# Robust Particle Filter by Dynamic Averaging of Multiple Noise Models


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# Outline

- **A Basic Framework of Particle Filter (PF)**
- **The Proposed Robust Particle Filter (RPF) Algorithm**
- **Simulation Results**
- **Conclusions**

# Problem

$$x_k = f(x_{k-1}) + u_k$$

$$y_k = h(x_k) + n_k$$

$\Rightarrow$

$p(x_k \mid x_{k-1})$ → State transition prior

$p(y_k \mid x_k)$ → Likelihood

We are interested in $p(x_k \mid y_{0:k})$, where $y_{0:k} = \{y_i\}_{i=0}^{k}$

How to estimate $p(x_k \mid y_{0:k})$ online, in presence of measurement outliers ?

# Bayes Filter

**Predict:**

$$p(x_k \mid y_{0:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid y_{0:k-1}) dx_{k-1}$$

**Update:**

$$p(x_k \mid y_k, y_{0:k-1}) = \frac{p(y_k \mid x_k)}{p(y_k \mid y_{0:k-1})} p(x_k \mid y_{0:k-1})$$

$$p_{k|k} = \frac{p(y_k \mid x_k) \int p(x_k \mid x_{k-1}) p_{k-1|k-1} dx_{k-1}}{p(y_k \mid y_{0:k-1})}$$

← **cumbersome, intractable integrals**

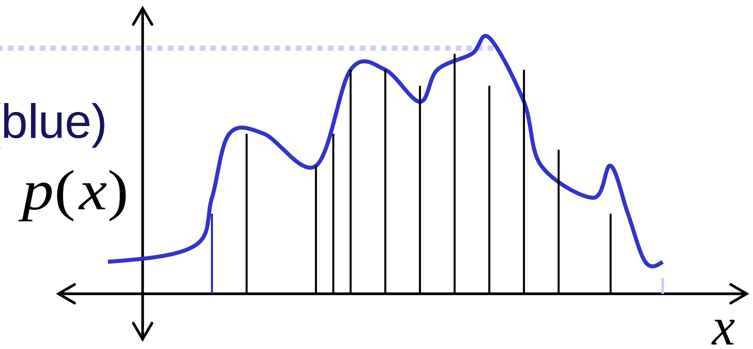
where $p_{k|k} = p(x_k \mid y_{0:k})$

**Solution:**

- **Approximate representation → particle filter**

# The basic idea uderlying PF

$p(x)$ → continuous probability distribution of interest (blue)




$$p(x) \approx \sum_{i=1}^{N} w^i \delta(x - x^i)$$

where $p(x)$ → probability distribution of interest (blue)

$x^i$ → the particles

$w^i$ → weights of the particles

$\delta(\bullet)$ → the Dirac delta function

$N$ → number of particles

$$p(x) \approx \chi = \left\{ x^i, w^i \right\}_{i=1}^{N}$$ → approximating random measure

# A Basic Framework of PF

**Starting from** $p(x_{k-1} \mid y_{0:k-1}) \approx \chi_{k-1} = \left\{ x_{k-1}^{i}, w_{k-1}^{i} \right\}_{i=1}^{N}$

- Sampling step. Sample $x_k^i \sim q\left( x_k \mid x_{k-1}^i, y_{0:k} \right)$
- Weighting step.

$$w_k^i = p\left( x_k^i \middle| y_{0:k} \right) \Big/ q\left( x_k^i \middle| x_{k-1}^i, y_{0:k} \right), \text{ set } w_k^i = w_k^i \Big/ \sum_{j=1}^{N} w_k^j$$

- Resampling step. Sample $x_k^i \sim \sum_{j=1}^{N} w_k^j \delta\left( x - x_k^i \right)$, set $w_k^i = 1/N$

**Output** $p(x_k \mid y_{0:k}) \approx \chi_k = \left\{ x_k^i, w_k^i \right\}_{i=1}^{N}$ **at time step k**

# PFs under model uncertainty

$$x_k = f\left(x_{k-1}\right) + u_k \qquad \Rightarrow \qquad p\left(x_k \mid x_{k-1}\right)$$

$$y_k = h\left(x_k\right) + n_k \qquad\qquad p\left(y_k \mid x_k\right)$$

- if there is uncertainty on $f$, how to modify the PF to adapt it? [1]
- if there is uncertainty on $h$, how to modify the PF to adapt it? [2]
- if there is uncertainty in the measurement noise model, how to modify the PF to adapt it?

[1] Liu, B., Instantaneous Frequency Tracking under Model Uncertainty via Dynamic Model Averaging and Particle Filtering, IEEE Trans. on Wireless Communications, vol.10, no.6, pp.1810-1819,2011.
[2] Dai, Y., Liu, B., Robust video object tracking via Bayesian model averaging based feature fusion, Optical Engineering, vol.55, no.8, pp.083102, 2016.

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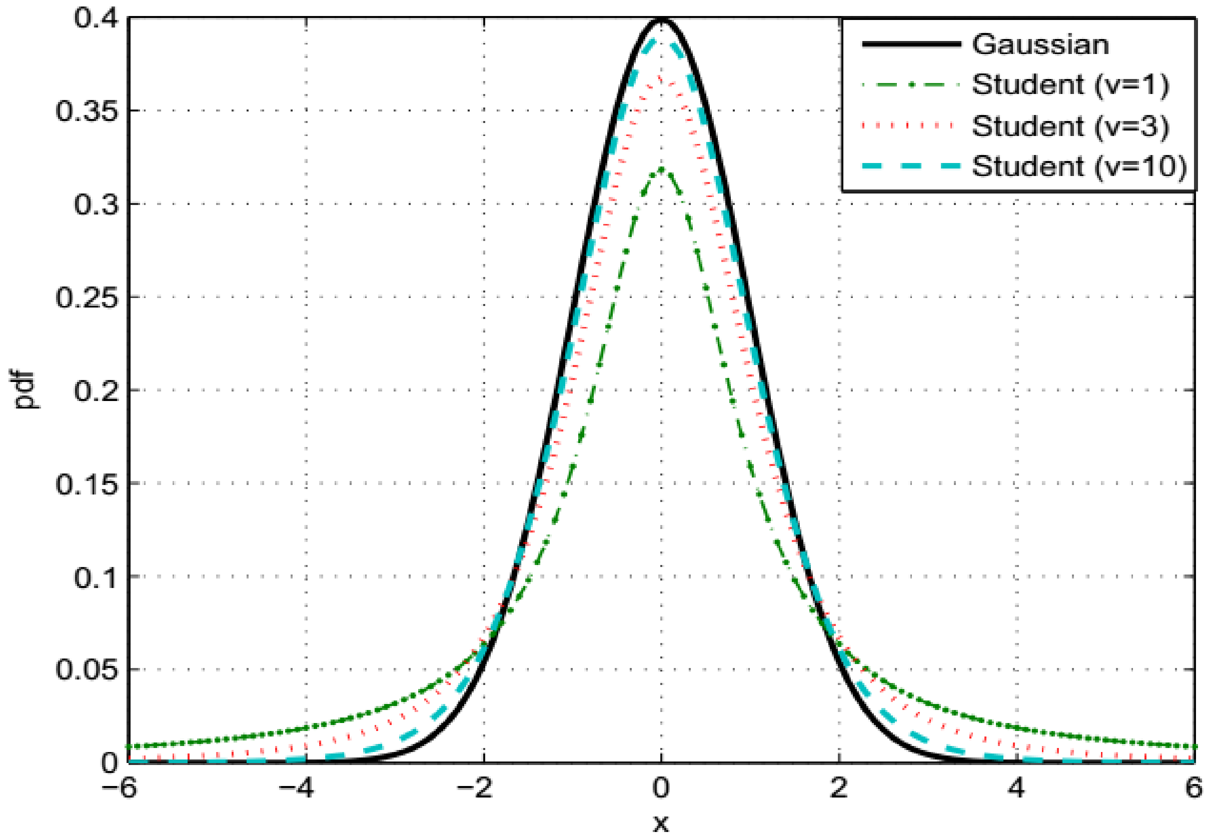
# Robustify PF by Employing Multiple Noise Models

Model the measurement noise $n_k$ by $M$ candidate models together



# Model Averaging Strategy to Handle Multiple Models

- Bayesian Model Averaging to compute $p_{k|k} = p(x_k \mid y_{0:k})$

$$p_{k|k} = \sum_{m=1}^{M} p_{m,k|k} \pi_{m,k|k}$$

where

$$p_{m,k|k} \triangleq p\left(x_k \middle| H_k = m, y_{0:k}\right) \text{ and } \pi_{m,k|k} \triangleq p\left(H_k = m \middle| y_{0:k}\right)$$

# Perform PF under each model hypothesis

**Starting from** $p_{k-1|k-1} \approx \chi_{k-1} = \left\{ x_{k-1}^{i}, w_{k-1}^{i} \right\}_{i=1}^{N}$

Sample $x_k^i$ from $q\left(x_k \middle| x_{k-1}, y_{0:k}\right)$ ; calculate its weight by

$$w_{m,k}^{i} \propto w_{k-1}^{i} p\left(x_k^i \middle| x_k^{i-1}\right) p_m\left(y_k \middle| x_k^i\right) \Big/ q\left(x_k^i \middle| x_{k-1}^{i-1}, y_{0:k}\right)$$

Then we have

$$p_{m,k|k} \approx \chi_{m,k} = \left\{ x_k^i, w_{m,k}^{i} \right\}_{i=1}^{N}$$

# Update the posterior prob. of each candidate model

Given $\pi_{m,k-1|k-1}$, we have

$$\pi_{m,k|k-1} = \frac{\pi^{\alpha}_{m,k-1|k-1}}{\sum_{m=1}^{M} \pi^{\alpha}_{m,k-1|k-1}}, \qquad (9)$$

where $\pi_{m,k|k-1} \triangleq p(\mathcal{H}_k = m|y_{0:k-1})$.

Then, employing Bayes' rule we have

$$\pi_{m,k|k} = \frac{\pi_{m,k|k-1}p_m(y_k|y_{0:k-1})}{\sum_{m=1}^{M} \pi_{m,k|k-1}p_m(y_k|y_{0:k-1})}, \qquad (10)$$

where $p_m(y_k|y_{0:k-1}) = \int p_m(y_k|x_k)p(x_k|y_{0:k-1})dx_k. \qquad (11)$ ← **cumbersome, intractable integral**

**Solution: approximate it by:**

$$p_m(y_k|y_{0:k-1}) \simeq \sum_{i=1}^{N} \omega^i_{k-1} p_m(y_k|\hat{x}^i_k). \qquad (12)$$

# The Proposed RPF Algorithm

**Starting from** $p(x_{k-1} \mid y_{0:k-1}) \approx \chi_{k-1} = \left\{x_{k-1}^i, w_{k-1}^i\right\}_{i=1}^N$

- Sampling step. Sample $x_k^i \sim q\left(x_k \mid x_{k-1}^i, y_{0:k}\right)$
- Weighting step.

$$w_{m,k}^i = p_m\left(x_k^i \middle| y_{0:k}\right) \Big/ q\left(x_k^i \middle| x_{k-1}^i, y_{0:k}\right), \quad \text{set} \quad w_{m,k}^i = w_{m,k}^i \Big/ \sum_{j=1}^N w_{m,k}^j$$

- Model averaging step. Compute $\pi_{m,k|k}$ using Eqns.(9)-(12)
- Resampling step. Sample $x_k^i \sim \sum_{j=1}^N w_k^j \delta\left(x - x_k^i\right)$, $w_k^j = \sum_{m=1}^M \pi_{m,k|k} w_{m,k}^j$;

  Set $w_k^i = 1/N$, $i = 1, \ldots, N$.

**Output** $p(x_k \mid y_{0:k}) \approx \chi_k = \left\{x_k^i, w_k^i\right\}_{i=1}^N$ **at time step k**

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# Simulation Setting

$$x_{k+1} = 1 + \sin(0.04\pi \times (k+1)) + 0.5x_k + u_k, \qquad (15)$$

$$y_k = \begin{cases} 0.2x_k^2 + n_k, & k \leq 30 \\ 0.2x_k - 2 + n_k, & k > 30 \end{cases} \qquad (16)$$

**Case I: filtering without the presence of outliers**

**Case II: filtering with the presence of outliers**

[3] R. Van Der Merwe, A. Doucet, N. De Freitas, and E. Wan, "The unscented particle filter," in *NIPS*, 2000, pp. 584–590

# Algorithm Performance Comparison for case I

| Algorithm | Time | MSE | |
|---|---|---|---|
| | | mean | var |
| PF: Generic | 1.561 | 0.350 | 0.056 |
| PF: MCMC move step | 3.275 | 0.371 | 0.047 |
| EKPF | 2.958 | 0.280 | 0.015 |
| EKPF: MCMC move step | 7.033 | 0.278 | 0.013 |
| UPF | 9.095 | 0.055 | 0.008 |
| UPF: MCMC move step | 19.735 | 0.052 | 0.008 |
| the proposed RPF | 5.509 | 0.018 | 0.0001 |

**Table 1:** Execution time (in seconds), Mean and variance of the MSE calculated over 30 independent runs for Case I.

# Algorithm Performance Comparison for case II

| Algorithm | MSE | |
|---|---|---|
| | mean | var |
| PF: Generic | 0.533 | 0.040 |
| PF: MCMC move step | 0.523 | 0.039 |
| EKPF | 22.663 | 0.343 |
| EKPF: MCMC move step | 22.668 | 0.358 |
| UPF | 19.804 | 0.289 |
| UPF: MCMC move step | 19.808 | 0.274 |
| the proposed RPF | 0.357 | 0.010 |

**Table 2:** Mean and variance of the MSE calculated over 30 independent runs for Case II.

# Parameter Sensitivity

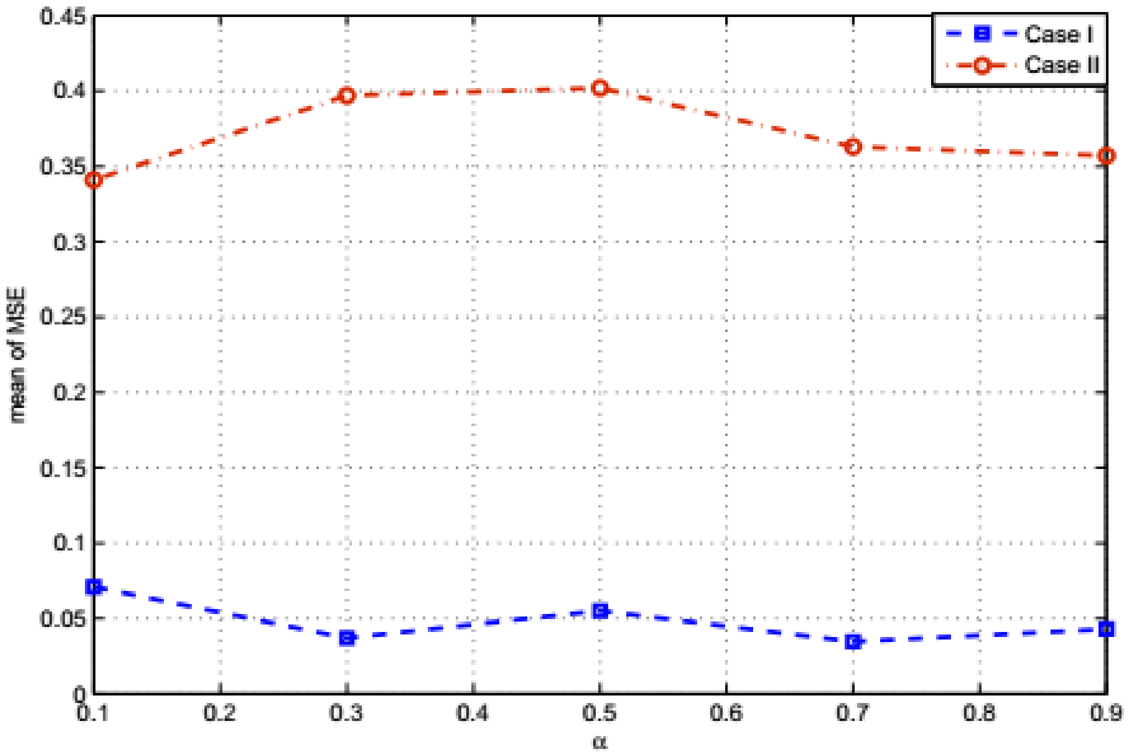
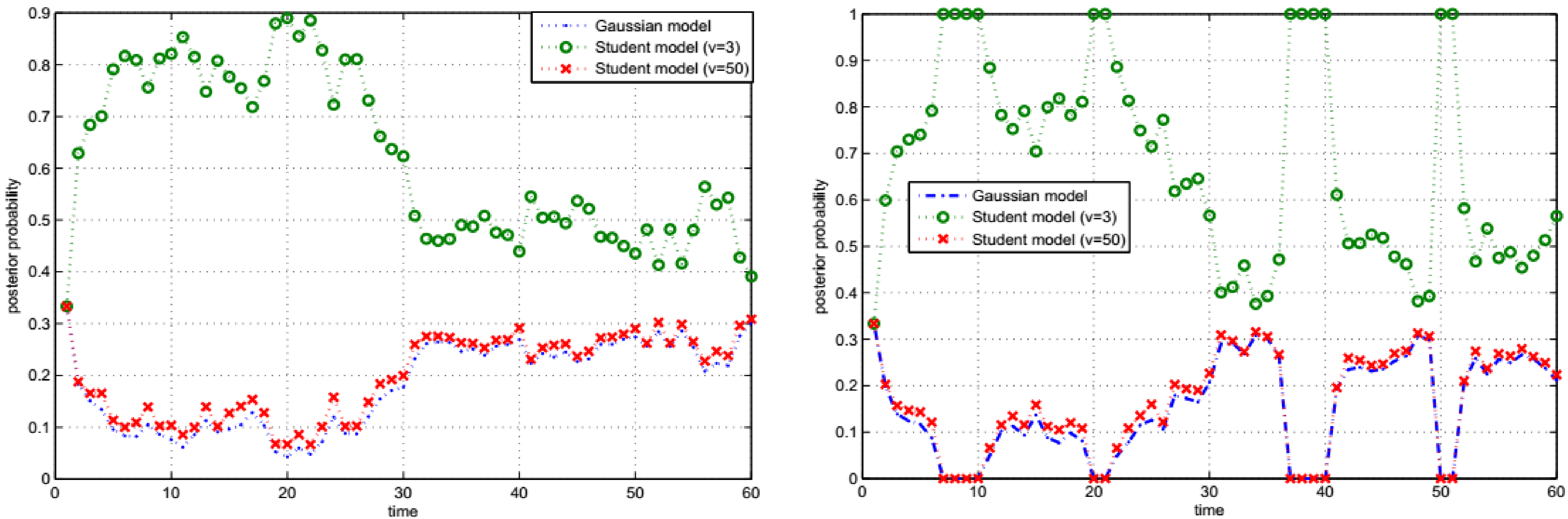



**Fig. 1**: Mean of the MSE calculated over 30 independent runs, in case of different $\alpha$ values, for both Case I and II.

# Simulation results




Averaged posterior probability of candidate models outputted by the proposed RPF method. The left and right sub-figures correspond to Case I and II, respectively.

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# Conclusions

- A multi-model based PF method, which is shown to be robust against the presence of outliers in the measurements.

- The usage of a mixture of heavier tailed Student's t distributions and a Gaussian distribution shows promises in modeling the measurement noise in the context of robust state filtering.

- Simple, while highly efficient !

- Future work: 1) consider uncertainties in $f$ , $h$, and the model of $n_k$ all together; possible usages of other types of mixing components; 3) real-life problems

# Thanks for your attention!

Q & A

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