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Hybrid Precoding Using Long-term Channel Statistics for Massive MIMO Systems

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- Background & motivation
- Proposed hybrid precoding techniques for massive MIMO
  - Unconstrained case w/o phase shifter constraint
  - Constrained case w/ phase shifter constraint
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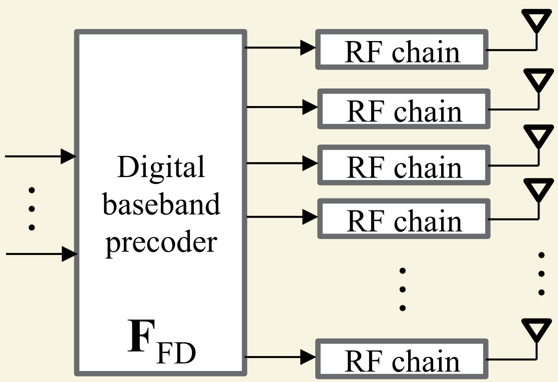
- Conclusions

Background: digital precoding vs. hybrid precoding

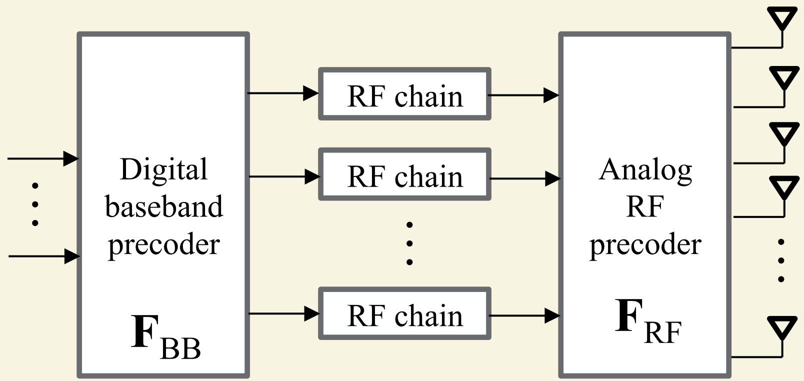
Digital baseband precoding for MU-MIMO
(Conventional MU-MIMO)



K
(# of users)

N
(# of antennas
= # of RF chains)

Hybrid analog/digital precoding for MU-MIMO



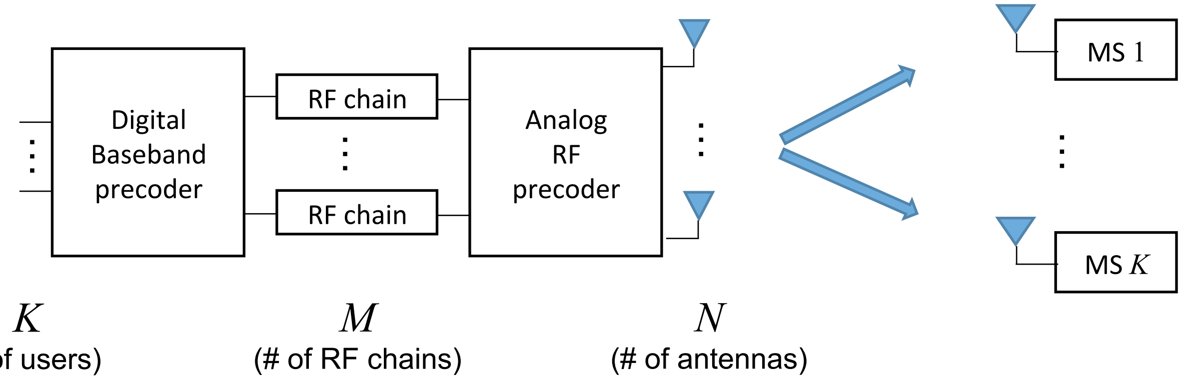
K
(# of users)

M
(# of RF chains)

N
(# of antennas)

Key feature1) $M < N$ (limited number of RF chains)
Key feature 2) $|\mathbf{F}_{\mathrm{RF}}(i,j)| = 1$ for $i=1,\ldots,N$, and $j=1,\ldots,M$ (implemented with phase shifters)

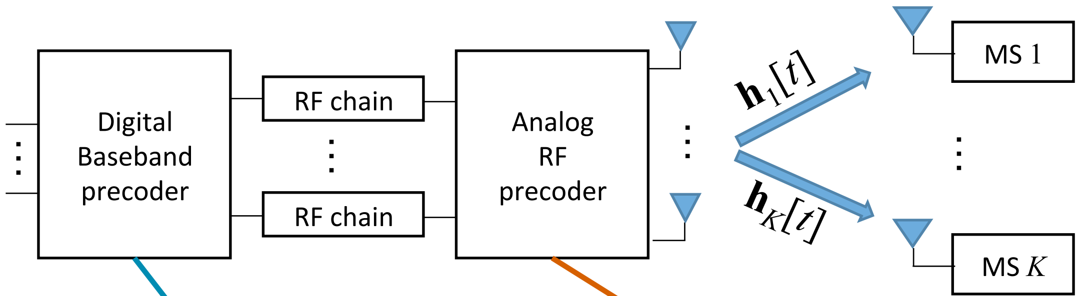
Motivation: using long-term channel statistics



	System model	Remark
Most prior work [1-3]	$\mathbf{y}[t] = \mathbf{H}[t]\mathbf{F}_{\mathrm{RF}}[t]\mathbf{F}_{\mathrm{BB}}[t]\mathbf{x}[t]+\mathbf{n}[t]$ t: frame (in time) index	Requires *instantaneous full CSIT* for $\mathbf{F}_{\mathrm{RF}}[t]$ design → May not be practical in commercial networks
Proposed	$\mathbf{y}[t] = \mathbf{H}[t]\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}[t]\mathbf{x}[t]+\mathbf{n}[t]$	Requires *only long-term CSIT* for $\mathbf{F}_{\mathrm{RF}}$ design → Can be more practical in commercial networks

1) L. Liang, W. Xu, and X. Dong, "Low-complexity hybrid precoding in massive multiuser MIMO systems," *IEEE Wireless Comm. Letters*, Dec. 2014.
2) A. Alkhateeb, G. Leus, and R. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," *IEEE Trans. on Wireless Comm.*, Nov. 2015.
3) F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE JSTSP*, Apr. 2016.

Overview of proposed techniques



Short-term baseband precoder design ($\mathbf{F}_{BB}[t]$)

Using instantaneous channel information of effective channel

→ Conventional MU-MIMO schemes like Regularized-ZF (R-ZF) can be used.

Long-term analog precoder design ($\mathbf{F}_{RF}$)

Using only long-term channel statistics [1)]

→ *A new technique is necessary.*

Proposed techniques for the analog precoder design

[Proposed technique T1]
Unconstrained case
w/o phase shifter constraint

Add the phase shifter constraint ($|\mathbf{F}_{RF}(i,j)|=1$)

[Proposed technique T2]
Constrained case
w/ phase shifter constraint

1) e.g. spatial channel covariance per user

[Proposed 1] Unconstrained case: Prior work

- Prior work
  - $\mathbf{F}_{\mathrm{RF}}$: Each column of $\mathbf{F}_{\mathrm{RF}}$ is a dominant eigenvector of each user's $\mathbf{R}_k$ matrix.

Channel model

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{w,1}\mathbf{R}_1^{\frac{1}{2}} \\ \vdots \\ \mathbf{h}_{w,K}\mathbf{R}_K^{\frac{1}{2}} \end{bmatrix}$$

$$\mathbf{h}_{w,k} : 1 \times N \ , \ \mathcal{CN}(\mathbf{0}, \mathbf{I})$$

$$\mathbf{R}_k = \mathbb{E}\left[\mathbf{h}_k^H \mathbf{h}_k\right] = \mathbf{V}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$$

(spatial channel covariance matrix of user k)

Hybrid precoding design [Prior work [1-2]]

$$\mathbf{F}_{\mathrm{RF}} = \left[\mathbf{v}_{1,max} \quad \cdots \quad \mathbf{v}_{K,max}\right]$$

where $\mathbf{v}_{k,max}$: dominant eigen vector of $\mathbf{R}_k$

$$\mathbf{F}_{\mathrm{BB}} = \left(\mathbf{F}_{\mathrm{RF}}^* \mathbf{H}\mathbf{H}^* \mathbf{F}_{\mathrm{RF}} + \frac{K\sigma^2}{P_{\mathrm{tx}}} \mathbf{I}_M\right)^{-1} \mathbf{F}_{\mathrm{RF}}^* \mathbf{H}$$

[Note] The index term $[t]$ is omitted for simplicity. Note that $\mathbf{h}_k$, $\mathbf{h}_{w,k}$, and $\mathbf{F}_{\mathrm{BB}}$ depend on t, and $\mathbf{R}_k$, $\mathbf{v}_{k,max}$, and $\mathbf{F}_{\mathrm{RF}}$ do not depend on t.

- Key idea: $\mathbf{F}_{\mathrm{RF}}$ *maximizes the long-term power of each user's desired signal* in the analog part.
- Shortcomings: Does not consider interference and can be used only when $M = K$.

1) L. Liang, Y. Dai, W. Xu, and X. Dong, "How to approach zero-forcing under RF chain limitations in large mmWave multiuser systems?," *IEEE/CIC*, Oct. 2014
2) A. Alkhateeb, G. Leus, and R. W. Heath, "Multilayer precoding for full-dimensional massive MIMO systems," *ASILOMAR*, Nov. 2014.

[Proposed 1] Unconstrained case: Proposed (1/5)

- Proposed
  - $\mathbf{F}_{\mathrm{RF}}$: The columns of $\mathbf{F}_{\mathrm{RF}}$ constitute a subspace that maximizes the performance of the baseband precoder.
    - Note: Each column is not assigned to a specific user.

Definition of *performance* : Asymptotic average SLNR [1)]
(a deterministic value as $N \to \infty$)

1) SLNR: signal-to-leakage-plus-noise ratio

	Main role of $\mathbf{F}_{\mathrm{RF}}$	Remark
Prior work	Maximize each user's long-term desired power in the analog part	Can be used only when $M = K$
Proposed approach	Maximize the performance of $\mathbf{F}_{\mathrm{BB}}$ in the analog part (i.e., help $\mathbf{F}_{\mathrm{BB}}$ to work well)	Can be used when $M \geq K$

- Key idea: $\mathbf{F}_{\mathrm{RF}}$ *helps* $\mathbf{F}_{\mathrm{BB}}$ *(R-ZF) to minimize the inter-user interference.*
- Can be applied to the case of $M \geq K$

[Proposed 1] Unconstrained case: Proposed (2/5)

1) System model (R-ZF is used in the baseband)

$$\mathbf{y} = \mathbf{H}^*\mathbf{x} + \mathbf{n} = \mathbf{H}^*\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{P}\mathbf{s} + \mathbf{n}$$

$$\mathbf{F}_{\mathrm{BB}} = \left(\mathbf{F}_{\mathrm{RF}}^*\mathbf{H}\mathbf{H}^*\mathbf{F}_{\mathrm{RF}} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_M\right)^{-1}\mathbf{F}_{\mathrm{RF}}^*\mathbf{H} = \begin{bmatrix}\mathbf{f}_{\mathrm{bb},1} & \cdots & \mathbf{f}_{\mathrm{bb},K}\end{bmatrix}$$

$$\mathbf{f}_{\mathrm{bb},k} = \left(\mathbf{F}_{\mathrm{RF}}^*\mathbf{H}\mathbf{H}^*\mathbf{F}_{\mathrm{RF}} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_M\right)^{-1}\mathbf{F}_{\mathrm{RF}}^*\mathbf{h}_k$$

$$\mathbf{P} = \mathrm{diag}\left(\begin{bmatrix}p_1 & \cdots & p_K\end{bmatrix}\right)$$

$$p_k = \frac{\sqrt{P_{tx}}}{\sqrt{K}\|\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{bb},k}\|} = \sqrt{\frac{P_{tx}}{K\mathbf{h}_k^*\mathbf{F}_{\mathrm{RF}}\mathbf{W}\mathbf{F}_{\mathrm{RF}}^*\mathbf{F}_{\mathrm{RF}}\mathbf{W}\mathbf{F}_{\mathrm{RF}}^*\mathbf{h}_k}}.$$

2) Let each column of $\mathbf{F}_{\mathrm{RF}}$ be a linear combination of orthonormal bases $\{\mathbf{v}_1, \ldots, \mathbf{v}_M\}$ (subspace)

$$\mathbf{F}_{\mathrm{RF}} = \mathbf{V}\mathbf{A}$$

where $\mathbf{A} \in \mathcal{C}^{M\times M}$ is an $M \times M$ matrix, and

$$\mathbf{V} = \begin{bmatrix}\mathbf{v}_1 & \cdots & \mathbf{v}_M\end{bmatrix} \in \mathcal{U}^{N\times M} \ (\mathbf{V}^H\mathbf{V} = \mathbf{I}_M)$$

3) Proposition

If $\mathbf{V}$ and P_{tx} are given, SLNR is maximized when $\mathbf{A}$ is unitary, which indicates that $\mathbf{F}_{\mathrm{RF}}$ must be a semi-unitary matrix to maximize SLNR.

[Proposed 1] Unconstrained case: Proposed (4/5)

4) If $\mathbf{A}$ is unitary, the SLNR becomes

$$\mathrm{SLNR}_k = \frac{\mathbf{h}_k^* \mathbf{V}\left(\mathbf{V}^*\mathbf{H}\mathbf{H}^*\mathbf{V} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k}{1 - \mathbf{h}_k^* \mathbf{V}\left(\mathbf{V}^*\mathbf{H}\mathbf{H}^*\mathbf{V} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k} = \mathbf{h}_k^* \mathbf{V}\left(\mathbf{V}^*\left(\sum_{i\neq k}^{K}\mathbf{h}_i\mathbf{h}_i^*\right)\mathbf{V} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k$$

5) Considering large antenna arrays, the SLNR converges to

$$\mathrm{SLNR}_k = N\mathbf{g}_k^H \mathbf{R}_k^{\frac{1}{2}}\mathbf{V}\left(N\sum_{i\neq k}^{K}\mathbf{V}^*\mathbf{R}_i^{\frac{1}{2}}\mathbf{g}_i\mathbf{g}_i^*\mathbf{R}_i^{\frac{1}{2}}\mathbf{V} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_N\right)^{-1}\mathbf{V}^*\mathbf{R}_k^{\frac{1}{2}}\mathbf{g}_k$$

$$\xrightarrow{a.s.} \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_k\mathbf{V}\left(N\sum_{i=1}^{K}\mathbf{V}^*\mathbf{R}_i^{\frac{1}{2}}\mathbf{g}_i\mathbf{g}_i^*\mathbf{R}_i^{\frac{1}{2}}\mathbf{V} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_N\right)^{-1}\right) \xrightarrow{a.s.} \gamma_k$$

where $\gamma_1, \ldots, \gamma_K$ are the unique nonnegative solution of

$$\gamma_k = \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_k\mathbf{V}\left(\left(\sum_{j=1}^{K}\frac{\mathbf{V}^*\mathbf{R}_j\mathbf{V}}{1+\gamma_j} + \frac{K\sigma^2}{P_{\mathrm{tx}}}\mathbf{I}_N\right)\right)^{-1}\right)$$

[Proposed 1] Unconstrained case: Proposed (5/5)

6) Optimization problem and its relaxation

$$\max \frac{1}{K}\sum_{k=1}^{K}\gamma_k, \quad \text{s.t.} \quad \gamma_k = \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_k\mathbf{V}\left(\sum_{j=1}^{K}\frac{\mathbf{V}^*\mathbf{R}_j\mathbf{V}}{1+\gamma_j}+\frac{K}{\rho}\mathbf{I}_M\right)^{-1}\right), \forall k$$

$$\boxed{\gamma_1 = \cdots = \gamma_K = \gamma = \frac{1}{K}\sum_{k=1}^{K}\gamma_k}$$

$$\max_{\mathbf{V}\in\mathbb{U}^{N\times M}} \gamma \text{ s.t.} \quad \gamma = \frac{1}{K}\sum_{k=1}^{K}\mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_k\mathbf{V}\left(\sum_{j=1}^{K}\frac{\mathbf{V}^*\mathbf{R}_j\mathbf{V}}{1+\gamma}+\frac{K}{\rho}\mathbf{I}_M\right)^{-1}\right)$$

$$= \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}\left(\frac{K\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}}{1+\gamma}+\frac{K}{\rho}\mathbf{I}_M\right)^{-1}\right)$$

where $\mathbf{R}_{\mathrm{tot}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R}_k$

8) Optimal solution

$$\mathbf{F}_{\mathrm{RF}} = \mathbf{V} = \text{dominant } M \text{ eigenvectors of } \sum_{k=1}^{K}\mathbf{R}_k$$

Proof. $\mu_1, \ldots \mu_M$: eigenvalues of $\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}$

$\lambda_1, \ldots \lambda_N$: eigenvalues of $\mathbf{R}_{\mathrm{tot}}$

$$\gamma = \frac{1}{K}\sum_{m=1}^{M}\frac{1}{\frac{1}{1+\gamma}+\frac{1}{\rho\mu_m}} \le \frac{1}{K}\sum_{m=1}^{M}\frac{1}{\frac{1}{1+\gamma}+\frac{1}{\rho\lambda_m}}$$

where the equality holds if

$\mathbf{V}$ is the M dominant eigenvectors of $\mathbf{R}_{\mathrm{tot}}$

by Cauchy's interlacing theorem

$\lambda_{N-M+i} \le \mu_i \le \lambda_i, \quad$ for $i = 1, \ldots, M$.

[Proposed 2] Constrained case: Prior work

- Prior work [1)]
  - Make $\mathbf{F}_{\mathrm{RF,C}}$ (constrained) as similar to $\mathbf{F}_{\mathrm{RF,UC}}$ (unconstrained) as possible

1) Problem formulation

$$\arg\min_{\mathbf{F}_{\mathrm{RF,C}}} \|\mathbf{F}_{\mathrm{RF,UC}} - \mathbf{F}_{\mathrm{RF,C}}\|_F \;\; \text{s.t.} \;\; \left|[\mathbf{F}_{\mathrm{RF,C}}]_{i,j}\right| = 1$$

2) Optimal solution

$$\left[\mathbf{F}_{\mathrm{RF,C}}^{\mathrm{opt}}\right]_{i,j} = e^{j\angle\left([\mathbf{F}_{\mathrm{RF,UC}}]_{i,j}\right)} \;\; \text{for} \;\; i = 1, \ldots, N, \;\; j = 1, \ldots, M$$

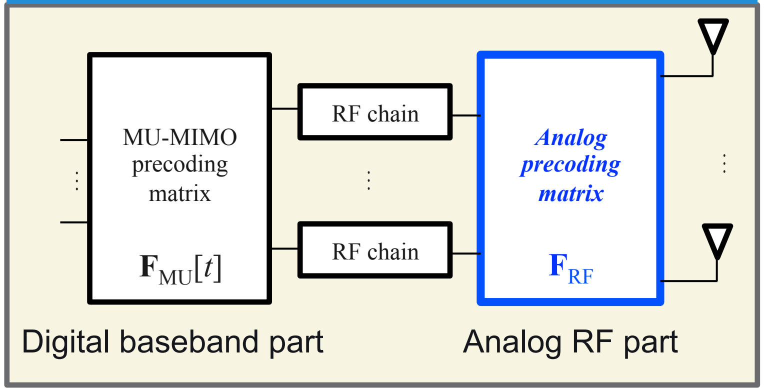
- Shortcomings of prior work
  - When $\mathbf{F}_{\mathrm{RF,UC}}$ is a semi-unitary matrix, the solution loses orthogonality. ($\mathbf{F}_{\mathrm{RF,C}}$ is not a semi-unitary matrix any more.)

1) L. Liang, W. Xu, and X. Dong, "Low-complexity hybrid precoding in massive multiuser MIMO systems," *IEEE Wireless Comm. Letters.*, Dec. 2014.

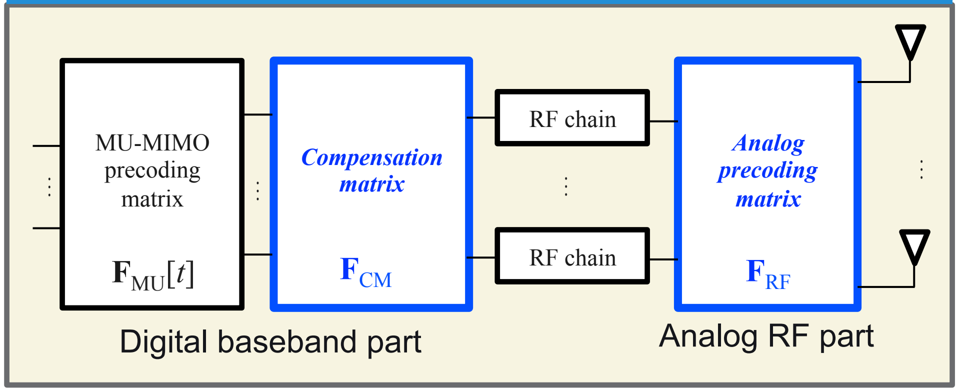
[Proposed 2] Constrained case: Proposed (1/2)

- Proposed
  - Use a *compensation matrix* that compensates the orthogonality lost in $\mathbf{F}_{\mathrm{RF,C}}$

Conventional hybrid structure



Proposed hybrid structure



$\mathbf{F}_{\mathrm{RF,UC}} = M$ dominant eigenvectors of $\sum_{k=1}^{K} \mathbf{R}_k$

$\mathbf{F}_{\mathrm{CM}} = (\mathbf{F}_{\mathrm{RF}}^{*}\mathbf{F}_{\mathrm{RF}})^{-\frac{1}{2}}$: *compensation matrix*

$$\min_{\mathbf{F}_{\mathrm{RF}}, |[\mathbf{F}_{\mathrm{RF}}]_{i,j}|=1, \mathbf{A}} \|\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A} - \mathbf{F}_{\mathrm{RF}}\|_{\mathrm{F}}^{2} \quad \text{for any invertible matrix } \mathbf{A}$$

Using an alternative minimization technique → $\mathbf{F}_{\mathrm{RF}}^{(n)} = \angle\left(\mathbf{F}_{\mathrm{RF,UC}}\mathbf{F}_{\mathrm{RF,UC}}^{*}\mathbf{F}_{\mathrm{RF}}^{(n-1)}\right)$

[Proposed 2] Constrained case: Proposed (2/2)

Step 1. Find an unconstrained analog precoding matrix without phase shifter constraint, $\mathbf{F}_{\mathrm{RF,UC}}$

$$\mathbf{F}_{\mathrm{RF,UC}} = M \text{ dominant eigenvectors of } \sum_{k=1}^{K} \mathbf{R}_k$$

Step 2. Find a constrained analog precoding matrix with phase shifter constraint, $\mathbf{F}_{\mathrm{RF,C}}$

Algorithm 1 Find $\mathbf{F}_{\mathrm{RF,C}}$

Input: $\mathbf{F}_{\mathrm{RF,UC}}$
Initialization: $\mathbf{F}_{(0)} = \measuredangle(\mathbf{F}_{\mathrm{RF,UC}}),\ n = 0$
repeat
$n \leftarrow n+1$
$\mathbf{F}_{(n)} = \measuredangle(\mathbf{F}_{\mathrm{RF,UC}}\mathbf{F}^*_{\mathrm{RF,UC}}\mathbf{F}_{(n-1)})$
until $\|\mathbf{F}_{\mathrm{RF,UC}}\mathbf{F}^*_{\mathrm{RF,UC}}\mathbf{F}_{(n-1)} - \mathbf{F}_{(n)}\|_F$ converges
Output: $\mathbf{F}_{\mathrm{RF,C}} = \mathbf{F}_{(n)}$

Step 3. Design a baseband compensation matrix, $\mathbf{F}_{\mathrm{CM}}$

$$\mathbf{F}_{\mathrm{CM}} = \left(\mathbf{F}^*_{\mathrm{RF,C}}\mathbf{F}_{\mathrm{RF,C}}\right)^{-\frac{1}{2}}$$

Step 4. Design a baseband multiuser MIMO precoding matrix, $\mathbf{F}_{\mathrm{MU}}[t]$

Regularized zero-forcing (RZF) with respect to the effective channel $\mathbf{H}^*_{\mathrm{eff}}[t] = \mathbf{H}^*[t]\mathbf{F}_{\mathrm{RF,C}}\mathbf{F}_{\mathrm{CM}}$

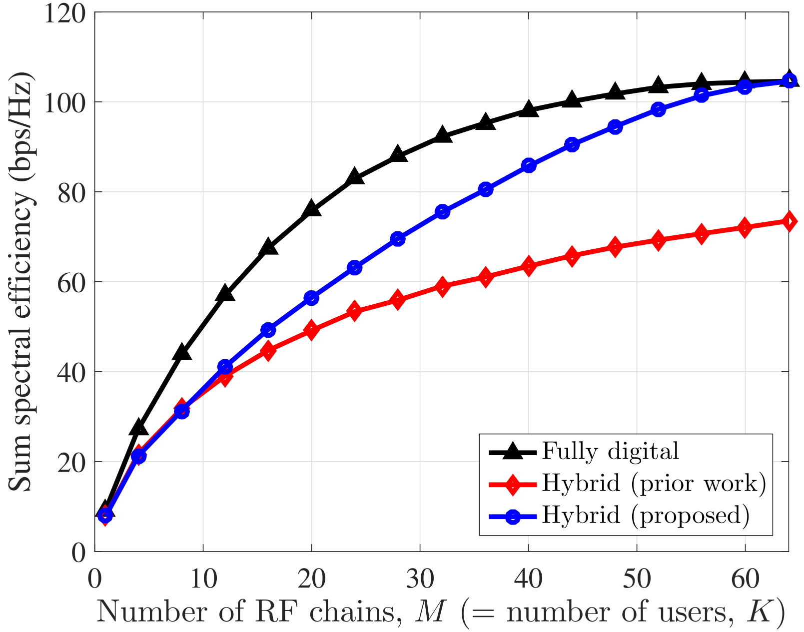
$$\mathbf{F}_{\mathrm{MU}}[t] = \left(\mathbf{H}_{\mathrm{eff}}[t]\mathbf{H}^*_{\mathrm{eff}}[t] + \beta\mathbf{I}\right)^{-1}\mathbf{H}_{\mathrm{eff}}[t]$$

Step 5. Design a final analog precoding matrix $\mathbf{F}_{\mathrm{RF}}$ and a baseband precoding matrix $\mathbf{F}_{\mathrm{BB}}[t]$

$$\mathbf{F}_{\mathrm{RF}} = \mathbf{F}_{\mathrm{RF,C}}, \quad \mathbf{F}_{\mathrm{BB}}[t] = \mathbf{F}_{\mathrm{CM}}\mathbf{F}_{\mathrm{MU}}[t]$$

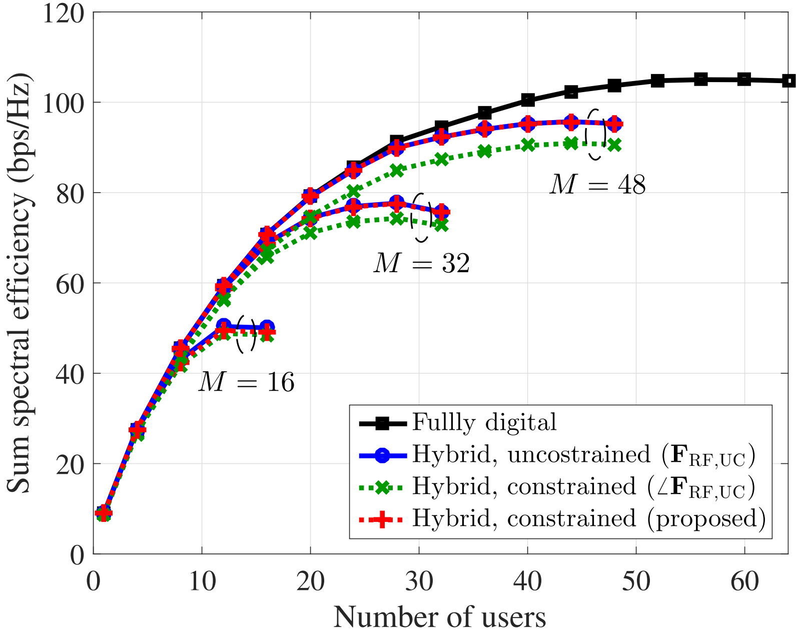
Simulation results

Sum spectral efficiency vs. M



- $N = 64$, $M = K$
- $L = 5$, $\sigma_{AS} = 10°$, SNR = 10 dB
- w/o phase shifter constraint

Sum spectral efficiency vs. K



- $N = 64$, $M = \{16, 32, 48\}$
- $L = 5$, $\sigma_{AS} = 10°$, SNR = 10 dB
- w/ & w/o phase shifter constraint

Conclusions

Our proposed hybrid precoding technique for massive MIMO systems

- Uses only long-term channel statistics for the analog precoder design
- Mitigates the loss caused by using phase shifters

Thank you !