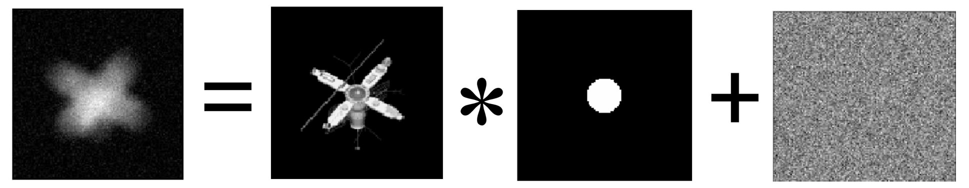
Blind Image Deconvolution Using Student's-t Prior With Overlapping Group Sparsity

In S. Jeon, Deokyung Kang, Suk I. Yoo
Dept. of Computer Science and Engineering, Seoul National University, Seoul, South Korea

INTRODUCTION

BLIND IMAGE DECONVOLUTION PROBLEM



Observed image y | Unknown true image x | Unknown blur kernel h | Unknown noise ε

GOAL: Given y recover both x and h

HARD: infinitely many possible combinations of x and h exist

We need to set priors on them...

BAYESIAN FORMULATION

$p(x,h \mid y) \propto p(y \mid x,h)\, p(x)\, p(h)$ — POSTERIOR of x, h

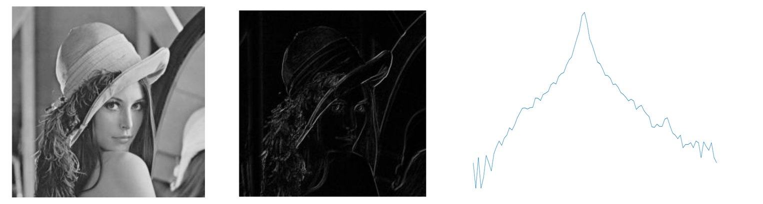
$p(y \mid x,h) = N(y \mid Hx, \sigma^2 I)$ — LIKELIHOOD assuming white Gaussian noise

$p(h) \propto 1$ — KERNEL PRIOR, we choose the flat prior

$p(x)$ — IMAGE PRIOR, main topic of this paper!

STUDENT'S-T IMAGE PRIOR

SPARSE IMAGE PRIOR

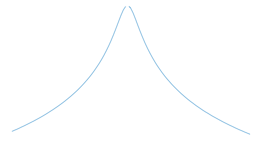


Natural image x | Filtered Image $g_m = F_m x$ | Histogram of log coefficients

KNOWLEDGE: when high-pass filters are applied to natural images, the resulting coefficients are sparse.

IMAGE PRIOR: $p(F_m x)$ is traditionally set to sparsity-enforcing priors (ex. total variation, hyper-Laplacian, and Student's-t).

STUDENT'S-T PRIOR



$$p(x,\gamma) \propto p(x \mid \gamma)\, p(\gamma)$$
$$\propto \prod_{m=1}^{M} N(x \mid 0, (F_m^T \Gamma F_m)^{-1}) \times \prod_{m=1}^{M}\prod_{i=1}^{N} \mathrm{Gamma}(\gamma_{m,i} \mid \alpha, \beta)^{-1}$$

We use **a hierarchical image prior**:
$p(F_m x_i \mid \gamma_{m,i})$ ~ Gaussian with precision γ
$p(\gamma_{m,i})$ ~ Gamma distribution with α, β.
Marginalization w.r.t γ is equivalent to Student's-t

DERIVATION OF OBJECTIVE

MAP estimation is equivalent to minimizing the negative log posterior.

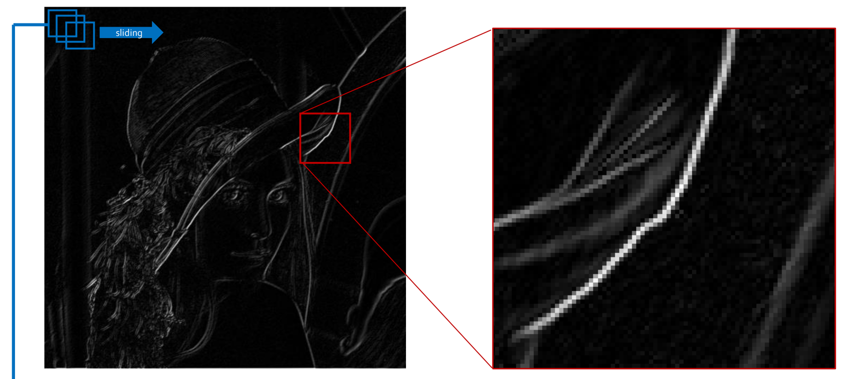
$$\min_{x,\gamma,h} -\log p(x,\gamma,h \mid y)$$
$$= \min_{x,\gamma,h} -\log p(y \mid x,h)\, p(x,\gamma)\, p(h)$$
$$= \min_{x,\gamma,h} -\log(p(y \mid x,h)) - \log(p(x,\gamma)) - \log(p(h))$$
$$= \min_{x,\gamma,h} \| Hx - y \|^2 + \lambda_1 \psi(x,\gamma)$$

where $\psi(x,\gamma) = \sum_{m=1}^{M} x^T F_m^T \Gamma(\gamma_{m,i}) F_m x + 2\sum_{m=1}^{M}\sum_{i=1}^{N}((1-\alpha)\log\gamma_{m,i} + \beta\gamma_{m,i})$

$\psi(x,\gamma)$ is regularization term obtained from Student's-t prior **promoting sparsity**.
However, it dose not take account the structural information among the coefficients.

OVERLAPPING GROUP SPARSITY

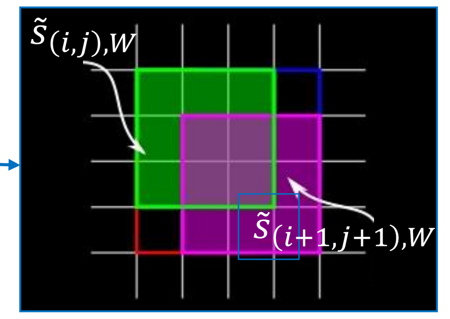
STRUCTURED INFORMATION



If we look at **the coefficients** carefully, we can see that **they are not really isolated**.

Instead, **they tend to group together**.

OVERLAPPING GROUP SPARSITY



To capture this property, we define $\tilde{s}_{(i,j),W}$: a group of $W \times W$ contiguous samples centered at coordinates (i,j).

Then, **overlapping group sparsity (OGS)** functional is
$$\varphi_{OGS}(s) = \sum_{(i,j)=1}^{n} \| \tilde{s}_{(i,j),W}(:) \|_2$$

FINAL PROBLEM FORMULATION

$$\min_{x,\gamma,h}(R(x) = \| Hx - y \|^2 + \lambda_1 \psi(x,\gamma) + \lambda_2 \phi(x))$$
where $\phi(x) = \sum_{m=1}^{M} \varphi_{OGS}(F_m x)$

If set $W = 1$, ϕ(x) is commonly used anisotropic TV prior.

If set $W > 1$, $\phi(x)$ is a group sparsity regularization term (or generalized TV).

INFERENCE

MAJORIZATION-MINIMIZATION

$$G(x,x') = \| Hx - y \|^2 + \lambda_1 \psi(x,\gamma) + \lambda_2 \phi'(x,x')$$
$$\geq R(x) = \| Hx - y \|^2 + \lambda_1 \psi(x,\gamma) + \lambda_2 \phi(x)$$

where $\phi'(x,x') = \sum_{m=1}^{M} x^T F_m^T \Lambda(F_m x') F_m x$

and $[\Lambda(u)]_{l,l} = \sum_{i,j=-(W-1)/2}^{W/2} \left[\sum_{k_1,k_2=-(W-1)/2}^{W/2} | u_{(r-i+k_1, t-j+k_2)} |^2 \right]^{-\frac{1}{2}}$

To efficiently solve the opt-problem, we iteratively **minimize an upper-bound** **G(x,x')** instead of minimizing **R(x)**. x' is the estimation of x at the previous iteration.

Equation for $[\Lambda(u)]_{l,l}$ looks terrifying but it's pretty simple in matlab.

$\Lambda(F_m x)$
= imfilter(1./sqrt(imfilter(dx.^2, boxfilt)),boxfilt)

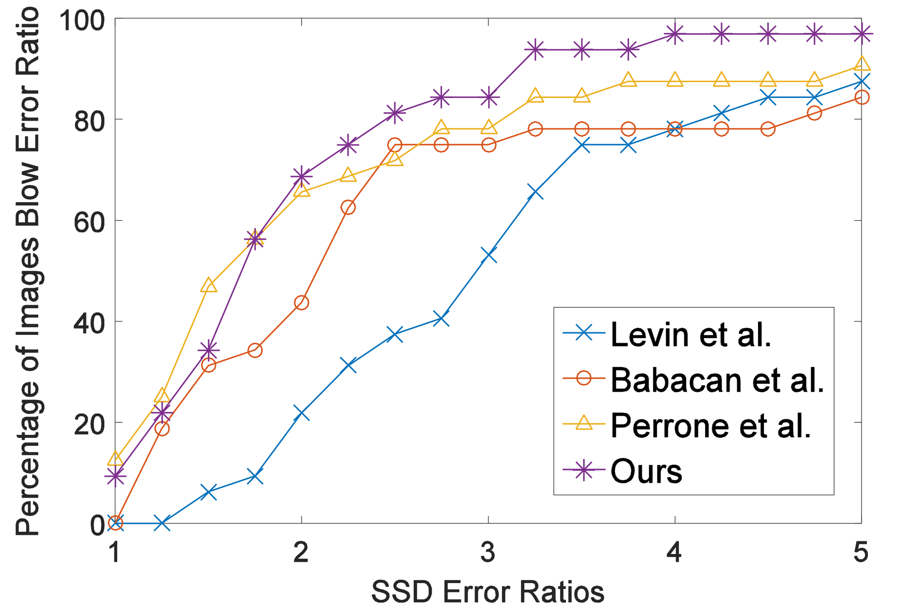
BLIND DECONVOLUTION ALGORITHM

1. $g_{m,i}^{(t)} = F_m x^{(t)}$, $\gamma_{m,i}^{(t+1)} = (\alpha + 1/2)/(\beta + (1/2)\left(g_{m,i}^{(t)}\right)^2)$ (get dx and update γ)
2. $x^{(t+1)} = ((H^T H)^{-1} + \sum_{m=1}^{M}(F_m^T \left(\lambda_1 \Gamma\left(\gamma_{m,i}^{(t)}\right) + \lambda_2 \Lambda\left(g_{m,i}^{(t)}\right)\right) F_m)^{-1}) H^T y$ (update x)
3. $h^{(t+1)} = (X^T X)^{-1} X^T y$ (update h)
4. $t = t + 1$
5. Go back to 1 until x converges (or t < max-iteration) (iteratively)

EXPERIMENT

Algorithm evaluation is based on the dataset from Levin et al. The dataset is made of 4 images of size 255x255 pixels blurred with 8 different blur kernels.

Algorithm	SSD Avg. Error	Std. SSD Error	Avg. SSD Ratio
Levin et al.	56.43	11.55	3.40
Babacan et al.	67.72	63.21	3.64
Perrone et al.	36.42	11.69	2.24
Ours	**31.84**	**5.20**	**1.93**



SSD, sum of squared distance between the recovered images and the ground truth images, measures **the quality of recovered images**.

SSD ratio (Levin et al.), $\sum_{i=1}^{N}(x_i^L - x_i^G)^2 / \sum_{i=1}^{N}(x_i^H - x_i^G)^2$, measures **the effectiveness of estimated blur kernels**.
(x_i^G:ground truth, x_i^L:deconvolution w/ estimated kernel, x_i^H:deconvolution w/ true kernel)

CONCLUSION

- In this paper, we presented a blind image deconvolution algorithm combining Student's-t image prior and the overlapping group sparsity (OGS).
- To the best of our knowledge, this is the first work that the structured group sparsity is employed to solve the "blind" image deconvolution problem.

REFERENCE

[1] A. Levin, Y. Weiss, F. Durand, and W. T. Freeman, "Efficient marginal likelihood optimization in blind deconvolution," in CVPR 2011
[2] S. D. Babacan, R. Molina, and M. N. Do, "Bayesian blind deconvolution with general sparse image priors," in ECCV, 2012.
[3] D. Perrone and P. Favaro, "A logarithmic image prior for blind deconvolution," Int. J. Comput Vis., 2015.