Large Inpainting of Face Images with Trainlets

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joint work with Michael Elad

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Image Inpainting

Degradation model

$$\mathbf{y} = \mathbf{M}\mathbf{z} + \mathbf{v}$$

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- Low-level image restoration methods
- Diffusion/content propagation methods
- Intrinsic need of a **global model**

Learning High Dimensional Model

Difficulties

- Computational hard problem
- Curse of dimensionality

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Related methods

- Manifold learning techniques
- Some global models
- Dictionary Learning

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Our solution

- We employ a high dimensional dictionary learning method to learn a global model of face images
- Solve an inverse problem regularized with a sparse prior

Contents

Sparse Representations

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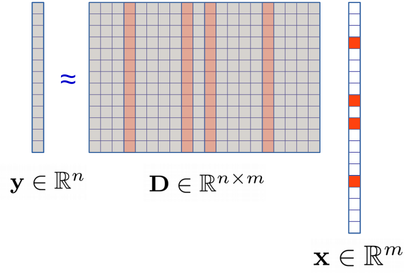

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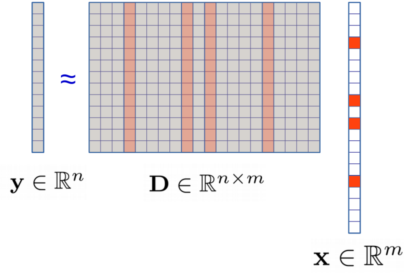

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$$\min_{\mathbf{x}} \ ||\mathbf{x}||_0 \text{ subject to } ||\mathbf{y} - \mathbf{D}\mathbf{x}||_2^2 \leq \epsilon^2,$$

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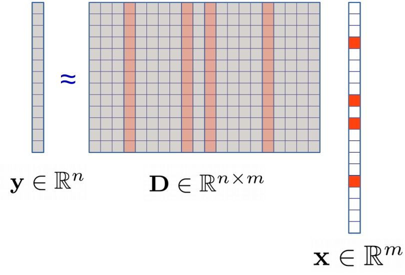

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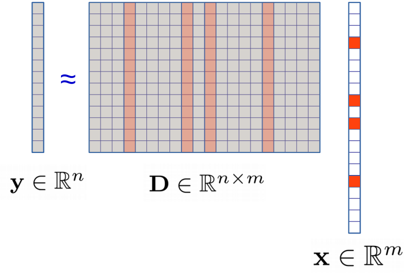

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- Relaxation Methods

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- Transforms
  - Structured Matrices (Fast Algorithms)
  - Fair sparsification

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- Many successful applications and results
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- Computational Constraints

Up-Scaling Dictionary Learning

Problem Formulation

$$\min_{\mathbf{A},\mathbf{X}} ||\mathbf{Y} - \boldsymbol{\Phi}\mathbf{A}\mathbf{X}||_F^2 \quad \text{subject to} \quad ||\mathbf{x}_i||_0 \leq p, \; ||\mathbf{a}_i||_0 \leq k$$

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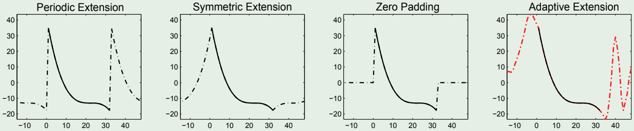
Φ: Cropped Wavelets

The transform for signal $\mathbf{f}$ is defined in terms of a pursuit over a **convolutional and multi-scale dictionary**, providing sparsest wavelet representations by optimally (implicitly) extending the signal borders.



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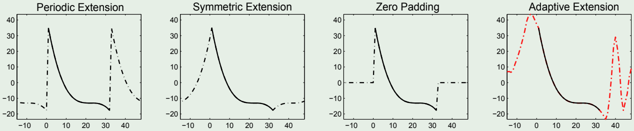

Online Learning

- Faster convergence
- Training on millions of examples

OSDL in practice

$$\min_{\mathbf{X},\mathbf{A}} \underbrace{\frac{1}{2}||\mathbf{Y}-\mathbf{\Phi}\mathbf{A}\mathbf{X}||_F^2}_{f(\mathbf{X},\mathbf{A})} \quad \text{s.t.} \quad \begin{cases} ||\mathbf{x}_i||_0 \leq p & \forall i \\ ||\mathbf{a}_j||_0 \leq k & \forall j \end{cases} .$$

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Data: Training samples $\{\mathbf{y}_i\}$, base-dictionary $\mathbf{\Phi}$, initial sparse matrix $\mathbf{A}^0$

for $t = 1, \ldots, T$ **do**

  Draw a mini-batch $\mathbf{Y}_t$ at random ;

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- Incorporate a momentum variable
- Analytical step-size
- Replace repeated/unused atoms

Trainlets for Data Approximation

- 64×64 Images.
- $\approx$ 12K training examples.
- Non-Redundant Dictionary ($\approx$ 4K atoms).
- Atoms Sparsity: 300.
- $\mathbf{\Phi}$: Db4 cropped-wavelets ($r = 1.37$, 1D) .

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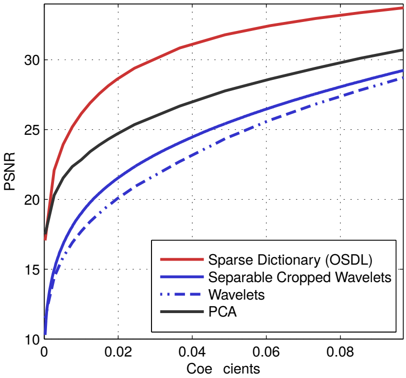

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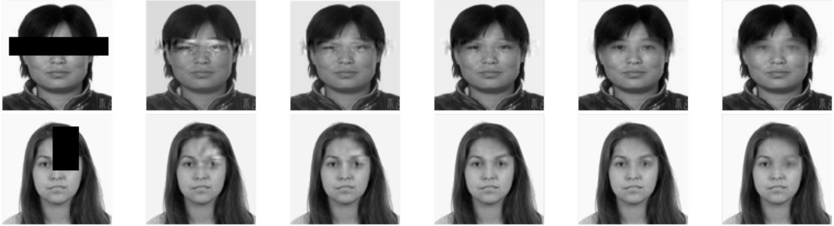

$$\lambda \rightarrow$$

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Inpainting Results

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Patch Propagation

PCA

SEDIL

Trainlets

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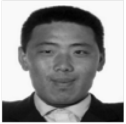

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Masked Image | Patch Propagation | PCA | SEDIL | Trainlets | Original Image

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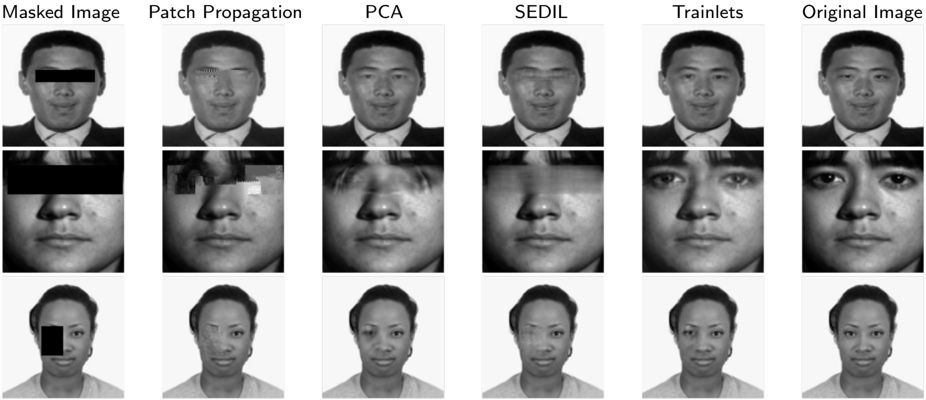

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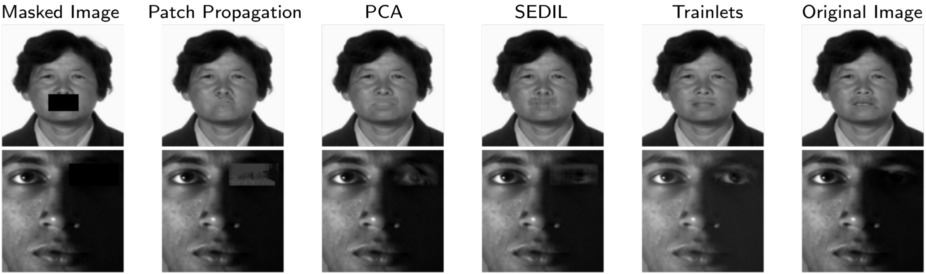

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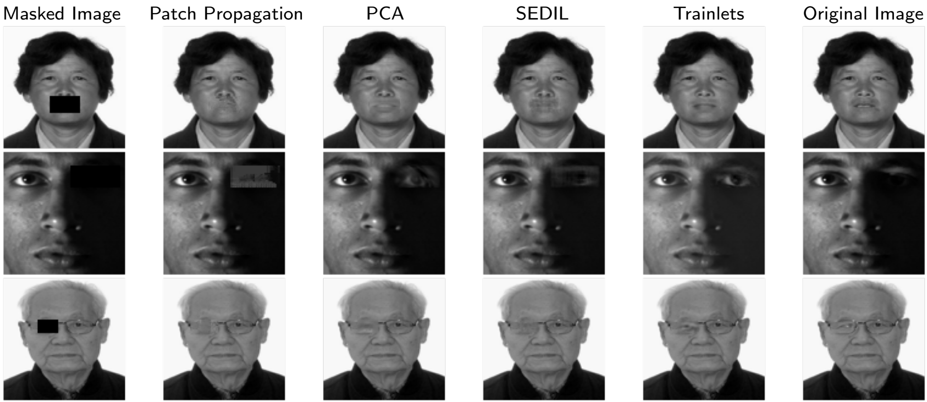

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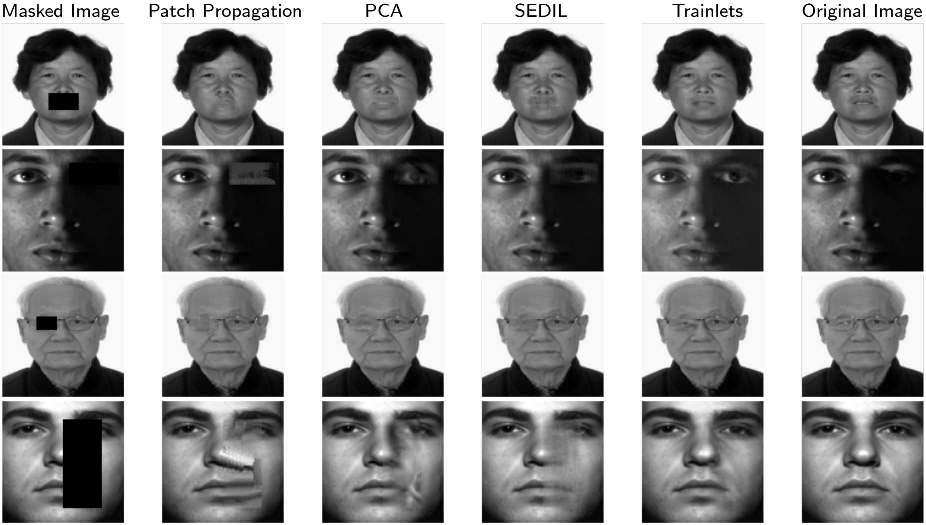

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- We exploit the representation power of Trainlets to learn a global model
- Very simple problem formulation
- No extra algorithmic manipulation are needed
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- Other type of inverse problems?

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Questions?

Code and model available at `jsulam.cswp.cs.technion.ac.il`