Jointly Optimal Power and Rate Allocation for Layered Broadcast Over Amplify-and-Forward Relay Channels

Mohamed A. Attia[†], Mohammad Shaqfeh[*], Karim G. Seddik[†], and Hussein Alnuweiri[*]

[†]American University in Cairo, Egypt
[*]Texas A&M University in Qatar

November 27, 2015

Motivation

- For single layer transmission, all transmitted bits are equally protected.
- Multi-layer transmission combines
  - Successive refinement layered source coding.
  - Ordered protection levels of the source layers.
- Therefore, the **base** source layer is given higher priority than the **enhancement** source layers.

Motivation (cont'd)

- As a result:
  - For **faded** channel: **Some** information is decoded.
  - For **good** channel: **All** information is decoded.
- Consequently, **outage probability** is decreased.

Motivation (cont'd)

- We are interested in multilayer transmission using broadcast approach:
  - source layers are protected using different channel codewords.
  - All source layers are jointly transmitted using superposition coding.
  - Then they are decoded using successive interference cancellation at the receiver.
- Our contribution is on the investigation of multilayer transmission on a relay channel.

Previous Work

Our Previous Work in terms of multilayer transmission on a relay channel:

- Optimal power allocation for 2-layer transmission over selection relaying decode-and-forward (SDF).
- Optimal power allocation for M-layer transmission over relaying Amplify-and-forward (AF).

Previous Work - SDF Relays

SDF Relays:

- We have investigated the 2-layer transmission.
- L_1 is the base layer, and L_2 is the enhancement layer.
- L_2 refines the description in L_1.
- We have applied the SDF strategy.
- We solved the power optimization problem over the 2 layers.
- We found that extending the solution for any number of layers becomes prohibitively complex.

Previous Work - AF Relays

AF Relays:

- We have investigated the Multi-layer transmission for any number of layers M.
- L_1 is the base layer, and the upper layers are the enhancement layers.
- Each layer refines the information form all lower layers successively (Successive Refinement SR).
- We have applied the AF strategy.
- An approximation was found for the end-to-end channel condition.
- This approximation allows for applying a previous algorithm for solving the power optimization problem.

Preliminaries

Utility function:

- It describes the user satisfaction.
- Function of total rate decoded successfully $\bar{R}$.
- For example:
  - Maximize expected rate $U(\bar{R}) = \bar{R}$
  - Minimize expected distortion $U(\bar{R}) = 1 - e^{-2\bar{R}}$

System Model

- We consider a system that consists of three nodes; source, destination and relay.
- We assume that the source is Gaussian and it is encoded into M layers with fixed rates.
- The relay is half-duplex and applies Amplify-and-Forward (AF) strategy.

System Model (cont'd)

- Therefore, the transmission is carried over two consecutive time slots of equal duration and bandwidth.
  - **The first time slot:** The source broadcasts the layers to the relay and the destination.
  - **The second time slot:** If the relay forwards the layers after amplifying to the destination.

System Model (cont'd)

- We denote the SNR over the three links of the relay channel using γ_{sr}, γ_{sd} and γ_{rd}.
- we assume that the source and the relay only know the statistics of the channels.

End-to-End Channel Condition

- The destination combines the two copies from the source and the relay using MRC.
- It was found the end-to-end channel quality γ as

$$\gamma = \gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1}.$$

- In order to decode a layer i, all previous layers should be first decoded successfully.

$$R_j \leq \frac{1}{2}\log\left(1 + \frac{\alpha_j}{\frac{1}{\gamma} + \sum_{m>j}^{M}\alpha_m}\right) \quad \forall j \leq i.$$

$$\gamma \geq \bar{\gamma}_i = \left\{\bar{\gamma}_{i-1}, \frac{1}{\frac{\alpha_i}{2^{2R_i}-1} - \sum_{m>i}^{M}\alpha_m}\right\}.$$

Channel Approximation

- The value of γ can be bounded as

$$\gamma_{sd} < \gamma \leq \gamma_{sd} + \min\left(\gamma_{sr}, \gamma_{rd}\right).$$

- Which can intuitively be written as

$$\gamma \approx \gamma_{sd} + k \min\left(\gamma_{sr}, \gamma_{rd}\right),$$

- Therefore for the fading of the channels is Rayleigh distributed, the CDF of γ:

$$F_{\gamma}(\gamma) = 1 - \frac{\beta_3}{\beta_3 - \beta'} e^{-\gamma(\beta')} + \frac{\beta'}{\beta_3 - \beta'} e^{-\gamma\beta_3}.$$

where $\beta' = \frac{\beta_2+\beta_3}{k}$, $\beta_1 = \frac{1}{\bar{\gamma}}$, $\beta_2 = \frac{1}{m_1\bar{\gamma}}$, and $\beta_3 = \frac{1}{m_2\bar{\gamma}}$.

Channel Approximation

- The appropriate value for k should be used $(0 < k \leq 1)$ such that the approximate CDF becomes as close as possible to the exact CDF of γ found numerically.

$m_1\bar{\gamma}$, $m_2\bar{\gamma}$	<0.5	0.5	0.8	1.2	2.2	4	7	10	25	50	70	150	250	500	1000	2000	6500	≥20000
<0.5	0.4	0.4	0.45	0.45	0.55	0.625	0.7	0.75	0.85	0.9	0.925	0.95	0.975	1	1	1	1	1
0.5	0.4	0.4	0.45	0.45	0.55	0.625	0.7	0.75	0.85	0.9	0.925	0.95	0.975	1	1	1	1	1
0.8	0.45	0.45	0.45	0.5	0.55	0.6	0.675	0.725	0.825	0.9	0.9	0.95	0.95	1	1	1	1	1
1.2	0.45	0.45	0.5	0.5	0.55	0.6	0.65	0.7	0.8	0.875	0.9	0.95	0.95	0.975	1	1	1	1
2.2	0.55	0.55	0.55	0.55	0.55	0.575	0.625	0.65	0.75	0.825	0.85	0.9	0.95	0.95	0.975	1	1	1
4	0.625	0.625	0.6	0.6	0.575	0.575	0.6	0.625	0.7	0.775	0.8	0.875	0.9	0.95	0.95	0.975	1	1
7	0.7	0.7	0.675	0.65	0.625	0.6	0.6	0.6	0.675	0.725	0.75	0.85	0.875	0.925	0.95	0.95	1	1
10	0.75	0.75	0.725	0.7	0.65	0.625	0.6	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	0.95	1	1
25	0.85	0.85	0.825	0.8	0.75	0.7	0.675	0.65	0.675	0.675	0.7	0.75	0.8	0.85	0.9	0.95	1	1
50	0.9	0.9	0.9	0.875	0.825	0.775	0.725	0.7	0.675	0.65	0.675	0.7	0.75	0.8	0.875	0.9	1	1
70	0.925	0.925	0.9	0.9	0.85	0.8	0.75	0.75	0.7	0.675	0.675	0.7	0.75	0.8	0.85	0.9	0.95	1
150	0.95	0.95	0.95	0.95	0.9	0.875	0.85	0.8	0.75	0.7	0.7	0.7	0.75	0.8	0.85	0.9	0.95	1
250	0.975	0.975	0.95	0.95	0.95	0.9	0.875	0.85	0.8	0.75	0.75	0.75	0.75	0.8	0.85	0.9	0.95	1
500	1	1	1	0.975	0.95	0.95	0.925	0.9	0.85	0.8	0.8	0.8	0.8	0.8	0.85	0.9	0.95	1
1000	1	1	1	1	0.975	0.95	0.95	0.95	0.9	0.875	0.85	0.85	0.85	0.85	0.85	0.9	0.95	1
2000	1	1	1	1	1	0.975	0.95	0.95	0.95	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.95	1
6500	1	1	1	1	1	1	1	1	1	1	0.95	0.95	0.95	0.95	0.95	0.95	0.95	1
≥20000	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Problem Formulation

- Our objective in this paper is to optimally allocate power for the layers in order to maximize the expected utility function

$$\max_{\alpha,R} \quad \int_0^\infty f_\gamma(\gamma) \quad U\left(\bar{R}(\gamma,\alpha,R)\right) d\gamma$$

$$\text{subject to} \qquad \sum_{i=1}^{M} \alpha_i = 1, \qquad \alpha_i \geq 0 \qquad \forall i,$$

- We start by doing the change of variables step

$$b_i = 2^{2R_i} - 1,$$

$$\bar{n}_i = \frac{1}{\bar{\gamma}_i}.$$

Problem Formulation

- Then we have the following problem

$$\begin{aligned} \max_{b,\bar{n}} \quad & \sum_{i=1}^{M} U(b_i)\left(F_n(\bar{n}_i) - F_n(\bar{n}_{i+1})\right) \\ \text{subject to} \quad & \sum_{i=1}^{M} b_i(\bar{n}_i - \bar{n}_{i+1}) = 1, \\ & 0 < \bar{n}_M \leq \bar{n}_{M-1} \leq \ldots \leq \bar{n}_1, \\ & b_M \geq b_{M-1} \geq \ldots \geq b_1 > 0, \end{aligned}$$

where $c_i = U_i - U_{i-1}$.

Problem Formulation

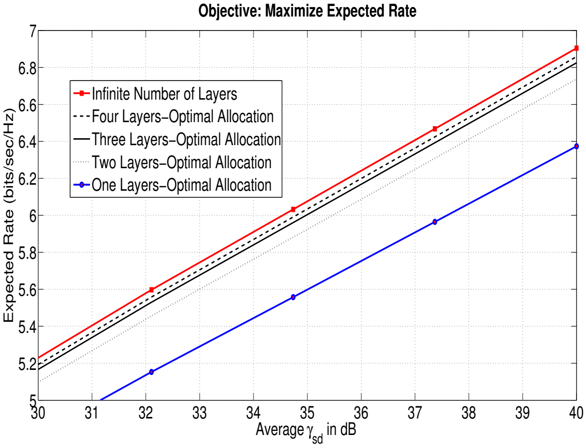
- It was found by [Shaqfeh '13] that a unique solution exists for this problem with using the full number of layers.
- Also, a strong duality between the primal and the dual problem is guaranteed.
- Then we have the following KKT conditions ($2M+1$ equations):

$$\frac{\Delta U_i}{\Delta b_i} f_n(\bar{n}_i) = \lambda \qquad \forall i,$$
$$\frac{\Delta F_i}{\Delta \bar{n}_i} U'(b_i) = \lambda \qquad \forall i,$$
$$\sum_{i=1}^{M} b_i(\bar{n}_i - \bar{n}_{i+1}) = 1,$$

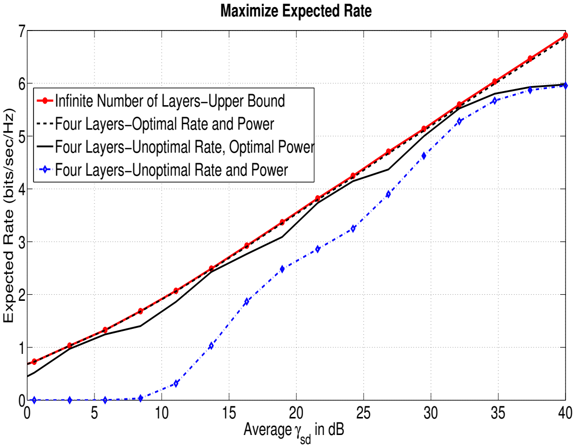
Problem Formulation

- We can solve this problem by doing 2-dimensional bisection search over λ and n_M to find $n_i's$ and $b_i's$.
- Hence we can find $\gamma_i's$ and $R_i's$ (Optimal rates and channel thresholds).

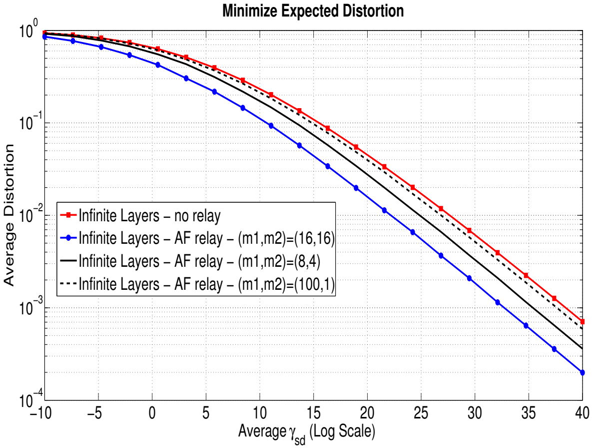
Maximize Expected Rate



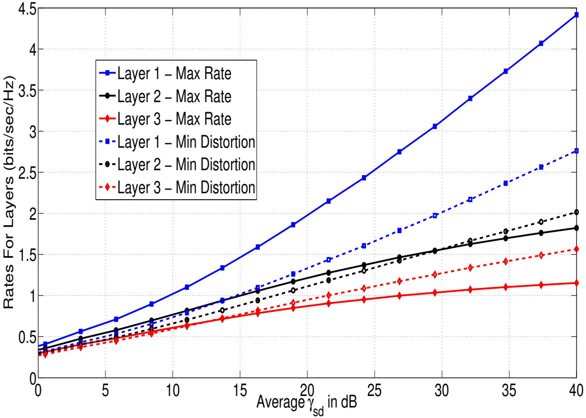
Maximize Expected Rate



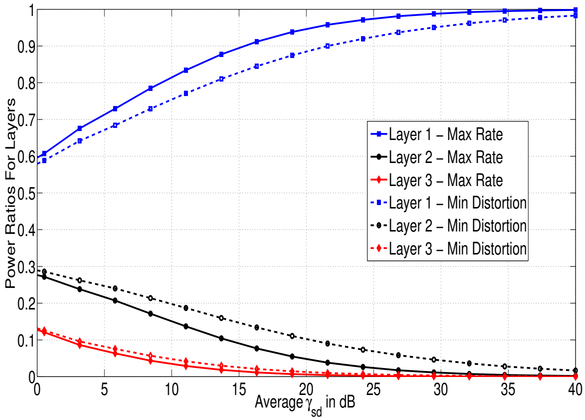
Minimize Expected Distortion



Optimal Rates for M=3



Optimal Power Ratios for M=3



Conclusions

- We have considered Multilayer transmission with M layers using the broadcast approach.
- A relay has been considered that applies AF strategy.
- We have proposed a simple approximation for the end-to-end channel statistics.
- We found a unique solution of using the full number of layers.
- We have shown that with a relatively small number of layers, we can approach the upper bound corresponding the infinite number of layers case.
- The numerical results demonstrate that for high values of SNR, the no-relay case may show better performance.

Thank you for your time and attention. **Questions?**