# Enhancing Observability in Power Distribution Grids

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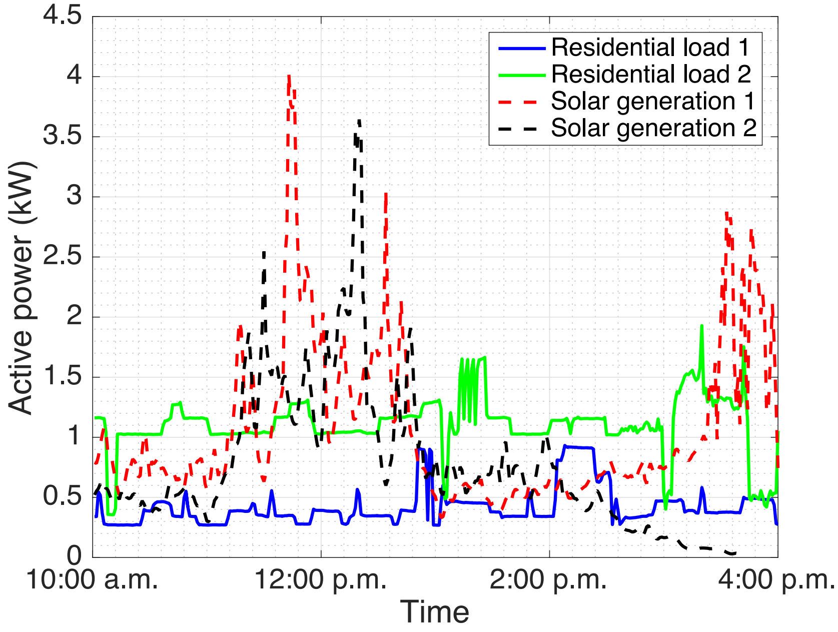
# Motivation

- Reduced observability due to limited metering infrastructure
- Leverage smart meter data **[1]**
- Smart PV inverters can be commanded within micro-sec.
- System state needed for grid dispatch tasks



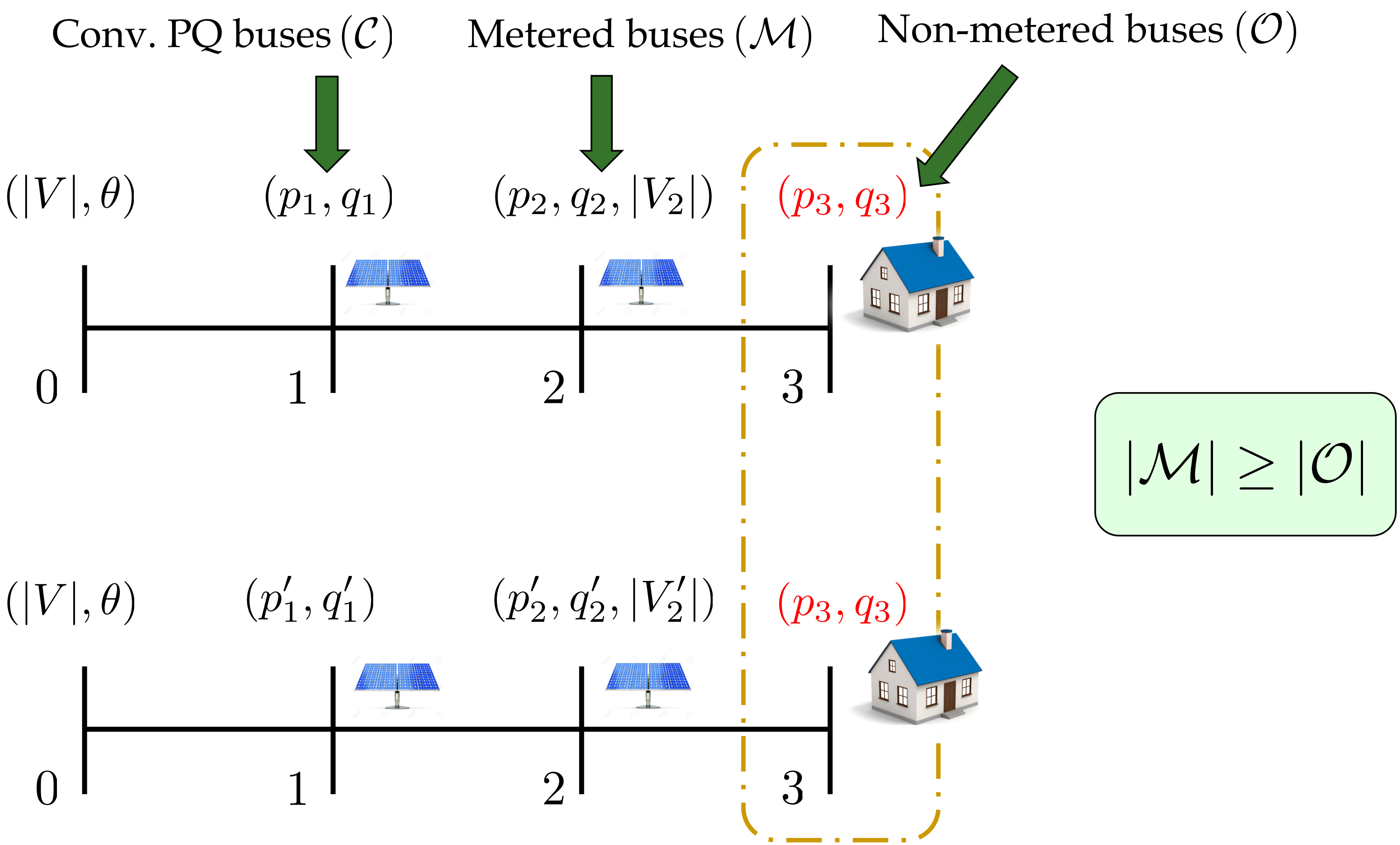
# Problem statement

- Define the following set of buses
  - Set $\mathcal{M}$ (metered buses): $(p_n, q_n, |V_n|^2)$ specified
  - Set $\mathcal{O}$ (non-metered buses): no information is available
  - Set $\mathcal{C}$ of conv. PQ buses: $(p_n, q_n)$ specified
  - Substation bus $\mathcal{S} = \{0\}$

$$\mathcal{N} := \{1, \ldots, N\} = \mathcal{C} \cup \mathcal{M} \cup \mathcal{O}$$

- Given smart meter data find the system state $\mathbf{v}$ , i.e. the vector of voltages and infer the unobserved loads in $\mathcal{O}$

# Coupled power flow (CPF)



# Coupling flows via grid probing

- Grid transitions from state $\mathbf{v} \rightarrow \mathbf{v}'$
- Grid sensing on $\mathcal{M}$ over two consecutive time instances

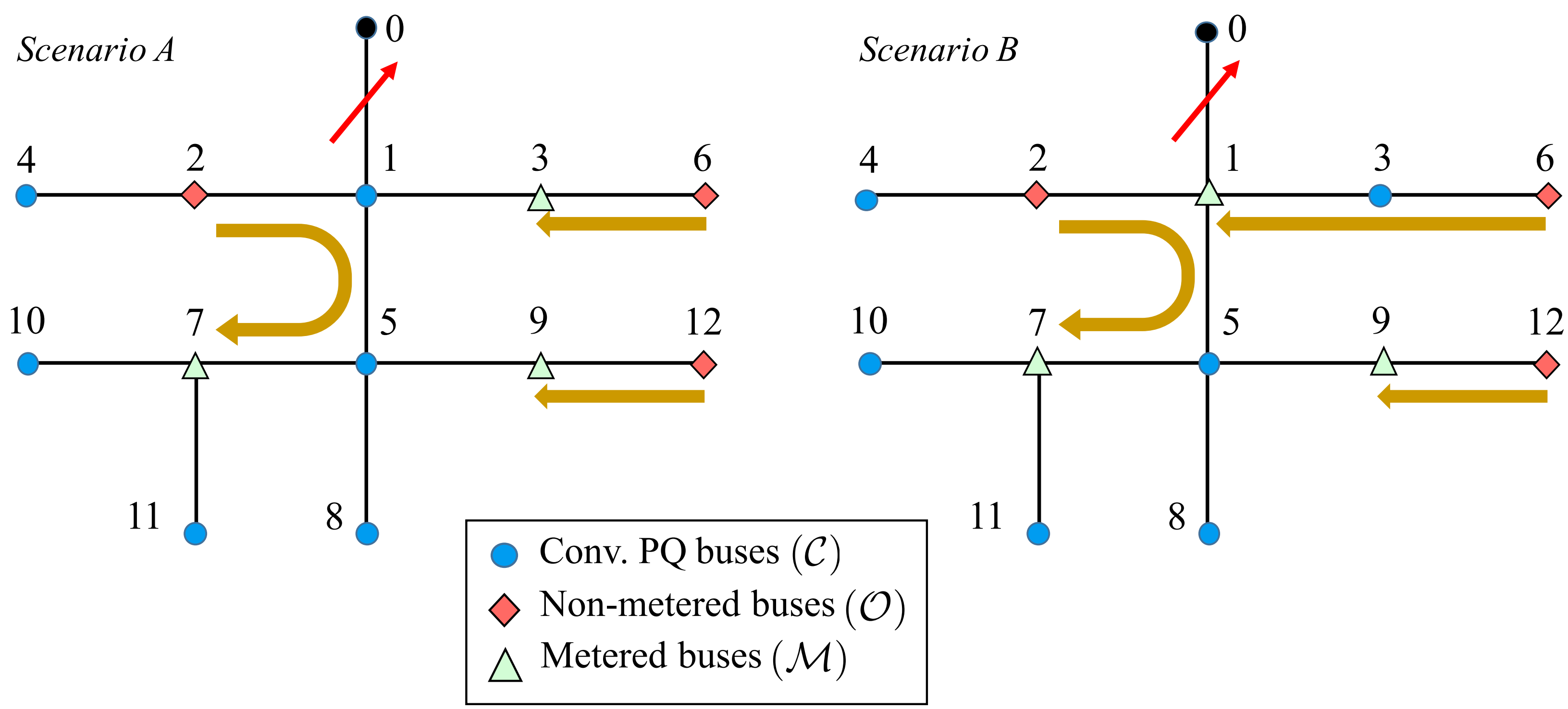
$$\left.\begin{aligned} v_{i,0}(\mathbf{v})^2 &= 0 & & \\ |V_n(\mathbf{v})|^2 &= |\hat{V}_n|^2 & & \forall n \in \mathcal{S} \cup \mathcal{M} \\ q_n(\mathbf{v}) &= \hat{q}_n & & \forall n \in \mathcal{C} \cup \mathcal{M} \\ p_n(\mathbf{v}) &= \hat{p}_n & & \forall n \in \mathcal{C} \cup \mathcal{M} \end{aligned}\right\} \text{ Time instance } t$$

$$\left.\begin{aligned} p_n(\mathbf{v}) &= p_n(\mathbf{v}') & & \forall n \in \mathcal{O} \\ q_n(\mathbf{v}) &= q_n(\mathbf{v}') & & \forall n \in \mathcal{O} \end{aligned}\right\} \text{ Coupling}$$

$$\left.\begin{aligned} v_{i,0}(\mathbf{v}')^2 &= 0 & & \\ |V_n(\mathbf{v}')|^2 &= |\hat{V}_n'|^2 & & \forall n \in \mathcal{S} \cup \mathcal{M} \\ q_n(\mathbf{v}') &= \hat{q}_n' & & \forall n \in \mathcal{C} \cup \mathcal{M} \\ p_n(\mathbf{v}') &= \hat{p}_n' & & \forall n \in \mathcal{C} \cup \mathcal{M} \end{aligned}\right\} \text{ Time instance } t'$$

# Criterion for solvability

*If there exists a set of vertex-disjoint paths from* $\mathcal{O} \rightarrow \mathcal{M}$ *in* $\mathcal{T} = (\mathcal{N}, \mathcal{L})$ *then the CPF equations are locally invertible in general.*



# Solving PF as an SDP

- **PF as a feasibility problem**

$$\begin{aligned} \text{find} \quad & \mathbf{v} \in \mathbb{C}^n \\ \text{s.t.} \quad & \mathbf{v}^H \mathbf{M}_k \mathbf{v} = \hat{s}_k, \quad k = 1, \ldots, m \end{aligned}$$

- **PF in standard form [2]**

$$\begin{aligned} \min_{\mathbf{V}} \quad & \operatorname{tr}(\mathbf{M}\mathbf{V}) \\ \text{s.t.} \quad & \operatorname{tr}(\mathbf{M}_k \mathbf{V}) = \hat{s}_k \quad k = 1, \ldots, m \\ & \mathbf{V} \succeq 0 \\ & \operatorname{rank}(\mathbf{V}) = 1 \end{aligned}$$

$$\mathbf{V} = \mathbf{v}\mathbf{v}^H$$

# Coupled PSSE

- Measurements are corrupted with noise and/or unknown loads may change over time.

$$\hat{s}_k = s(\mathbf{v}) + \epsilon_k$$

- Rather than solving exactly the CPF equations, minimize a least-squares cost

$$\underbrace{\sum_k (\hat{s}_k - \mathbf{v}^H \mathbf{M}_k \mathbf{v})^2}_{\text{Time instance } t} + \underbrace{\sum_k (\hat{s}_k - \mathbf{v}'^H \mathbf{M}_k \mathbf{v}')^2}_{\text{Time instance } t'} + \underbrace{\sum_k (\mathbf{v}^H \mathbf{M}_k \mathbf{v} - \mathbf{v}'^H \mathbf{M}_k \mathbf{v}')^2}_{\text{Coupling}}$$

# SDP-based PSSE

$$\min_{\mathbf{V},\mathbf{V}',\boldsymbol{\epsilon}} \sum_{\ell=1}^{2L+2O} f_\ell(\epsilon_\ell) + \alpha[\mathrm{tr}(\mathbf{M}\mathbf{V}) + \mathrm{tr}(\mathbf{M}\mathbf{V}')]$$

$$\begin{aligned} \text{s.t. } & \mathrm{tr}(\mathbf{M}_\ell \mathbf{V}) + \epsilon_\ell = \hat{s}_\ell, \quad \ell = 1, \ldots, L \\ & \mathrm{tr}(\mathbf{M}_\ell \mathbf{V}') + \epsilon_\ell = \hat{s}'_\ell, \quad \ell = L+1, \ldots, 2L \\ & \mathrm{tr}(\mathbf{M}_\ell (\mathbf{V}' - \mathbf{V})) = \epsilon_\ell, \; \ell = 2L+1, \ldots, 2L+2O \\ & \mathbf{V} \succeq \mathbf{0}, \mathbf{V}' \succeq \mathbf{0}. \end{aligned}$$

- **Least-Squares (LS) [3],[4],[5],[6]**

$$f(\boldsymbol{\epsilon}) = \sum_{\ell=1}^{2L+2O} \epsilon_\ell^2$$

- **Least-Absolute Value (LAV)**

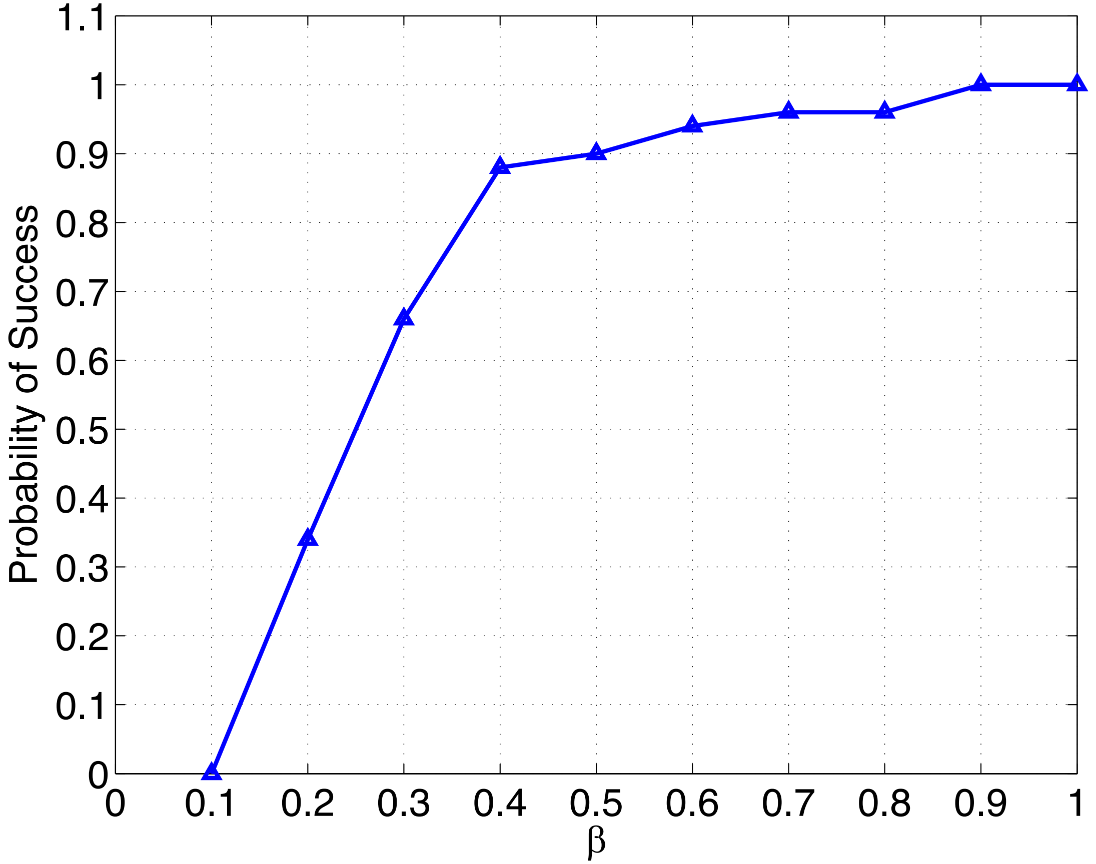
$$f(\boldsymbol{\epsilon}) = \sum_{\ell=1}^{2L+2O} |\epsilon_\ell|$$

- **Strengthen SDP relaxation**

$$\mathrm{tr}(\mathbf{M}_{pn}\mathbf{V}) < 0 \qquad \forall n \in \mathcal{O}$$

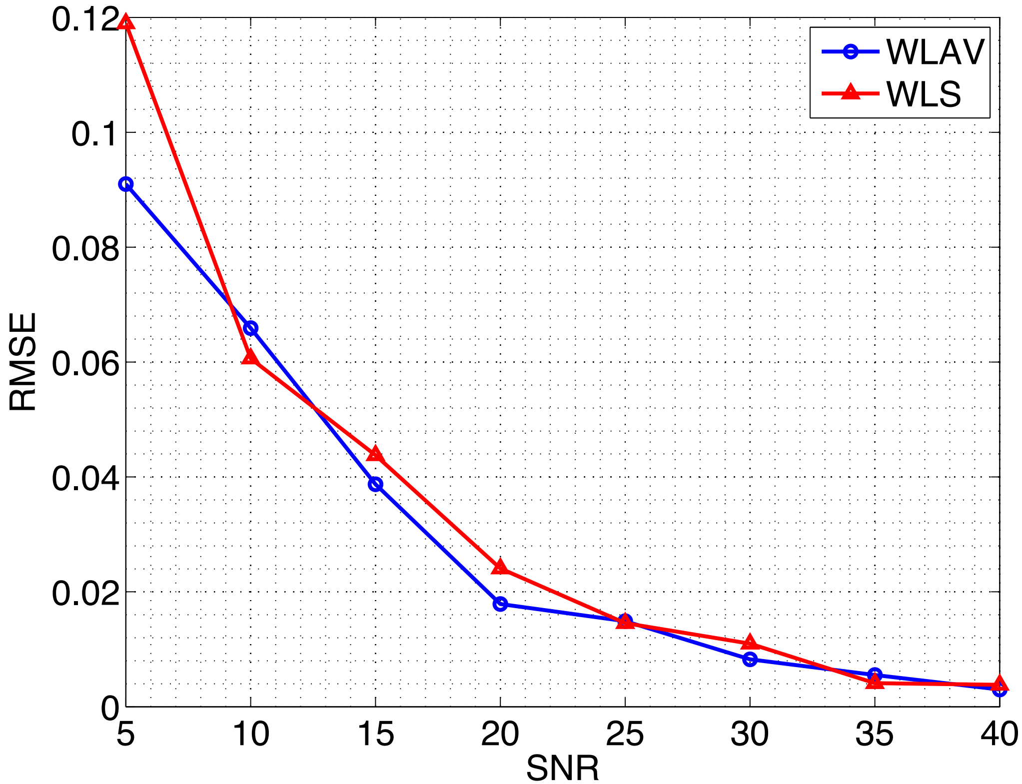
$$\mathrm{tr}(\mathbf{M}_{pn}\mathbf{V}) < 0 \qquad \forall n \in \mathcal{O}$$

# Probability of success



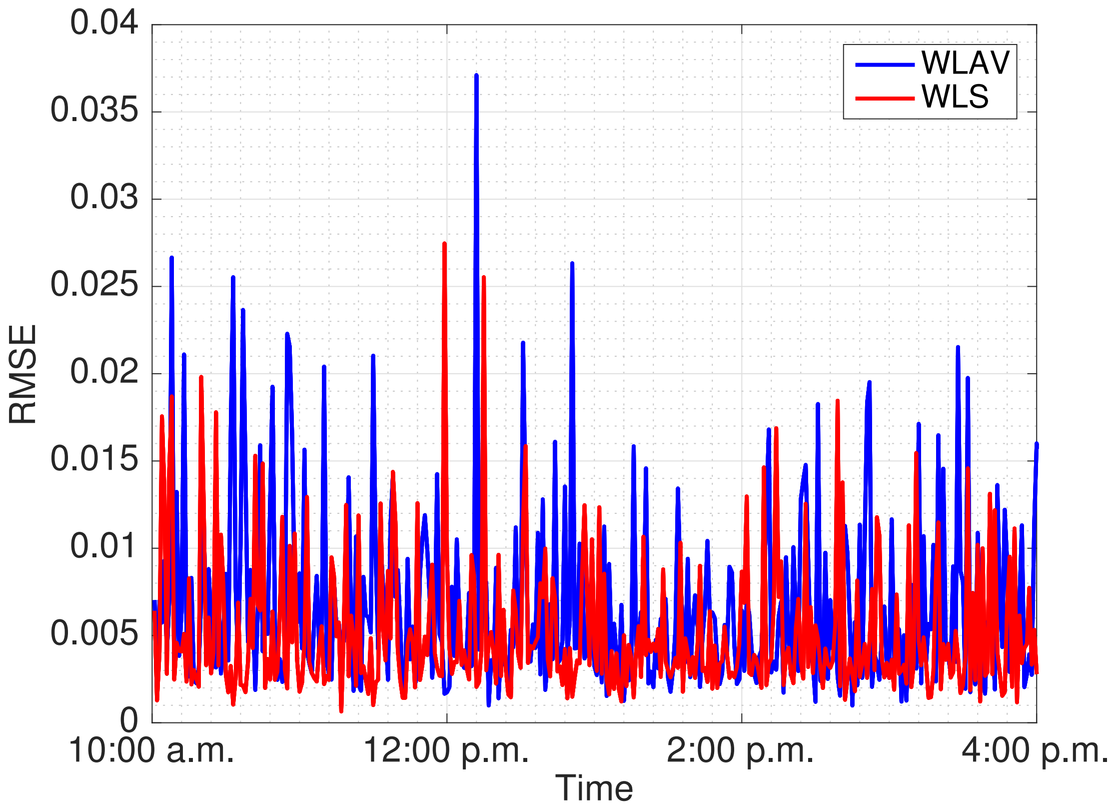

- $\beta$ captures the $\ell_2$-norm between the two states $(\mathbf{v}, \mathbf{v}')$ **[6]**
- As $||\mathbf{v} - \mathbf{v}'||_2$ increases, grid probing changes system state

# Estimation error




- Averaged over **20** simulations for each step **[6]**
- RMSE: $\sqrt{\sum_{t=1}^{N} \frac{(\hat{v}_t - v_{t,true})^2 + (\hat{v}'_t - v'_{t,true})^2}{2N}}$

# Numerical tests with real data




- Simulation over one day with **real data** [1],[6]
- Grid probing to change system state
- RMSE is very low

# Conclusions

- Used smart inverters for grid probing
- Novel technique using time coupling
- Fewer measurements needed to estimate the system state
- Characterized local invertibility via an intuitive **criterion**
- Developed an SDP-based solver for coupled power flow (CPF) and coupled power system state estimation (CPSSE).

# References


- **[1]** Pecan street Inc. Dataport 2013. Available: https://dataport.pecanstreet.org/
- **[2]** R. Madani, J. Lavaei, and R. Baldick, “Convexification of power flow problem over arbitrary networks,” in Proc. IEEE Conf. on Decision and Control, Osaka, Japan, Dec. 2015.
- **[3]** H. Zhu and G. B. Giannakis, "Power System Nonlinear State Estimation Using Distributed Semidefinite Programming," *IEEE Journal of Selected Topics in Signal Processing*, vol. 8, no. 6, pp. 1039-1050, Dec. 2014.
- **[4]** C. Klauber and H. Zhu, “Distribution system state estimation using semidefinite programming,” in Proc. North American Power Symposium, Charlotte, NC, Oct. 2015.
- **[5]** R. Madani, A. Ashraphijuo, J. Lavaei, and R. Baldick, “Power system state estimation with a limited number of measurements,” in Proc. IEEE Conf. on Decision and Control, Las Vegas, NV, Dec. 2016.
- **[6]** S. Bhela, V. Kekatos, and S. Veeramachaneni, "Enhancing Observability in Distribution Grids using Smart Meter Data," *IEEE Trans. on Smart Grid*, Feb. 2016 (accepted).