

# Low-Complexity MIMO Detector with 1024-QAM

Authors

**Hadi SARIEDDEEN**

**Prof. Mohammad M. MANSOUR**

**Dr. Louay M.A. JALLOUL**

**Prof. Ali CHEHAB**

December 15, 2015



# Outline:

---

## **Introduction**

- Motivation
- System Model
- Popular Detectors

## **Proposed Work**

- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

## **Results**

- Complexity Study
- Simulation Scenario
- Simulation Results

## **Summary & future work**

# Motivation

---

- Quadrature Amplitude Modulation (QAM) is rising
  - 1024QAM and beyond
  - Mainly in microwave backhaul but also in WiFi
- Broadcom announced new 5G WiFi chips
  - NitroQAM™ (1024-QAM) technology
  - 8x8 MU-MIMO
- Detection with 1024QAM
  - Near-optimal detectors are complex
  - Their low complexity versions degrade performance
- Low complexity LORD detector has limitations
  - Optimize search region
  - Optimize LLR saturation



# System Model

---

MIMO system combined with OFDM

Received signal at resource element is given by:  $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$

$\mathbf{H} = N_r \times N_t$  channel matrix

$\mathbf{x}$  transmitted QAM symbols

$\mathbf{n}$  complex additive white Gaussian noise with zero mean and variance  $\sigma^2 = \frac{N_t}{\text{SNR}}$

We consider the case  $N_r = N_t = 2$

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}$$

$\mathbf{h}_1$ : channel coefficients of user of interest

$\mathbf{h}_2$ : channel coefficients of interferer

$$E[x_1 \cdot x_1^*] = E[x_2 \cdot x_2^*] = 1$$

$x_1$  and  $x_2$  are drawn from a 1024QAM constellation  $\mathcal{M}$



# Maximum Likelihood (ML) Detection

---

A hard-output ML detector solves:

$$\min_{\mathbf{x} \in S} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

where  $S$  is the lattice of symbol vectors ( $|S| = |\mathcal{M}|^2$ )

Let  $\mathbf{b}_x = [x_b]_{b=1}^K$  be the bit vector of  $\mathbf{x}$ ,  $x_b \in \{0,1\}$  and  $K = \log_2(|S|)$

A soft-output ML detector calculates the log-likelihood ration (LLR) of bit  $b$  as:

$$\lambda_b = \min_{\mathbf{x} \in S_{b,1}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2} - \min_{\mathbf{x} \in S_{b,0}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2}$$

$S_{b,1}$  corresponds to points in  $S$  having in the bit position  $b$  a value of 1

$S_{b,0}$  corresponds to points in  $S$  having in the bit position  $b$  a value of 0



# Minimum Mean Square Error (MMSE) Detector

---

The MMSE detector solves for an equalized output  $\hat{\mathbf{y}}$ :

$$\hat{\mathbf{y}} = (\mathbf{H}^* \mathbf{H} + (1/\text{SNR}) \mathbf{I}_2)^{-1} \mathbf{H}^* \mathbf{y}$$

And the LLRs can be computed as:

$$\lambda_{\hat{b}}^t = \min_{\mathbf{x}(t) \in \mathcal{M}_{\hat{b},t,1}} \frac{|\hat{\mathbf{y}}(t) - \mathbf{x}(t)|^2}{\sigma_{\text{MMSE}}^2} - \min_{\mathbf{x}(t) \in \mathcal{M}_{\hat{b},t,0}} \frac{|\hat{\mathbf{y}}(t) - \mathbf{x}(t)|^2}{\sigma_{\text{MMSE}}^2}$$

$t \in \{1,2\}$  is the symbol index

$\sigma_{\text{MMSE}}^2 = \sigma^2 \mathbf{W}(t, t)$  is a scaled variance, where  $\mathbf{W} = (\mathbf{H}^* \mathbf{H} + (1/\text{SNR}) \mathbf{I}_2)^{-1}$

$\mathcal{M}_{\hat{b},t,1}$  corresponds to points in  $\mathcal{M}$  having in the bit position  $\hat{b}$  of symbol  $t$  a 1

$\mathcal{M}_{\hat{b},t,0}$  corresponds to points in  $\mathcal{M}$  having in the bit position  $\hat{b}$  of symbol  $t$  a 0



# Layered Orthogonal Lattice Detector (LORD)

---

QR decomposition in the preprocessing step:

$$\tilde{\mathbf{y}} = \mathbf{Q}^* \mathbf{y} = \mathbf{R} \mathbf{x} + \mathbf{Q}^* \mathbf{n} = \mathbf{R} \mathbf{x} + \tilde{\mathbf{n}}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} \\ 0 & r_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}$$

Exhaustively search layer 2, for each possibility  $\bar{x}_2$ ,  $\bar{x}_1 = \text{slice} \left( (\tilde{y}_1 - r_{1,2} \bar{x}_2) / r_{1,1} \right)$

The searched lattice of vectors  $\bar{\mathbf{x}} = [\bar{x}_1, \bar{x}_2]$  is  $\hat{S}$  ( $|\hat{S}| = |\mathcal{M}|$ )

Searching layer 2, LLRs of  $x_2$  can be computed

$$\lambda_{\hat{b}}^2 = \min_{\mathbf{x} \in \hat{S}_{\hat{b},2,1}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2} - \min_{\mathbf{x} \in \hat{S}_{\hat{b},2,0}} \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2}$$

$\hat{S}_{\hat{b},2,1}$  corresponds to points in  $\hat{S}$  having in the bit position  $\hat{b}$  of symbol 2 a value 1

$\hat{S}_{\hat{b},2,0}$  corresponds to points in  $\hat{S}$  having in the bit position  $\hat{b}$  of symbol 2 a value 0

To compute LLRs of  $x_1$  the layers should be swapped and the same operation is repeated

Output identical to ML detector with 2x2 MIMO



# Turbo LORD (T-LORD)

---

T-LORD is a generalization of LORD

It builds on the maximum-a-posteriori (MAP) detector instead of the ML detector

Used with iterative detection and decoding ( $T$  iterations)

MAP detector accepts a-priori LLRs  $\xi$  from the decoder

The modified distance metric is:

$$\varphi(\mathbf{x}) = \frac{\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2}{\sigma^2} + \sum_{k=1}^K \mathbf{b}_x(k)\xi(k)$$

The a-posteriori LLRs can then be calculated as:

$$\lambda_b^t = \max_{\mathbf{x} \in \hat{S}_{b,t,1}} \varphi(\mathbf{x}) - \max_{\mathbf{x} \in \hat{S}_{b,t,0}} \varphi(\mathbf{x})$$





# Outline:

---

## Introduction

- Motivation
- System Model
- Popular Detectors

## Proposed Work

- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

## Results

- Complexity Study
- Simulation Scenario
- Simulation Results

## Summary & future work

# Low Complexity LORD (LC-LORD)

---

Searching  $|\mathcal{M}| = 1024$  lattice points is still computationally demanding

LC-LORD only explores a subset of the constellation at the root layer

- A reduced QAM  $\theta$
- A square subset centered on equalized output  $\tilde{y}_2/r_{2,2}$

LLRs can not be computed when all points in  $\theta$  have the same bit value at a specific bit

An LLR saturation mechanism is required

Especially for high order bits when Gray mapping is employed

LC-LORD need not be applied on all carriers

- Worst carriers can be isolated and treated with regular LORD
- This depends on the implementation constraints
- Criteria for sorting worst carriers is:

$$\min_{l=1,2} r^l(2,2)$$

where  $l$  denotes the antenna index at the root layer



# Optimizing Search Region Location

LC-LORD fails when the actual transmitted symbol lies outside  $\theta$

This is worse with correlated channels

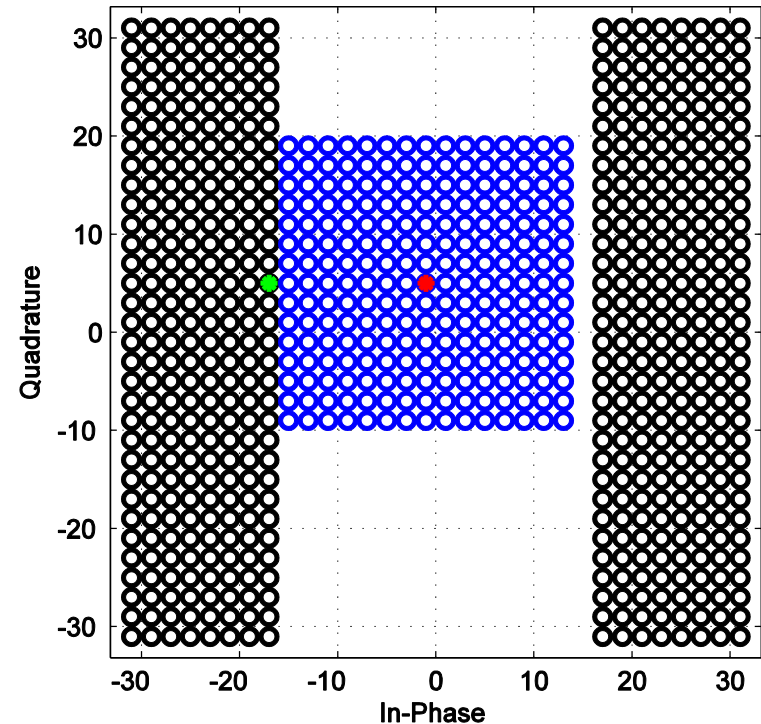
- $\mathbf{H}$  ill-conditioned
- $r_{2,2}$  tends to zero

One solution uses the hard-output of MMSE detection as a center of search on both layers

This is called MMSE-LC-LORD

Note that operations on both layers are now dependent

- Can not be fully parallelized



Constellation Schematic - Black Circles Indicate that Third MSB is 1



# Layer-Ordered LC-LORD (LO-LC-LORD)

---

This proposed solution is based on:

- Layer ordering
- Zero-forcing with decision feedback (ZFDF)

Find equalized output on layer 2

$$\bar{x}_2^1 = \text{slice}(\tilde{y}_2/r_{2,2})$$

Get its corresponding projection on layer 1

$$\bar{x}_1^1 = \text{slice}\left(\left(\tilde{y}_1 - r_{1,2}\bar{x}_2^1\right)/r_{1,1}\right)$$

We obtain  $\bar{\mathbf{x}}^1 = [\bar{x}_1^1, \bar{x}_2^1]$

Permute layers and apply same procedure to obtain  $\bar{\mathbf{x}}^2 = [\bar{x}_1^2, \bar{x}_2^2]$

The centers of reduced search on both layers are the components of  $\mathbf{x}_{\text{center}}$

$$\mathbf{x}_{\text{center}} = \min_{\mathbf{x} \in \{\bar{\mathbf{x}}^1, \bar{\mathbf{x}}^2\}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$



# Iterative LC-LORD (Iter-LC-LORD)

This solution adds an iterative behavior

The Center Generator is a hard-output LC-LORD

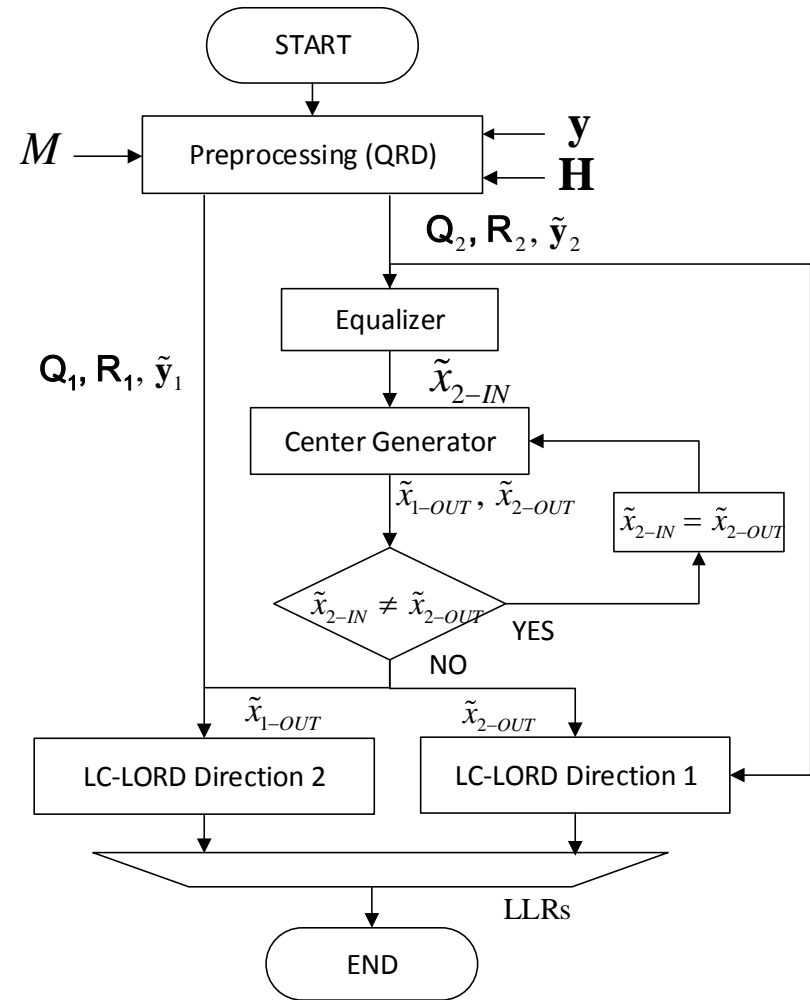
- single layer operation
- hard output constitutes updated centers of search

Updated search centers are closer to ML hard output

Might get stuck in local minima

Algorithm halts after a maximum of  $J$  iterations

With T-LORD center updates can take place on every detection/decoding iteration



# Region-Thresholding LC-LORD (RegTh-LC-LORD)

With LC-LORD one of the two terms below can go missing

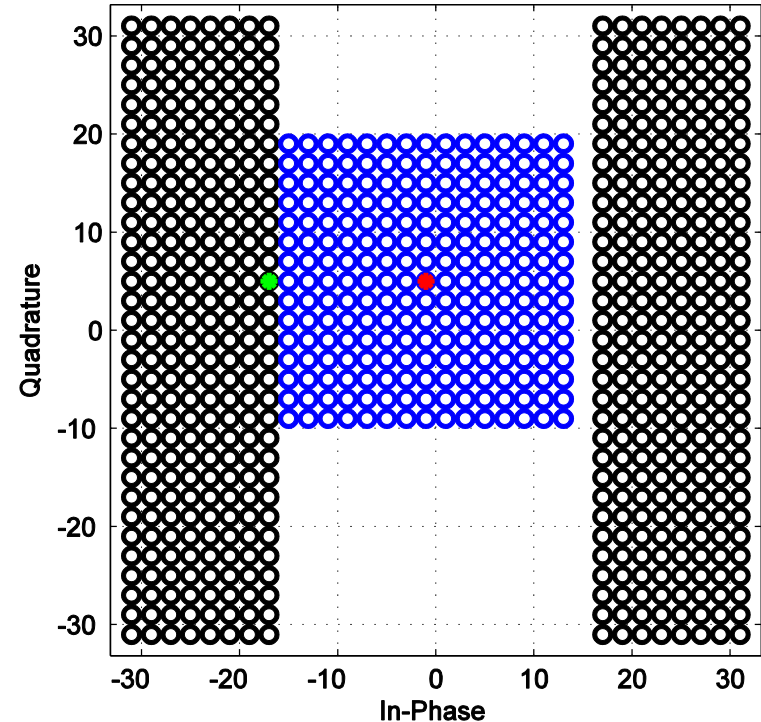
$$\lambda_b^t = \max_{\mathbf{x} \in \hat{S}_{b,t,1}} \varphi(\mathbf{x}) - \max_{\mathbf{x} \in \hat{S}_{b,t,0}} \varphi(\mathbf{x})$$

LLR saturation in Literature:

- Saturate LLR to a threshold value
- Substitute missing term by maximum Euclidean norm within  $\theta$

Proposed approach (RegTh-LC-LORD):

- Locate the closest point to the center of  $\theta$  having opposite bit value (in green)
- Project on other layer + slice
- Substitute missing term by the distance from resultant vector to received vector



Constellation Schematic - Black Circles Indicate that Third MSB is 1



# Outline:

---

## Introduction

- Motivation
- System Model
- Popular Detectors

## Proposed Work

- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

## Results

- Complexity Study
- Simulation Scenario
- Simulation Results

## Summary & future work

# Complexity Study

## Preprocessing complexity

- QR decomposition (can be avoided in 2x2 MIMO)
- Handling search region boundaries

## Search routine complexity

- Summarized in table
- In terms of Euclidean distance computations (visited nodes)
- Table shows the worst case
- When search center is close to boundaries of  $\mathcal{M}$ ,  $\theta$  gets clipped
- In Iter-LC-LORD  $\theta$ s of subsequent iterations partially overlap and computations can be saved

Approach	Description	Nodes Visited
ML	Full Complexity LORD	$2 \times  \mathcal{M} $
LC-LORD	Low Complexity LORD	$2 \times  \theta $
LO-LC-LORD	Layer Ordered LC-LORD + Region Thresholding	$2 \times  \theta $
Iter-LC-LORD	Iterative LC-LORD + Region Thresholding	$(J + 2) \times  \theta $
MMSE-LC-LORD	MMSE-based LC-LORD + Region Thresholding	$2 \times  \theta $
RegTh-LC-LORD	LC-LORD + Region Thresholding	$2 \times  \theta $
MMSE	Soft-output MMSE	$ \mathcal{M} $





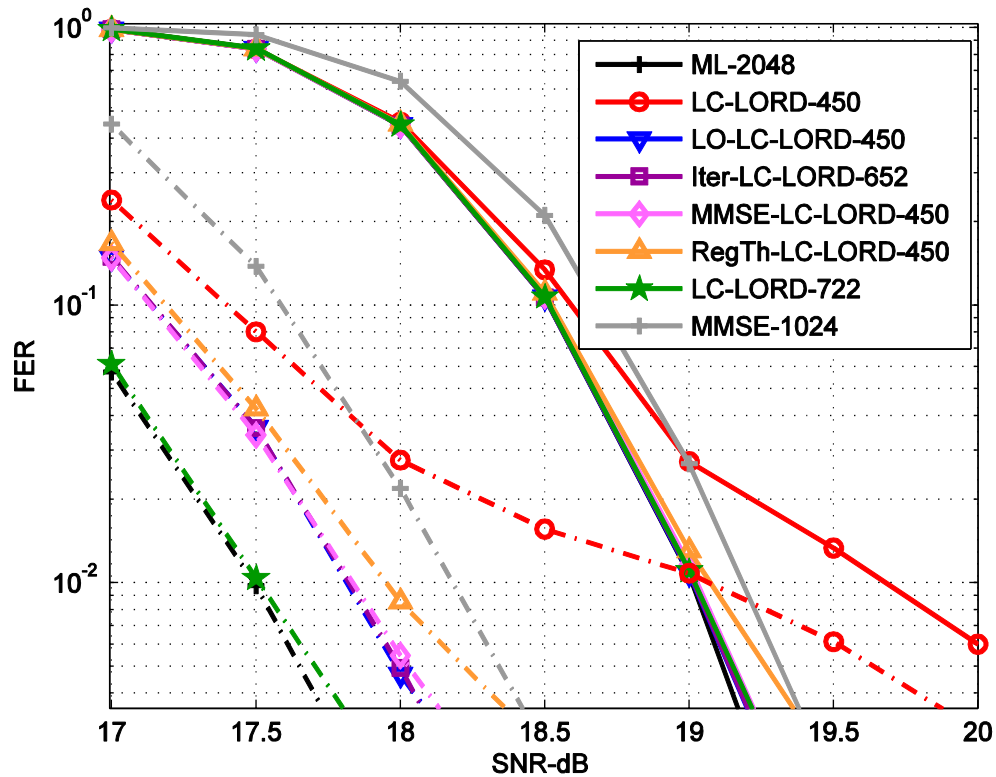
# Simulation Scenario

---

- A 2x2 MIMO simulation chain was implemented
  - System model in introduction
  - All studied detectors were implemented
  - Iterative detection/decoding
- Turbo coding/decoding
  - Code rate 1/2
  - 8 iterations
- Two channel types
  - Uncorrelated (rich scattering)
  - Highly correlated ( $\alpha = 0.9$ )
- Performance measure
  - Frame Error Rate (FER)
- Parameters
  - $|\theta| = 225$
  - $J = 8$  (1.8 on average)
  - $T = 4$



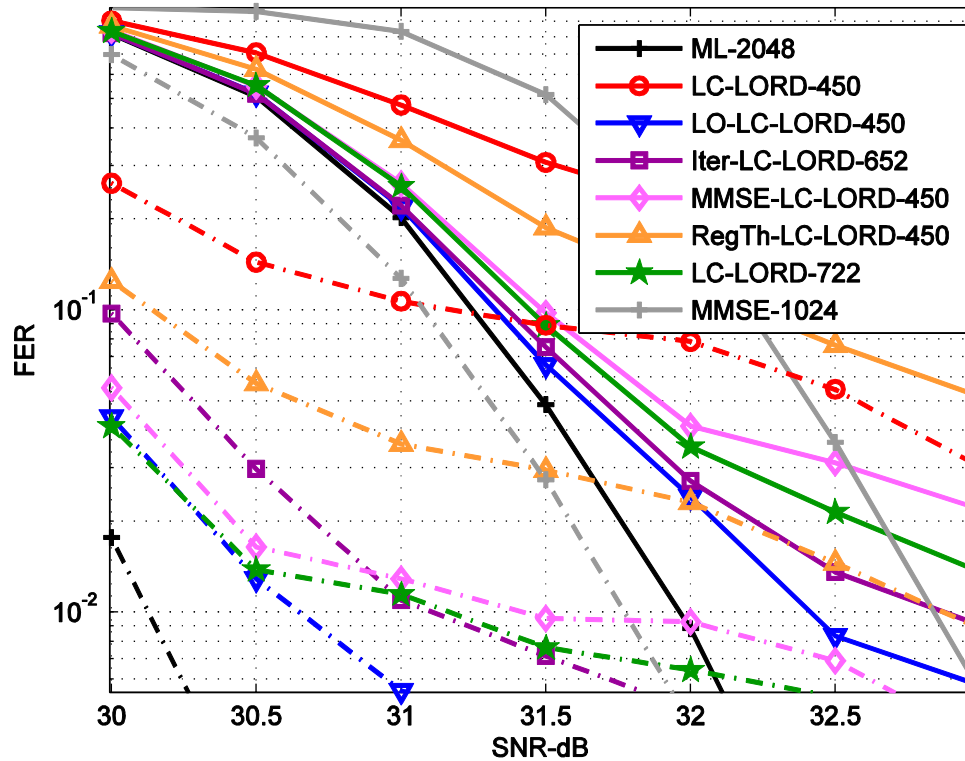
# FER – Uncorrelated



Detectors Performance with Uncorrelated Channels and 15% Full Complexity Carriers, for  $T = 1$  (solid) and  $T = 4$  (dotted)



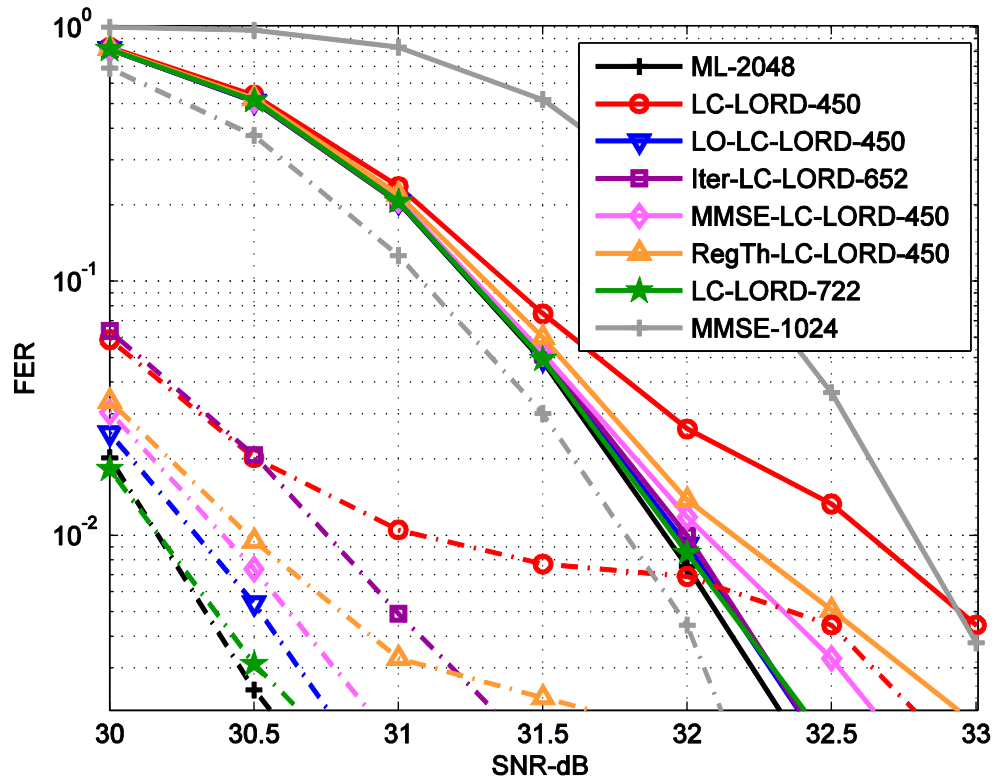
# FER – Correlated



Detectors Performance with Correlated Channels and 15% Full Complexity Carriers, for  $T = 1$  (solid) and  $T = 4$  (dotted)



# FER – Correlated



Detectors Performance with Correlated Channels and 30% Full Complexity Carriers, for  $T = 1$  (solid) and  $T = 4$  (dotted)



# Outline:

---

## Introduction

- Motivation
- System Model
- Popular Detectors

## Proposed Work

- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

## Results

- Complexity Study
- Simulation Scenario
- Simulation Results

## Summary & future work

# Summary and Future Work

---

- ❑ **2×2 MIMO** systems that use **1024-QAM** were studied.
- ❑ Building on the **LORD** detector, several algorithms were proposed.
- ❑ Optimizing the **location** of a reduced **region of search**.
- ❑ Optimizing **LLR saturation**.
- ❑ The optimizations resulted in an **enhanced performance**, at a **reduced complexity**.
- ❑ The proposed approaches are to be studied with **higher order MIMO**, where **LORD loses optimality**.

# Thanks for listening

**Hadi SARIEDDEEN**

**Prof. Mohammad M. MANSOUR**

**Dr. Louay M.A. JALLOUL**

**Prof. Ali CHEHAB**

