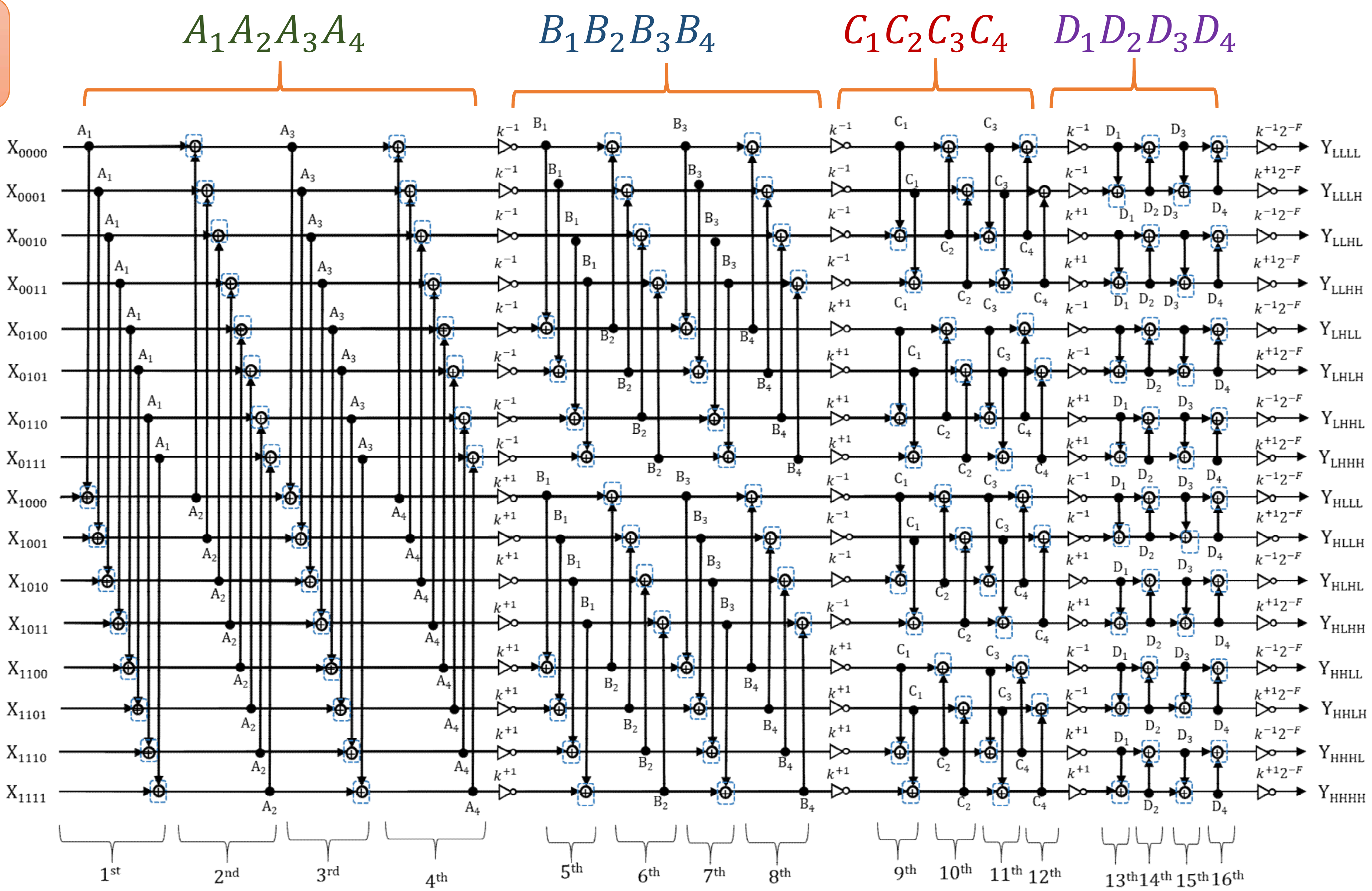
Non-separable 1D and 2D for 4D (Proposed):

$$A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$$

Structure	Rounding Operators
Separable 4D (Existing I)	192
Non-separable 3D (Existing II)	96
Non-separable 2D (Proposed)	96

Separable 4D (Existing I)

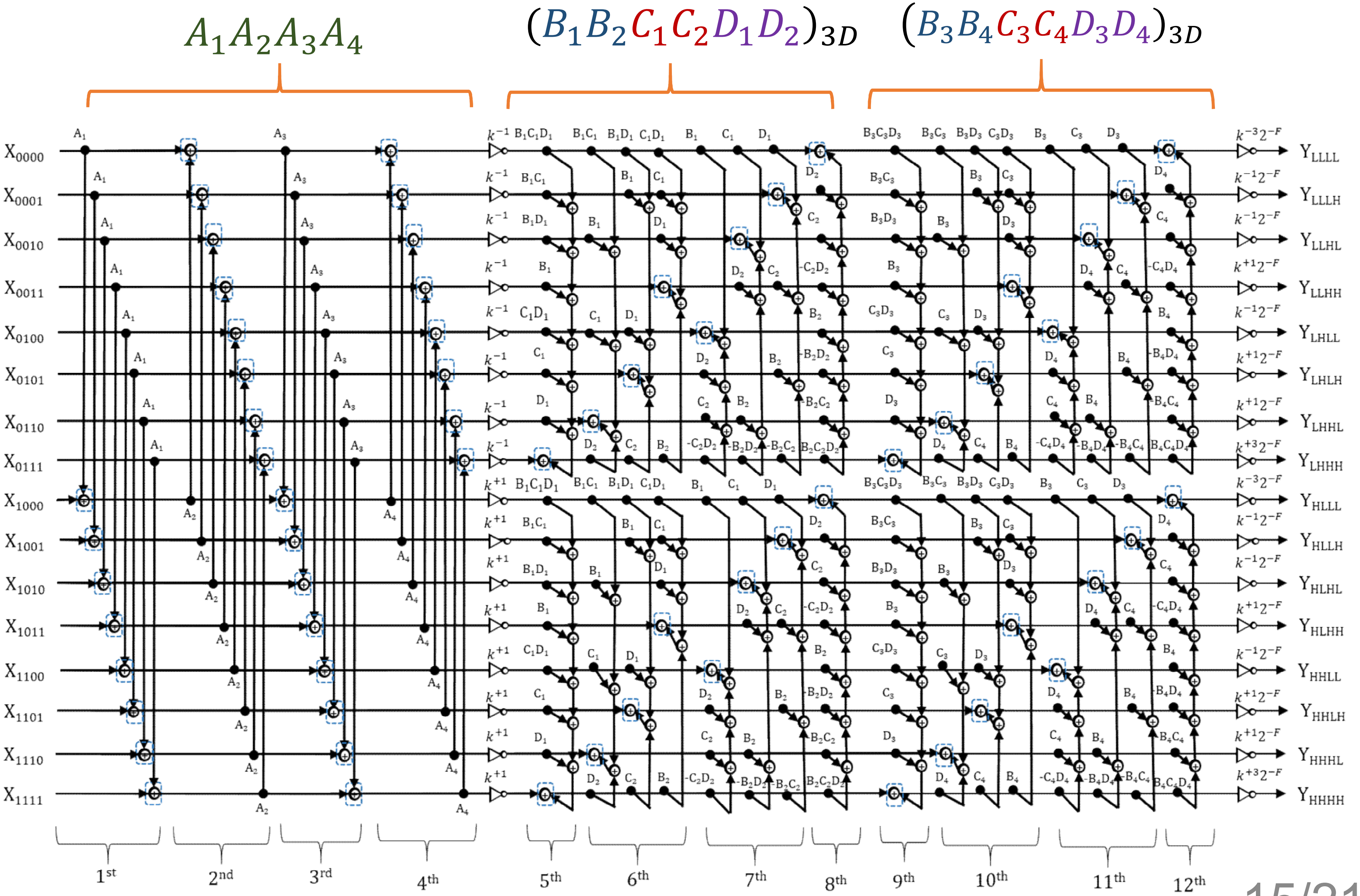
Cascade of 1D



192 rounding operators

4D Wavelet composed of 1D and
Non-separable 3D (Existing II)

Cascade of 1D and two NonSep 3Ds

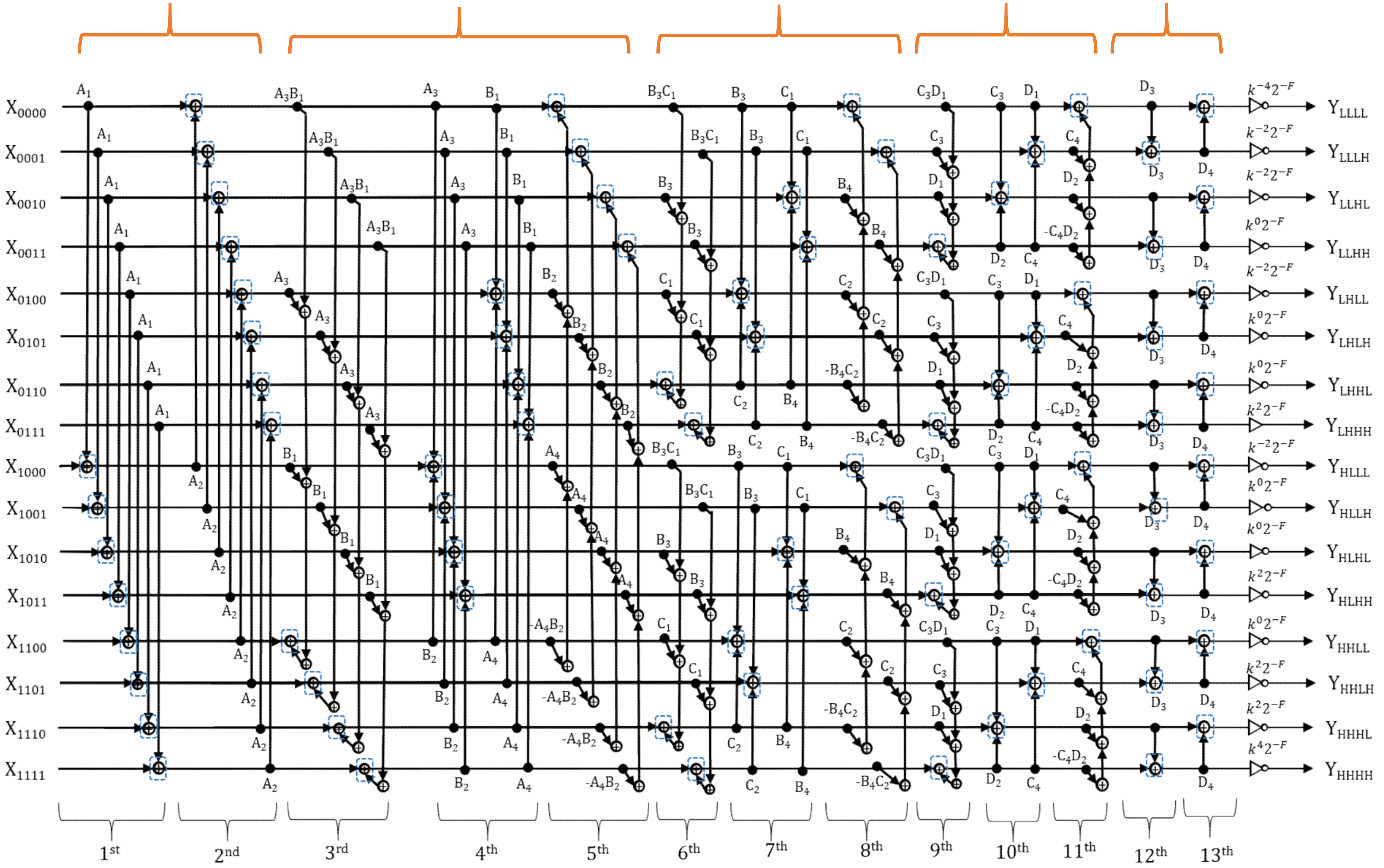


96 rounding operators

4D Wavelet Composed of 1D and Non-separable 2D (Proposed)

Cascade of 1D and three NonSep 2Ds

$$A_1A_2 \quad (A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$$

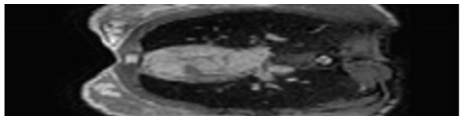


96 rounding operators

Input Data

MRI data

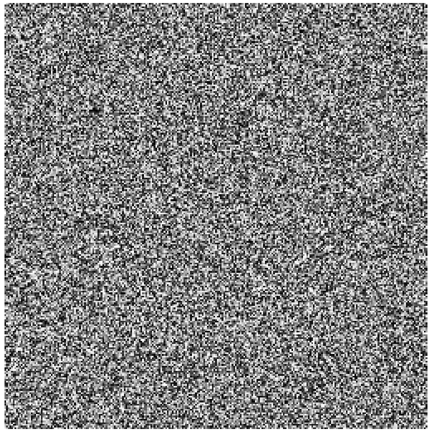
Size: 50 x 224 x 224 x 16



Example

Random signal

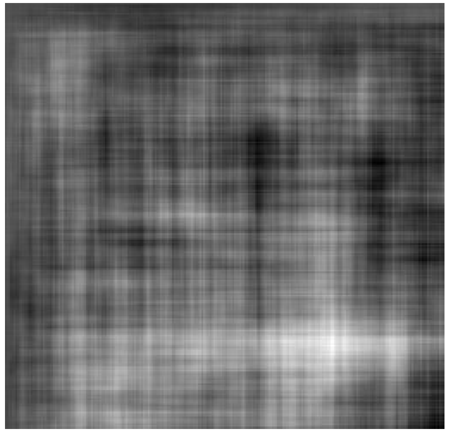
Size: 128 x 128 x 32 x16



Example

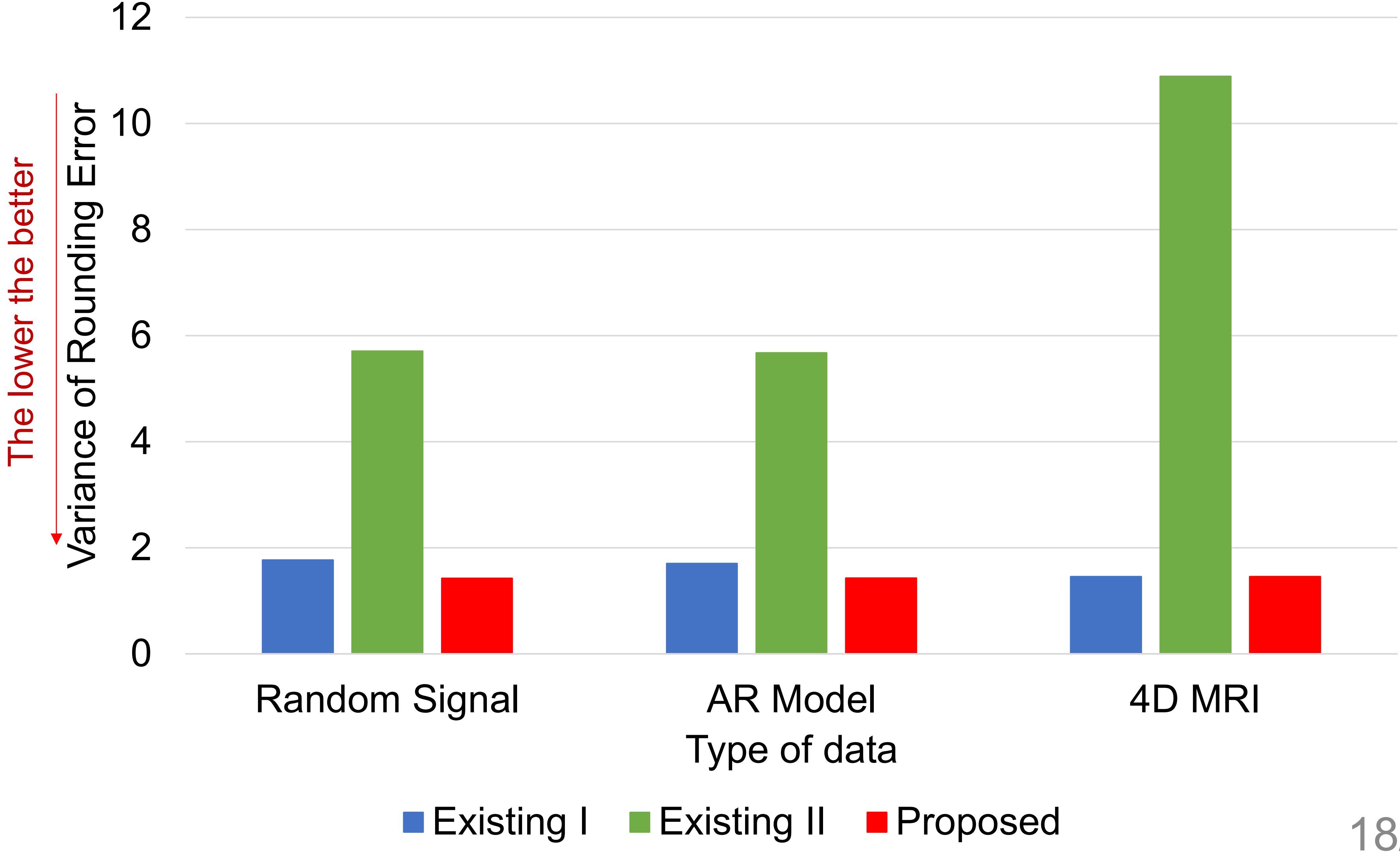
Auto Regressive Model (AR)

Size: 256 x 256 x 32 x 16

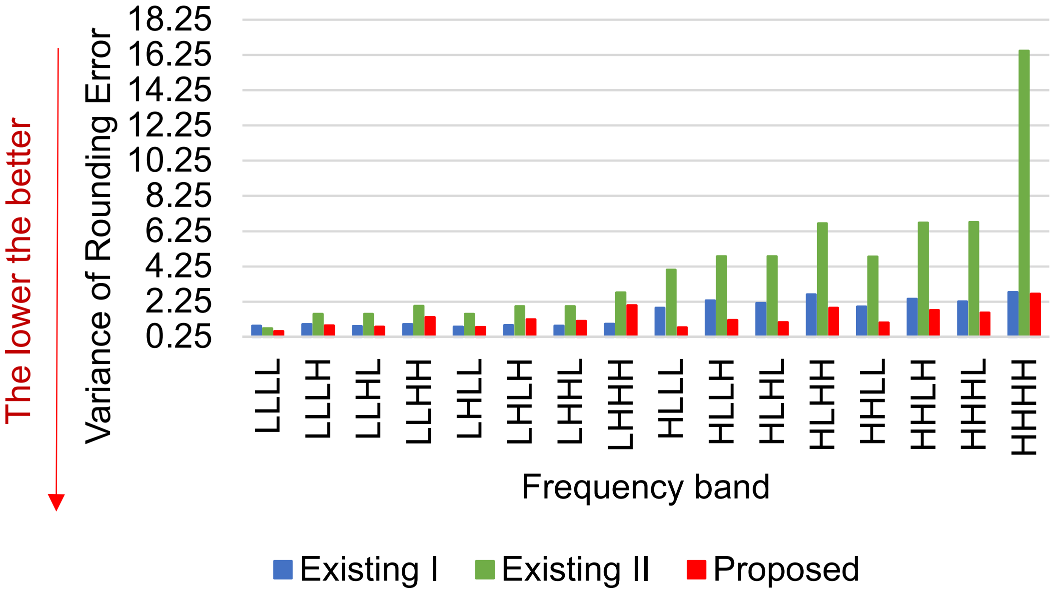


Example

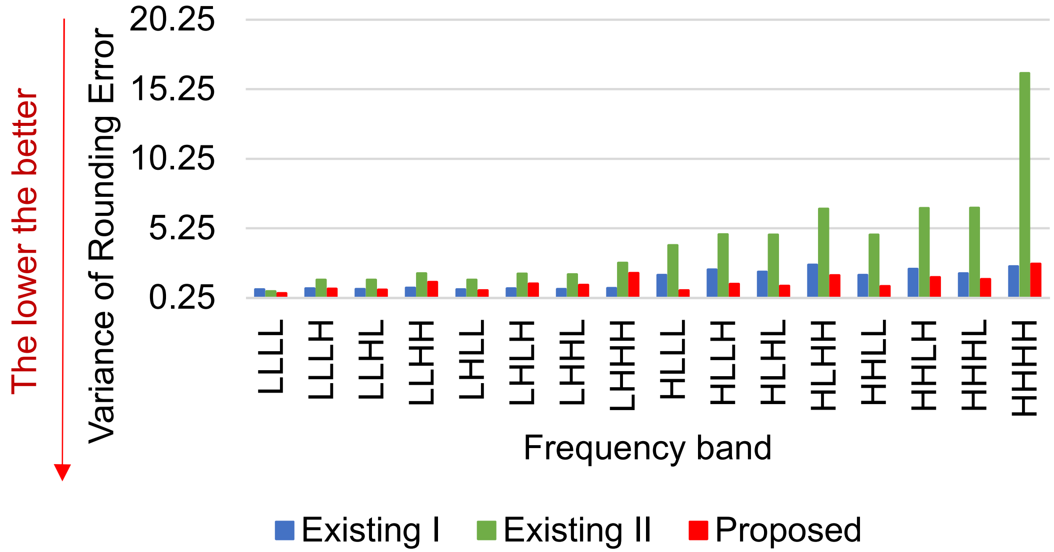
Evaluation: Average variance of rounding noise in each frequency band



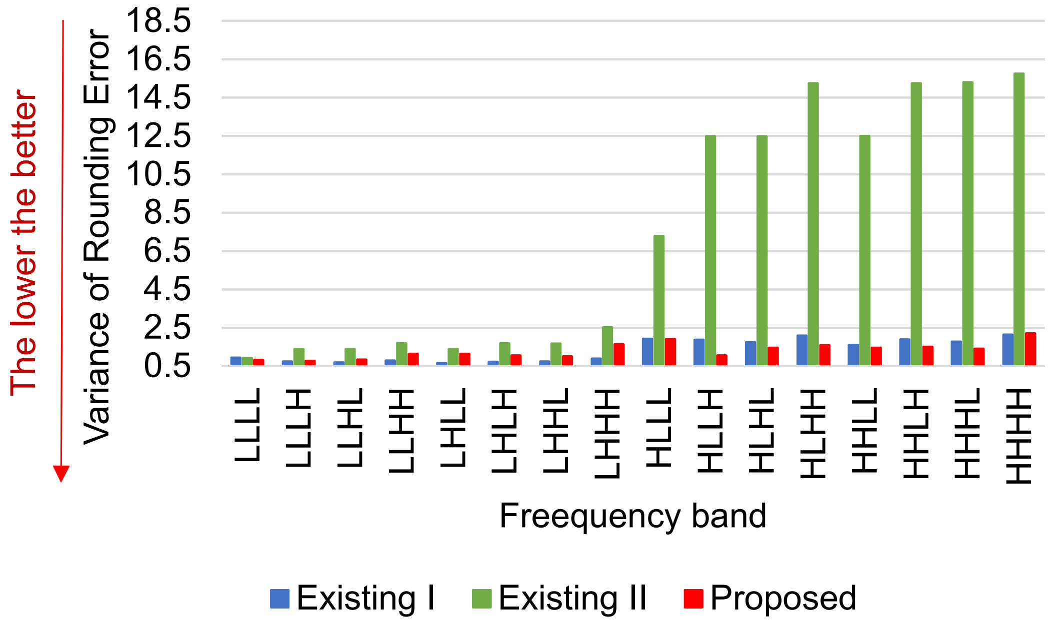
Evaluation: Rounding noise in each frequency band



(a) 4D Random Signal

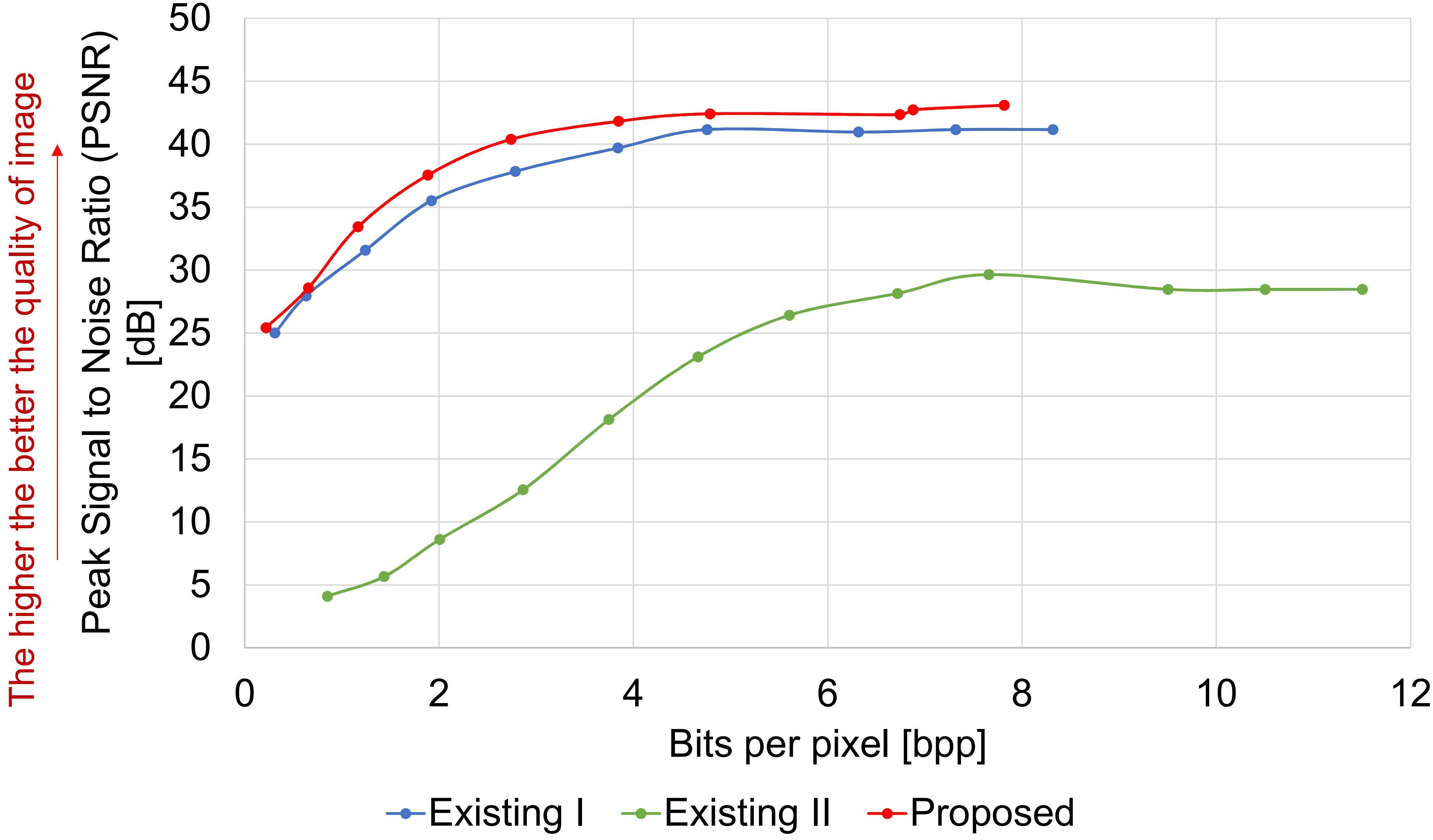


(b) 4D AR Model



(c) 4D MRI

Evaluation: Coding performance in lossy mode



Test data: 4D AR Model

Conclusion

Structure	Rounding Noise	Coding Performance
Separable 4D (Existing I)	☆☆	☆☆
Non-separable 3D (Existing II)	☆	☆
Non-separable 2D (Proposed)	☆☆☆	☆☆☆

Between the Existing I and Proposed methods,

Number of rounding operators: 50 [%]↓

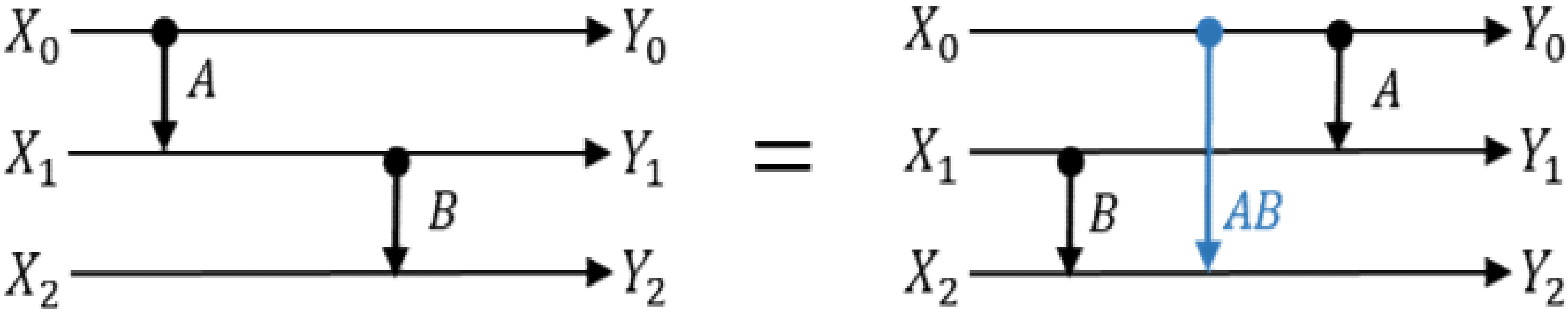
Variance of rounding noise: 16.09 [%] ↓

Coding performance: 2.57[dB]↑

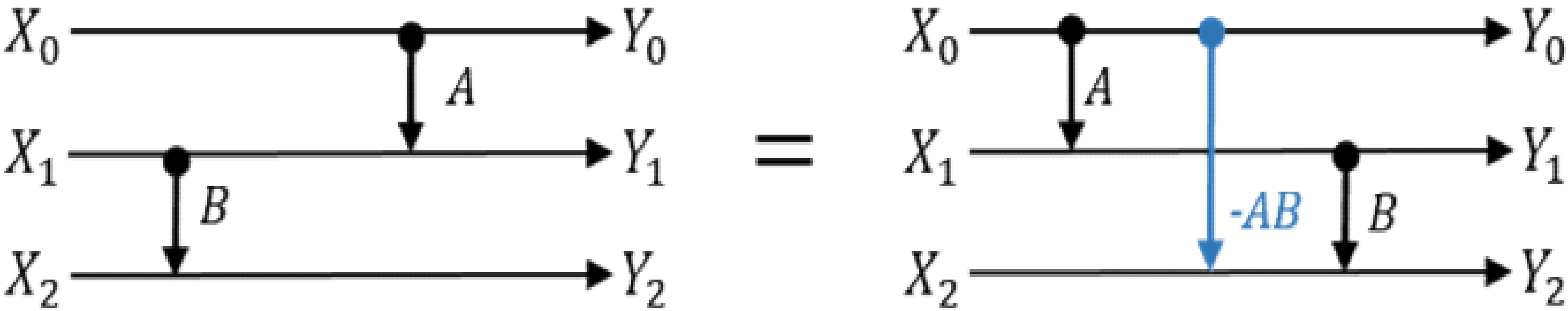
APPENDIX

How to derive the non-separable?

Derivation of the ‘non-separable’ structure: Basic properties for modification



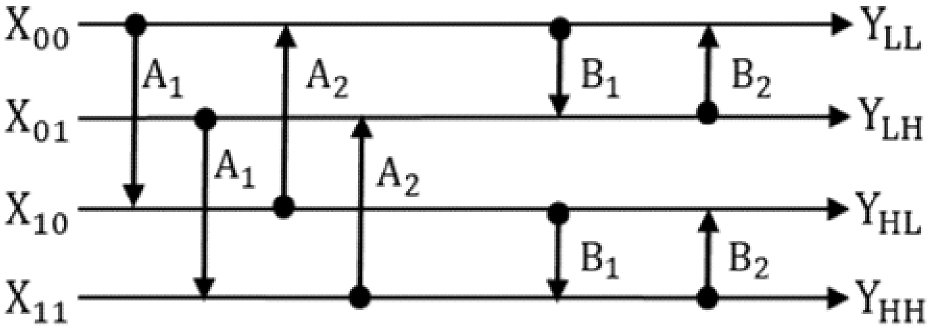
Property I



Property II

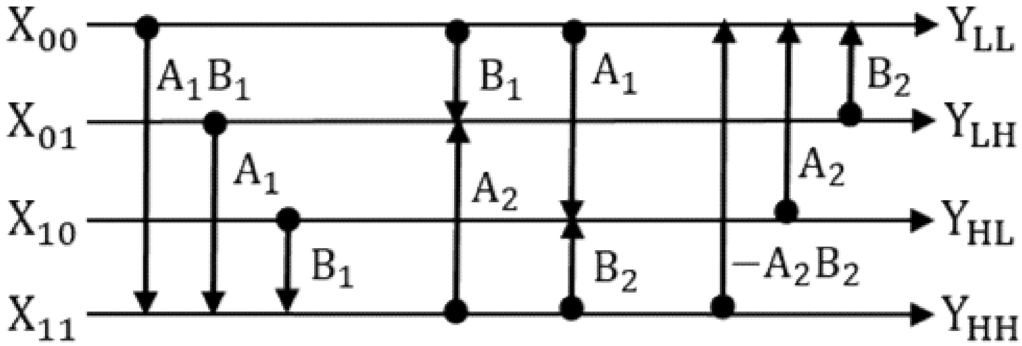
Derivation of the ‘non-separable’ structure: Derivation process for 2D

$A_1A_2B_1B_2$

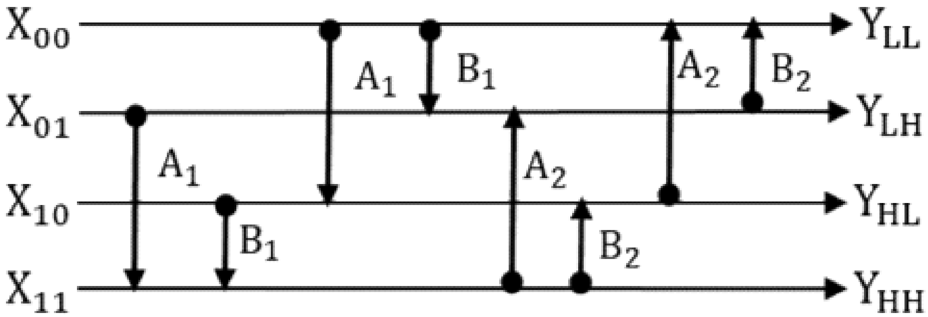


‘Separable’ 2D

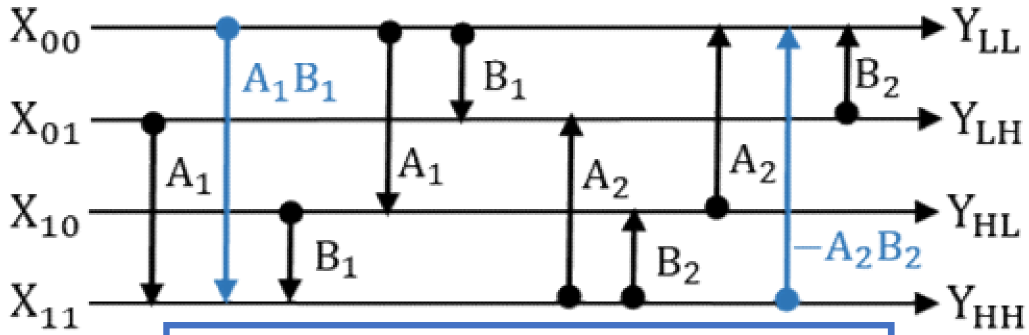
$(A_1A_2B_1B_2)_{2D}$



‘Non-separable’ 2D



Rearranging



Applying property I and II

Same process is applied to 3D and 4D

How to find the best structure from the 2.092×10^{13} structures?

Create six **"Rules"**
to exclude Unnecessary structures

↓

focus on **only 7** candidates

↓

find the **Best** structure
which has the minimum rounding noise

The six "Rules"

$A_1 A_2 A_3 A_4 B_1 B_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$ — Original Separable 4D

Excluded — Reasons

1. $B_1 B_2 A_3 A_4 A_1 A_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$ — Same rounding noise
2. $A_2 A_1 A_3 A_4 B_1 B_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$ — Not (9,7) quadruple wavelet
3. $A_1 A_2\ B_1 B_2\ C_1 C_2\ D_1 D_2\ A_3 A_4\ B_3 B_4\ C_3 C_4\ D_3 D_4$ — Large rounding noise

Permitted

4. $(A_1 A_2)(A_3 A_4)(B_1 B_2)(B_3 B_4)(C_1 C_2)(C_3 C_4)(D_1 D_2)(D_3 D_4)$
5. $A_1 A_2 (A_3 A_4 B_1 B_2)_{2D} B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$
6. $A_1 A_2 (A_3 A_4 B_1 B_2 C_1 C_2)_{3D} B_3 B_4 C_3 C_4 D_1 D_2 D_3 D_4$

7 Possible Structures

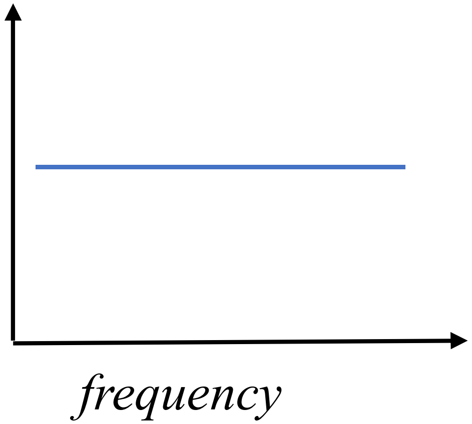
$A_1A_2A_3A_4B_1B_2B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4$ ← Original

1. $A_1A_2(A_3A_4B_1B_2)_{2D}B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4$
2. $A_1A_2A_3A_4B_1B_2(B_3B_4C_1C_2)_{2D}C_3C_4D_1D_2D_3D_4$
3. $A_1A_2A_3A_4B_1B_2B_3B_4C_1C_2(C_3C_4D_1D_2)_{2D}D_3D_4$
4. $A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}C_3C_4D_1D_2D_3D_4$
5. $A_1A_2A_3A_4B_1B_2(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$
6. $A_1A_2(A_3A_4B_1B_2)_{2D}B_3B_4C_1C_2(C_3C_4D_1D_2)_{2D}D_3D_4$
7. $A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$ ← Proposed

The rounding noise is experimentally investigated

Autoregressive model

AR model is created to make spectrum random input to become the same as image spectrum

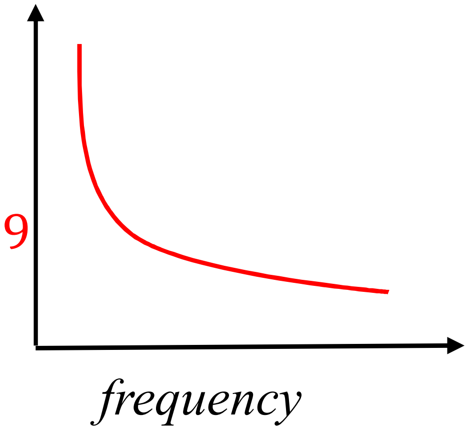


W(z) : White noise

AR Model

$$\frac{W(z)}{1-\rho z^{-1}} = H(z)$$

$H(z)$:
$\rho = 0.99$

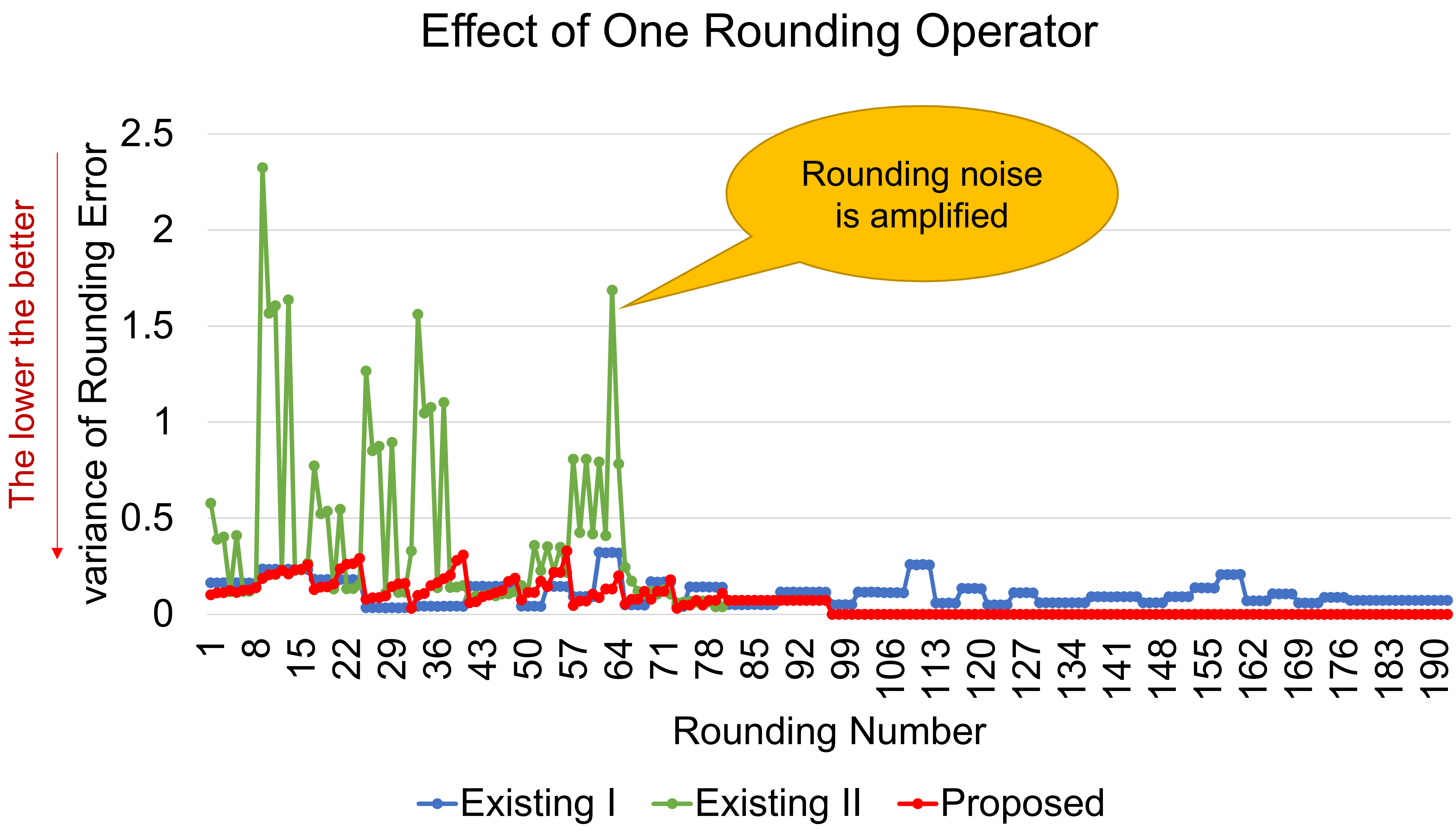


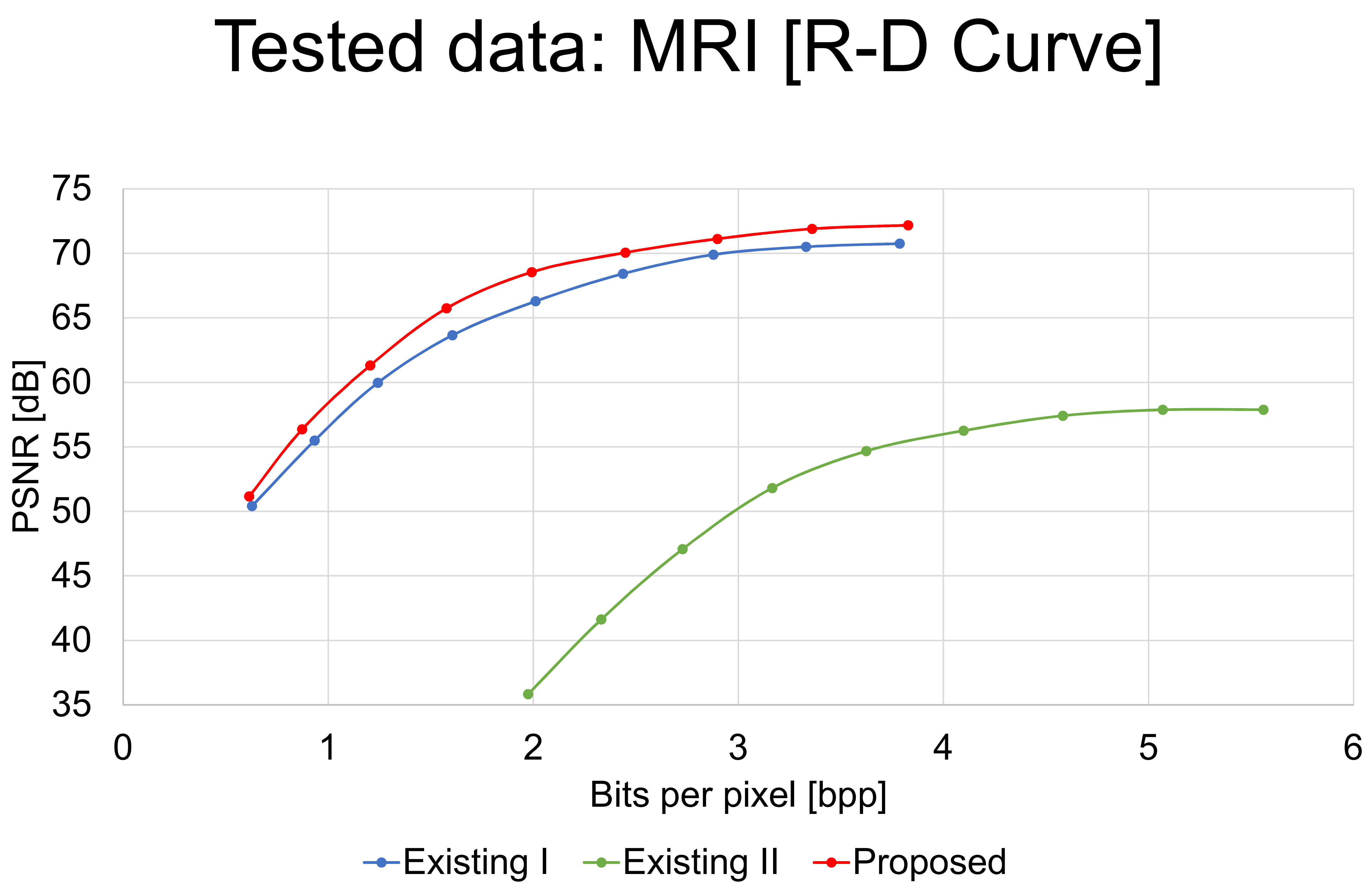
ρ=0

ρ=0.9

AR Model Spectrum = Image Spectrum

Analysis on Experimental Results





Tested data: MRI [R-D Curve]
75
70
65
60
55
50
45
40
35
PSNR [dB]
0
1
2
3
4
5
6
Bits per pixel [bpp]
Existing I
Existing II
Proposed

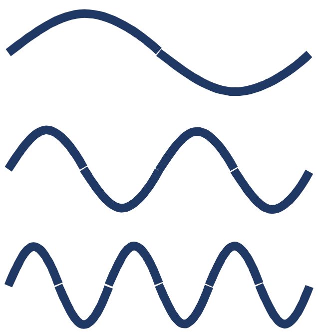
Why Wavelet?

International Standard of Image Compression

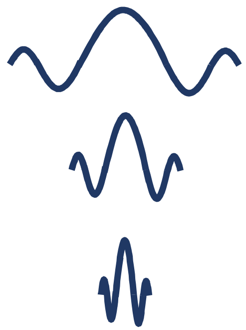
JPEG
- Discrete Cosine Transform (DCT)

JPEG2000
- Discrete Wavelet Transform (DWT)

All basis have the same length



Long in low frequency, Short in high frequency



- long gradation

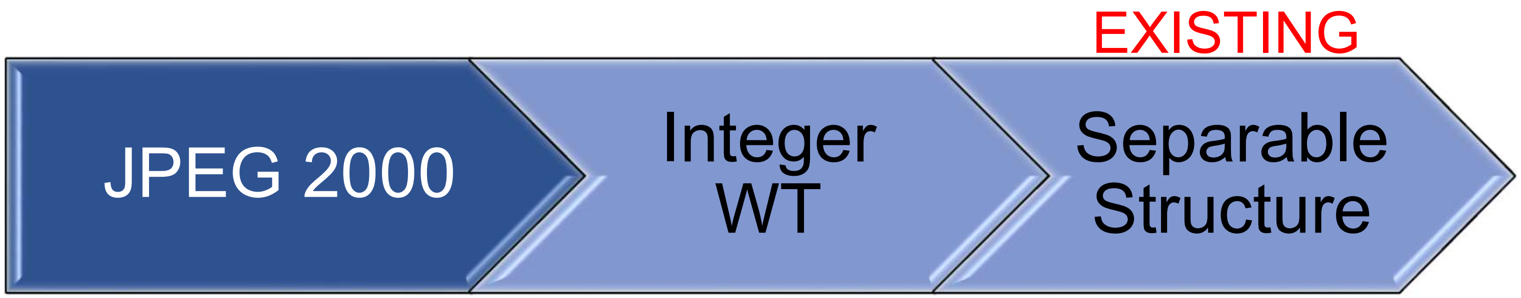
- small edges

Data can be compressed

[M. Vetterli and C. Herley, "Wavelet and Filter Banks: Theory and Design", IEEE Trans. On Signal Processing, Vol. 40, No. 9, 1992]

Existing 4D Data Compression Methods

- fMRI Image
  - Method: Motion Compensation *[V. Sanchez et al, 2009]*
- fMRI Image
  - Method: JPEG 2000 *[H. G. Lalgudi et al, 2005]*
- 4D Remote Sensing
  - Method: JPEG 2000 *[J. M. Gomez et al, 2010]*
- 4D Geometry
  - Method: Lifting Wavelet Transform *[Y. Wang et al, 2006]*



Proposal

		Quadruple (9,7)
2D	Separable	JPEG 2000
	Non-separable	ICIP ’09
3D	Separable	JPEG 2000
	Non-separable	APSIPA ’13
4D	Separable	JPEG 2000
	Non-separable	ICASSP ‘17

- **[ICIP '09]** M. Iwahashi, H. Kiya, "Non Separable 2D Factorization of Separable 2D DWT for Lossless Image Coding," *IEEE Proc. International Conference on Image Processing (ICIP),* pp.17-20, Nov. 2009.
- **[APSIPA ‘13]** M. Iwahashi, T. Orachon, H. Kiya, “Non Separable 3D Lifting Structure Compatible with Separable Quadruple Lifting DWT”, Asia-Pacific Signal and Information Processing Association 2013 Annual Summit and Conference (APSIPA ASC), OS.26, IVM.11, no.4, pp.1-4, Oct. 2013.

Implementation of Integer Wavelet Transform in JPEG 2000 with Lifting Structure

Advantage	• Perfect reconstruction • Signal is recovered without loss
Problem	• Rounding noise existed in the lifting structure
Type	• Lossless => Double (5,3) • Lossy => Quadruple (9,7)

Data trends: The increased of data dimensions

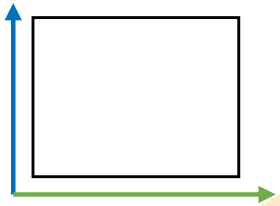
x-dimension (spatial)
y-dimension (spatial)
z-dimension (spatial)
t-dimension (temporal)

Large File Size

Compression is necessary

1D

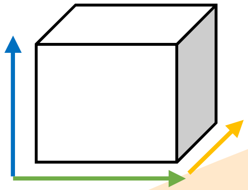
- Text
- Sound



2D

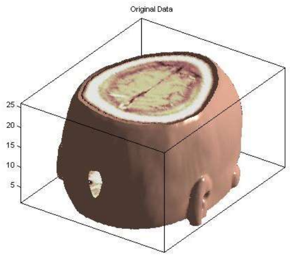
- Still Image

LENA still image

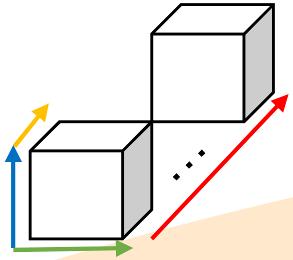


3D

- Video
- Medical Image
- Light Field Image
- Hyperspectral Image

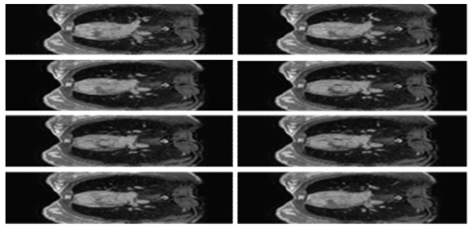


MRI Image



4D

- Medical Image
- 3D stereo video
- Hyperspectral image



fMRI image