Practical Optimization Algorithms for Image Processing

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Introduction and Problem

Several problems in image processing can be casted as following optimization problem:

$$x^* \leftarrow \operatorname*{argmin}_{x\in\mathbb{R}^n} f(x) + g(x), \text{ where } g(x) = r(x) + i_\Omega(x)$$

f: convex and smooth, r: convex but nonsmooth, i_Ω: indicator function for convex set Ω

Augmented Lagrangian Approach

- Introduce variable splitting, $z = x$, and write the problem as:

$$\operatorname*{argmin}_{x,z} \{f(x) + g(z)\}, \text{ such that } x = z$$

- Write augmented Lagrangian (AL) of the problem as:

$$\mathcal{L}_\gamma(x, z, u) = f(x) + g(z) + \frac{\gamma}{2}\|x - z + u\|_2^2, \gamma > 0, \text{is } \textit{penalty parameter}.$$

- The AL can be minimize by either **Method of Multipliers** (MM) or **Alternating Directions Method of Multipliers** (ADMM).

ADMM

- ADMM is faster than all first-order methods given that i). it introduces enough variable splittings such that each subproblems in ADMM iterations have closed-form solutions, and ii). the AL penalty parameter associated with each augmented terms are tunned optimally[2].
- However, there is no universal rule to select optimal values of AL penalty parameters.

Proposed Algorithm [1]

- Proposed Algorithm is an instance of MM
- Joint-minimization is solved in hierarchical way:

$$x^* := \operatorname*{argmin}_{x} \mathcal{L}_\gamma(x, z^*, u), \text{ where } z^*(x) = \operatorname*{argmin}_{z} g(z) + \frac{\gamma}{2}\|x - z + u\|_2^2$$

by quasi-Newton method (e.g. VMLM-B [3]), where the gradient is given as:

$$\nabla_x \mathcal{L}_\gamma(x, z^*(x), u) = \nabla f(x) + \gamma(x - z^*(x) + u).$$

ADMM Iterations

repeat while not converged

1. $x^{(k+1)} \leftarrow \operatorname*{argmin}_{x} f(x) + \frac{\gamma}{2}\|x - z^{(k)} + u^{(k)}\|_2^2$
2. $z^{(k+1)} \leftarrow \operatorname*{argmin}_{z} g(z) + \frac{\gamma}{2}\|x^{(k+1)} - z + u^{(k)}\|_2^2$
3. $u^{(k+1)} \leftarrow u^{(k)} + x^{(k+1)} - z^{(k+1)}$
4. $k \leftarrow k + 1$

Proposed Iterations

repeat while not converged

1. $\{x^{(k+1)}, z^{(k+1)}\} \leftarrow \text{VMLM-B}(x^{(k)}, \text{F}, \text{G})$

$$\text{where} \left\{ \begin{array}{l} \text{F}(x) = f(x) + g(z^*(x)) \\ \quad + \frac{\gamma}{2}\|x - z^*(x) + u^{(k)}\|_2^2, \\ \text{G}(x) = \nabla f(x) + \gamma(x - z^*(x) + u^{(k)}), \\ z^*(x) \leftarrow \min_z g(z) + \frac{\gamma}{2}\|x - z + u^{(k)}\|_2^2 \end{array} \right\}$$

2. $u^{(k+1)} \leftarrow u^{(k)} + x^{(k+1)} - z^{(k+1)}$
3. $k \leftarrow k + 1$

Illustrations and Experimental Results

1. A Toy Problem: $x^* := \operatorname*{argmin}_{x\in\Omega} \{x^T Q x + b^T x + \lambda\|x\|_1\}$

The two algorithms:

	ADMM	Proposed
Variable splittings:	$z = x$	$z = x$
Penalty parameters:	ρ	ρ

2. Image Deblurring with TV: $x^* := \operatorname*{argmin}_{x\geq 0} \{\|y - Hx\|_W^2 + \lambda\, \text{TV}(x)\}$

where y is blurred and noisy image, H is blurring operator, W is inverse of noise covariance (diagonal) matrix.

The three algorithms:

	ADMM-1x	ADMM-3x	Proposed
Variable splittings:	$v = \nabla x$	$\xi = y - Hx, v = \nabla x, z = x$	$v = \nabla x$
Penalty parameters:	γ	ρ, ν, γ	γ

3. Poissonian Image Deblurring with TV [4]: $x^* := \operatorname*{argmin}_{x} \{\varphi(CHx) + i_+(x) + \lambda \text{TV}(x)\}$

where $\varphi(t) = \mathbf{1}^T t + i_+(t) - y^T \log(t)$, C being chopping operator.

The three algorithms:

	ADMM-1x	ADMM-4x	Proposed
Variable splittings:	$v = \nabla x$	$w = Hx, \xi = Cw, v = \nabla x, z = x$	$v = \nabla x$
Penalty parameters:	γ	ρ, ν, γ, η	γ

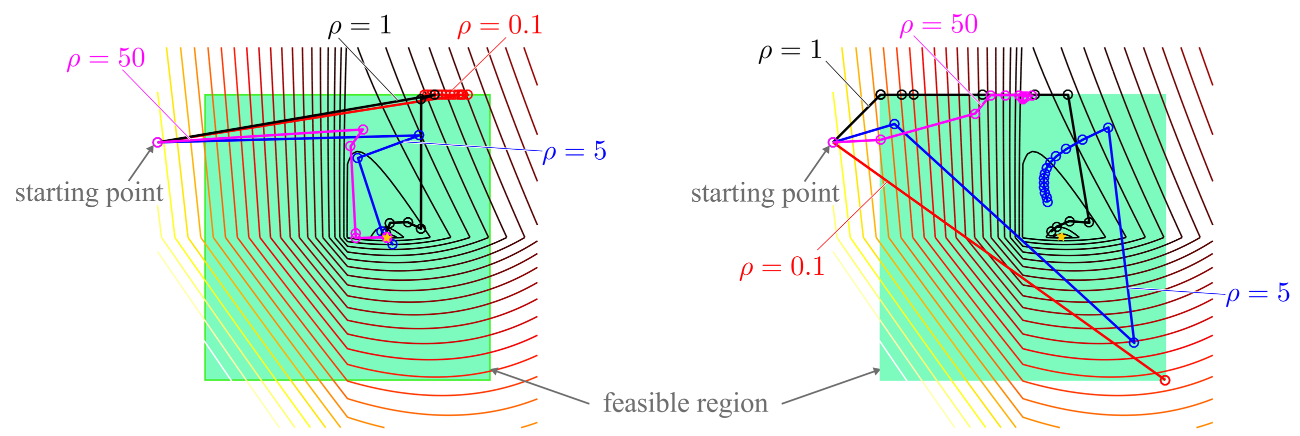

Fig.1 Iterations of the two algorithms on contour plot for the Problem 1: Left is Proposed, right is ADMM.

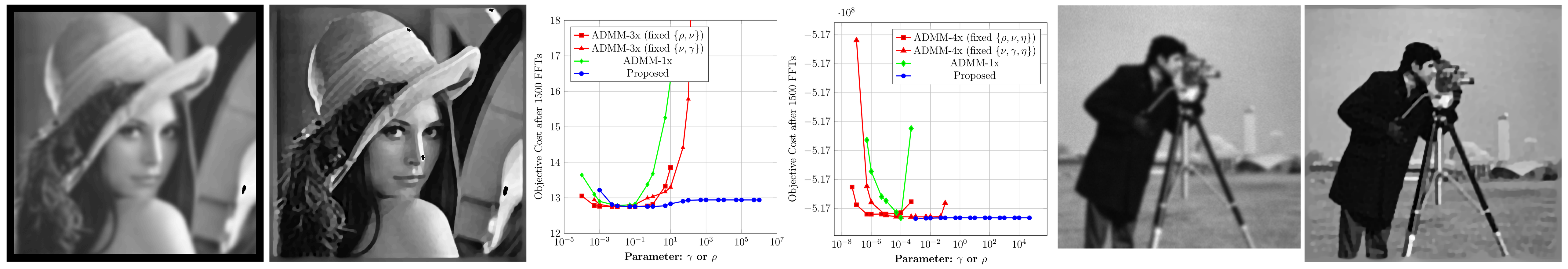

Fig.2 Image deblurring results for two problems, and influence of penalty parameter on convergence speed: Left is for Problem 2, and right is for Problem 3.

Conclusion

- Proposed algorithm requires lesser variable splittings, and is as fast as ADMM with multiple variable splitting and optimally tunned penalty parameters.
- Proposed algorithm requires no penalty parameter tunning.

References

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