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Acoustic imaging of sparse sources with Orthogonal Matching Pursuit and clustering of basis vectors

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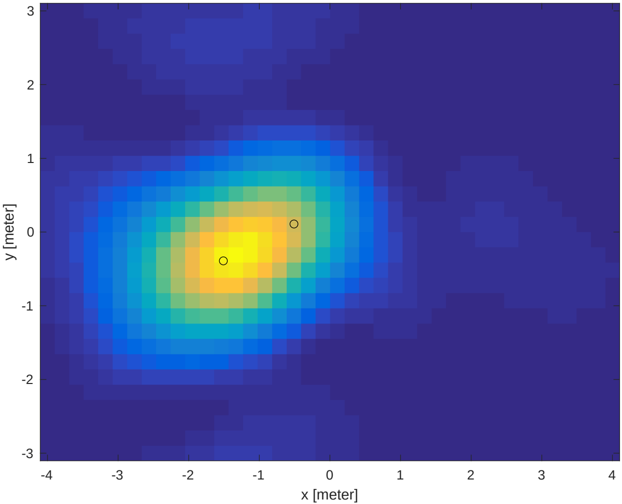
SQUAREHEAD TECHNOLOGY

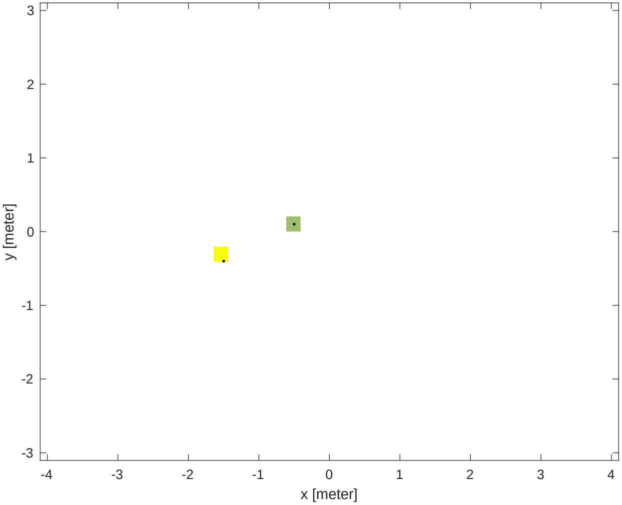
Presentation outline

1. Brief description of deconvolution (DAMAS)
2. Proposed deconvolution algorithm
3. Simluations & experimental results

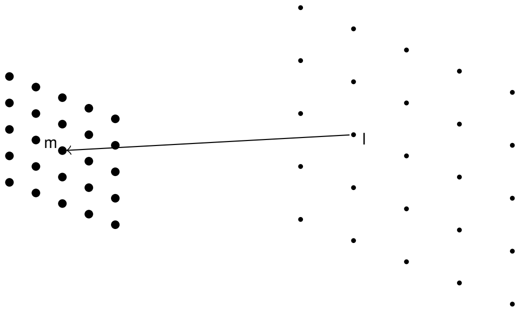
Single-frequency acoustic image



Deconvolved source map



Sound field model

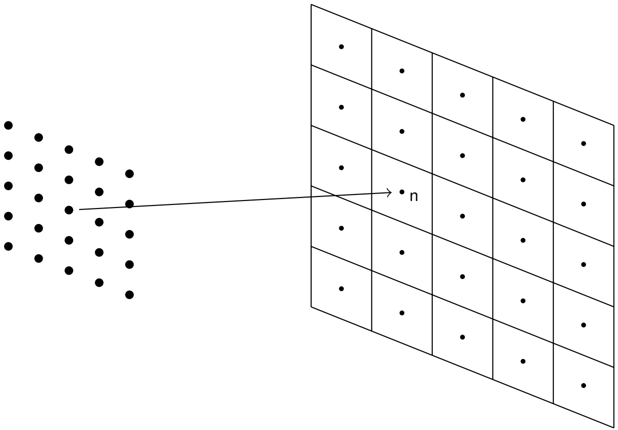


$$\begin{bmatrix} x_1(\omega) \\ \vdots \\ x_m(\omega) \\ \vdots \\ x_M(\omega) \end{bmatrix} = \begin{bmatrix} | & & | & & | \\ G_1(\omega) & \cdots & G_l(\omega) & \cdots & G_L(\omega) \\ | & & | & & | \end{bmatrix} \begin{bmatrix} s_1(\omega) \\ \vdots \\ s_l(\omega) \\ \vdots \\ s_L(\omega) \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_l \\ \vdots \\ \eta_L \end{bmatrix}$$

$$x(\omega) = G(\omega)s(\omega) + \eta \tag{1}$$

M: Number of array elements, L: Number of model sources

Acoustic imaging model



$$Y_n = \frac{1}{M^2} w_n^H R_x w_n = \frac{1}{M^2} w_n^H E\left\{xx^H\right\} w_n = \frac{1}{M^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \left[w_n^H E\left\{(G_i s_i + \eta_i)(G_j s_j + \eta_j)^*\right\} w_n \right] \qquad (2)$$

$0 \leq n \leq N$, N: Number of image points

Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS)

Uncorrelated sources and i.i.d. Gaussian noise:

$$Y_n = \frac{1}{M^2} \sum_{l=1}^{L} \left[\left| w_n^H G_l \right|^2 E\left\{ |s_l|^2 \right\} \right] + \frac{1}{M^2} E\left\{ |\eta|^2 \right\} \tag{3}$$

$$Y = AQ \ \left(+ \sigma^2 \mathbb{1}_N \right) \tag{4}$$

Point spread function (PSF): $$A_{[n,l]} = \frac{1}{M^2} \left| w_n^H G_l \right|^2 \in \mathbb{R}_+ \tag{5}$$

Source map: $$Q_l = E\left\{ |s_l|^2 \right\} \in \mathbb{R}_+ \tag{6}$$

Reference: Brooks, Thomas F., and William M. Humphreys. “A Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS) Determined from Phased Microphone Arrays.” Journal of Sound and Vibration 294, no. 4 (2006): 856–879.

A as a frame (redundant basis) / dictionary

$Y = \sum_l A_l Q_l$
A_l: Column l of A, basis vector (or dictionary atom)
Q_l: Expansion coefficient

Assumptions

1. N_s, number of sources is known
2. Number of sources is small, i.e. Q is sparse:
   $N_s = |\Gamma| = |\mathrm{supp}\,(Q)| << L$

Goal

Reconstruction method that is fast (near-realtime) and accurate (position/DOA & power) $\rightarrow$ greedy pursuit

Orthogonal Matching Pursuit DAMAS (OMP-DAMAS) [1]

for $s = 1$ **to** N_s **do**

$\Gamma \leftarrow \Gamma \cup \arg\max_k \left|(A^H \rho)_k\right|$

$Q \leftarrow \arg\min_q ||Y - A_\Gamma q||_2 = \left(A_\Gamma^H A_\Gamma\right)^{-1} A_\Gamma^H Y$

$\rho \leftarrow Y - A_\Gamma Q = \left[1 - A_\Gamma \left(A_\Gamma^H A_\Gamma\right)^{-1} A_\Gamma^H\right] Y$

if stopping criterion fulfilled **then**

exit loop

end if

end for

Problems with direct application of OMP

- Due to noise and basis mismatch: $Y \notin \mathrm{colsp}\{A\}$
- Greedy pursuit is inherently suboptimal

[1] Padois, Thomas, and Alain Berry. "Orthogonal Matching Pursuit Applied to the Deconvolution Approach for the Mapping of Acoustic Sources Inverse Problem." The Journal of the Acoustical Society of America 138, no. 6 (December 1, 2015): 3678–85. doi:10.1121/1.4937609.

Proposed reconstruction algorithm: Clustered Orthogonal Matching Pursuit - DAMAS (COMP-DAMAS)

Two stages:

1. Exhaustive greedy pursuit.
2. Spatial clustering of basis vectors such that $N_c \geq N_s$, and least-squares fitting of each cluster to a single-source model.

COMP-DAMAS Stage 1

Exhaustive greedy pursuit

for $s = 1$ **to** L **do**
 $\Gamma \leftarrow \Gamma \cup \arg\max_k \left|(A^\dagger \rho)_k\right|$
 $Q \leftarrow (A_\Gamma^H A_\Gamma)^{-1} A_\Gamma^H Y$
 $\rho \leftarrow Y - A_\Gamma Q$
 if $\left(\left\|\rho^{(s-1)}\right\|_2 - \left\|\rho^{(s)}\right\|_2\right) / \left\|\rho^{(s-1)}\right\|_2 \leq \delta$ **then**
  Exit loop
 end if
end for

Index selection rule

Maximal least-squares coefficient (Enhanced-OMP [1]):

$$A^\dagger \rho \approx \arg\min_q ||\rho - Aq||_2$$

$$A^\dagger = \left(A^H A + \lambda \sigma_{\max}(A) \mathbb{1}_L\right)^{-1} A^H$$

λ: Regularization parameter

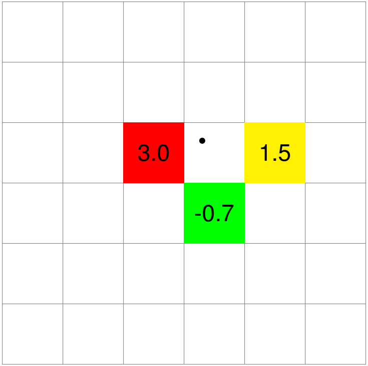
Termination

ρ is dominated by noise

[1] Osamy, Walid, Ahmed Salim, and Ahmed Aziz. "Sparse Signals Reconstruction via Adaptive Iterative Greedy Algorithm." International Journal of Computer Applications 90, no. 17 (March 26, 2014): 5–11. doi:10.5120/15810-4715.

Stage 2: Clustering & merging

Due to exhaustive pursuit, real sources *may be* represented by several basis vectors and expansion coefficients



Bottom-up spatial clustering

Distance function:

$$\begin{aligned} d_{i,j} &= 1 - \frac{1}{2}\left(\frac{\left|w^H(\vec{x}_i)G_j\right|^2}{\left|w^H(\vec{x}_j)G_j\right|^2} + \frac{\left|w^H(\vec{x}_j)G_i\right|^2}{\left|w^H(\vec{x}_i)G_i\right|^2} \right) \\ &= 1 - \frac{1}{2}\left(\frac{A_{[i,j]}}{A_{[j,j]}} + \frac{A_{[j,i]}}{A_{[i,i]}} \right), \quad 0 \leq d_{i,j} \leq 1 \end{aligned} \tag{7}$$

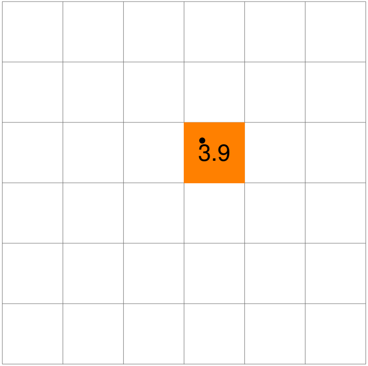
Continue while:

$$\min_{i \neq j} d_{i,j} < \gamma = 0.85 \quad \text{AND} \quad N_c \geq N_s$$

Stage 2: Clustering & merging

Fitting cluster to single source model with

index $\hat{k}$ and power $\hat{q}$



Fitting cluster to a single source model

$$\left(\hat{k}, \hat{q}\right)_\Gamma = \underset{(k,q)}{\arg\min} \, ||A_k q - A_\Gamma Q_\Gamma||_2 \, , \quad k \in \mathbb{Z}, \; q \in \mathbb{R}_+ \tag{8}$$

$$\hat{k} = \underset{k}{\arg\min} \left\| A_k \left(\sum_{j \in \Gamma} Q_j \right) - A_\Gamma Q_\Gamma \right\|_2 \tag{9}$$

$$\hat{q} = (A_{\hat{k}}^H A_{\hat{k}})^{-1} A_{\hat{k}}^H A_\Gamma Q_\Gamma \tag{10}$$

Simulations & Experiments

Benchmark methods

1. Orthogonal Matching Pursuit DAMAS (OMP-DAMAS)
2. CLEAN based on Source Coherence (CLEAN-SC)
3. Robust DAMAS with Sparse Constraint (SC-RDAMAS)
4. Generalized Inverse Beamforming (GIB)

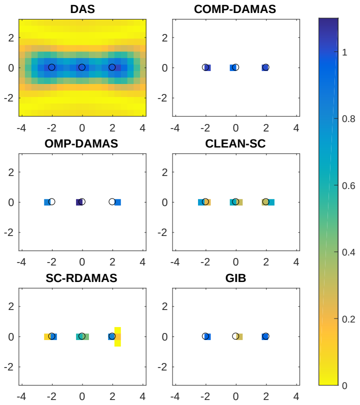
References:

1. Padois, Thomas, and Alain Berry. “Orthogonal Matching Pursuit Applied to the Deconvolution Approach for the Mapping of Acoustic Sources Inverse Problem.” The Journal of the Acoustical Society of America 138, no. 6 (December 1, 2015): 3678–85. doi:10.1121/1.4937609.
2. Sijtsma, Pieter. “CLEAN Based on Spatial Source Coherence.” International Journal of Aeroacoustics 6, no. 4 (December 1, 2007): 357–74. doi:10.1260/147547207783359459.
3. Chu, Ning, José Picheral, Ali Mohammad-djafari, and Nicolas Gac. “A Robust Super-Resolution Approach with Sparsity Constraint in Acoustic Imaging.” Applied Acoustics 76 (February 2014): 197–208. doi:10.1016/j.apacoust.2013.08.007.
4. Suzuki, Takao. “L1 Generalized Inverse Beam-Forming Algorithm Resolving Coherent/Incoherent, Distributed and Multipole Sources.” Journal of Sound and Vibration 330, no. 24 (November 21, 2011): 5835–51. doi:10.1016/j.jsv.2011.05.021.

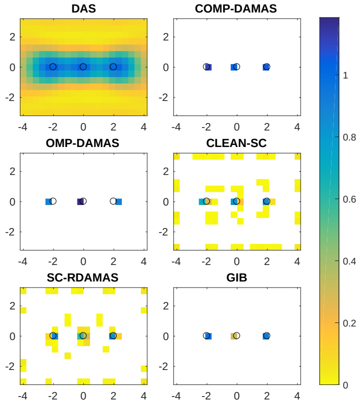
Simulation setup

- Single frequency delay-and-sum at 2kHz, sampled at 16kHz
- White noise sources, 100 snapshots of 128 samples
- 8m x 6m imaging area, 20x15=300 grid points, distance 4.3m from array
- 16x16-element array, 39cm x 39cm

Simulation results, scenario 1

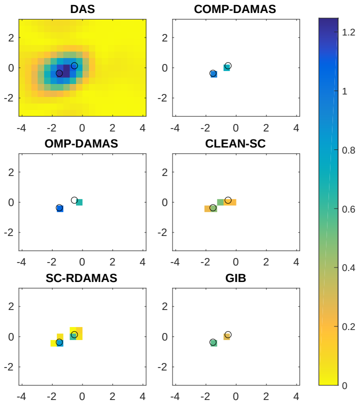


No noise

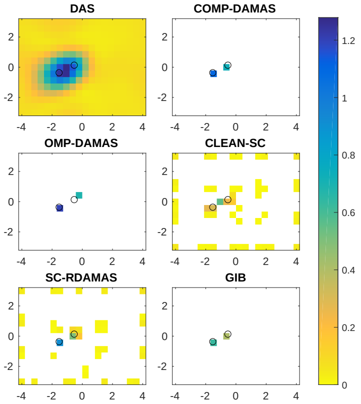


Single-channel SNR = -9dB

Simulation results, scenario 2

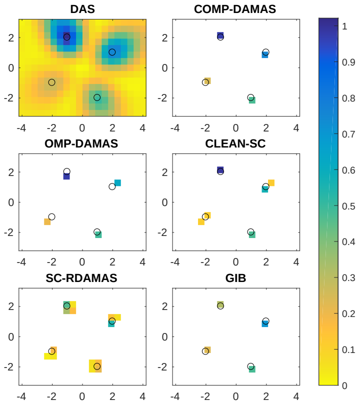


No noise

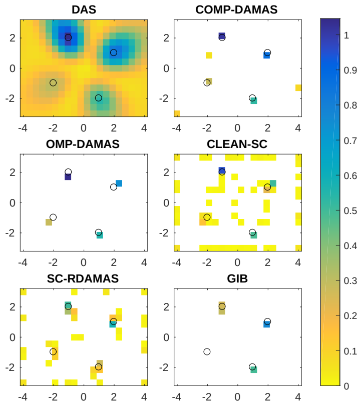


Single-channel SNR = -9dB

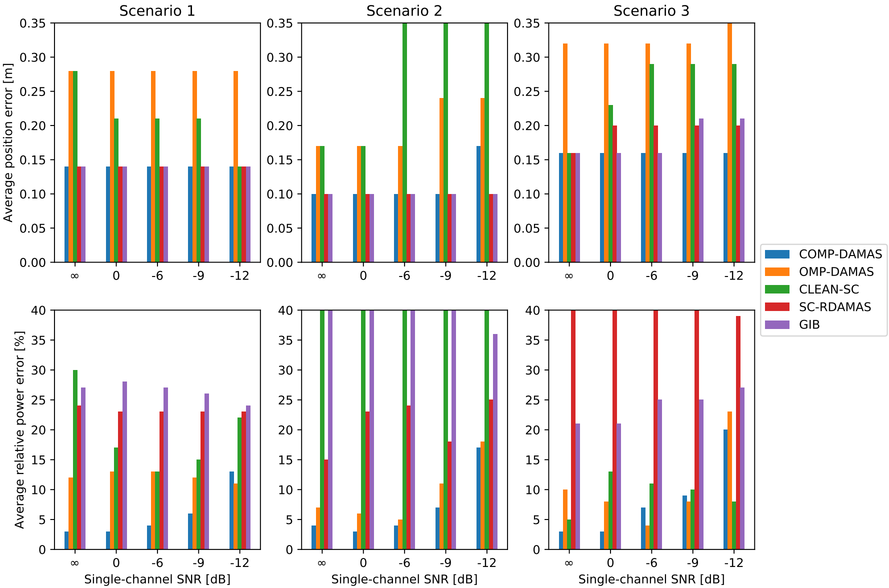
Simulation results, scenario 3



No noise

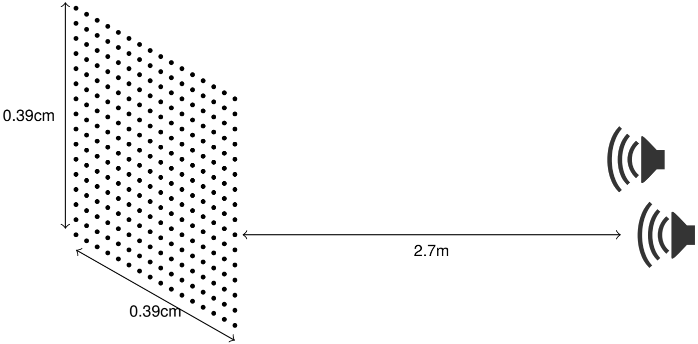


Single-channel SNR = -9dB

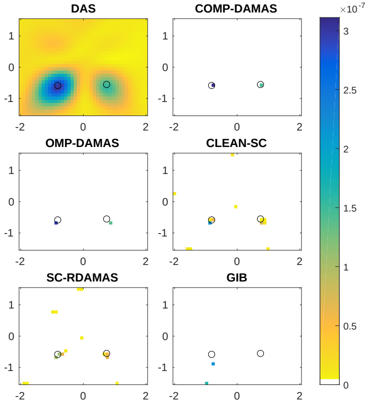

Scenario 1
Scenario 2
Scenario 3
Average position error [m]
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
∞
0
-6
-9
-12
Average relative power error [%]
40
35
30
25
20
15
10
5
0
Single-channel SNR [dB]
COMP-DAMAS
OMP-DAMAS
CLEAN-SC
SC-RDAMAS
GIB

Experimental setup

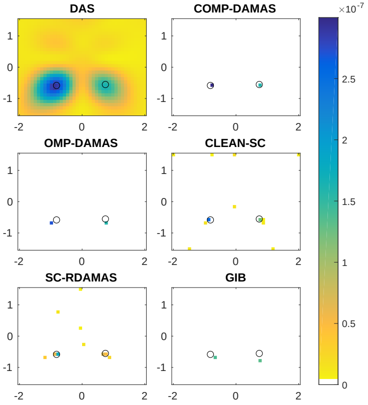
- **Single frequency delay-and-sum at 2.4kHz, sampled at 44.1kHz**
- **Two white noise sources, 10, 100 & 1000 snapshots of 128 samples**
- **4m x 3m imaging area, 40x30=1200 grid points, distance 2.7m from array**
- **16x16-element array, 39cm x 39cm**



Experimental results



30ms (10 snapshots)



3s (1000 snapshots)

Experimental accuracy

	Average position error [m]		
	$K = 10$	$K = 100$	$K = 1000$
COMP-DAMAS	**0.03**	0.07	**0.03**
OMP-DAMAS	0.13	0.15	0.15
CLEAN-SC	0.10	**0.04**	0.04
SC-RDAMAS	0.07	0.07	0.07
GIB	$0.28^{\dagger}$	$0.08^{\dagger}$	0.18

Average relative runtimes (300 grid points, deconvolution only)

COMP-DAMAS	OMP-DAMAS	CLEAN-SC	SC-RDAMAS	GIB	(Beamforming)
2.5	1	160	800	1900	20

Conclusion

- Lower overall position & power error than benchmark methods
- Low runtime and short required integration time => suitable for realtime implementation

Future work

- Parameter-free regularization
- Simultaneous fitting of multiple sources in stage 2

Thank you for listening.