# SAR Image Despeckling By Combination of Fractional-Order Total Variation and Nonlocal Low Rank Regularization


Gao Chen[1], Gang Li[1], Yu Liu[2], Xiao-Ping Zhang[3], Li Zhang[1]

[1] Department of Electronic Engineering, Tsinghua University, Beijing, China

[2] School of Electronic and Information Engineering, Beihang University, Beijing, China

[3] Department of Electrical & Computer Engineering, Ryerson University, Toronto, Canada


## Abstract


This paper proposes a combinational regularization model for synthetic aperture radar (SAR) image despeckling. In contrast to most of the well-known regularization methods that only use one image prior property, the proposed combinational regularization model includes both fractional-order total variation (FrTV) regularization term and nonlocal low rank (NLR) regularization term. By characterizing the smoothness and nonlocal self-similarity property of the SAR image simultaneously, the proposed model, on the one hand, can better remove the noise in homogeneous regions of a noisy image, on the other hand, can better preserve edges and geometrical features of the images during the despeckling process. Afterwards, an alternating direction method (ADM) is derived to efficiently solve the optimization problem in the proposed model. Experimental results demonstrate the good performance of the proposed model, both in removing SAR image speckles and preserving image texture and details.


## Introduction

- Despeckling is an important task in SAR image processing. The regularization method is an effective tool for SAR image despeckling.
- Existing regularization methods for SAR image despeckling include:
  - Total variation regularization (e.g., AA, SO)
  - Nonlocal regularization (e.g., PPB, SAR-BM3D)
- They have their respective advantages and disadvantages.

- Contributions: In order to inherit both of their advantages and improve the performance of SAR despeckling, in this paper we propose a combinational regularization model for speckle reduction (CRM-SR), in which a fractional-order total variation regularization term and a nonlocal low rank regularization term are included. Besides, an alternating direction method is derived to efficiently solve the optimization problem in the proposed model.

## Despeckling model

The speckle in SAR images is characterized by the multiplicative noise model:

$$\boldsymbol{f} = \boldsymbol{u}\boldsymbol{n} \tag{1}$$

where $\boldsymbol{u}$ is the noisy free SAR image and $\boldsymbol{n}$ is the speckle noise that can be modeled by a gamma distribution. To simplify the problem, the multiplicative model is transformed into the additive model by logarithmic transformation $\boldsymbol{w} = \log \boldsymbol{u}$ and the problem of despeckling is converted into recovering $\boldsymbol{w}$ from the noisy observation $\log \boldsymbol{f}$.

Regularization method for SAR image can be formulated as:

$$argmin\ H(\boldsymbol{w},\boldsymbol{f}) + \rho\varphi(\boldsymbol{w}) \tag{2}$$

where $H(\boldsymbol{w},\boldsymbol{f}) = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\boldsymbol{w_{i,j}} + \boldsymbol{f_{i,j}}exp(-\boldsymbol{w_{i,j}})\right)$ is the data fitting term and $\varphi(\boldsymbol{w})$ is the regularization term which includes prior information about the original image.

In this paper, the regularization term $\varphi(\boldsymbol{w})$ is elaborately designed which reflects the smoothness and the nonlocal self-similarity of the SAR image simultaneously, it is formulated as:

$$\varphi(\boldsymbol{w}) = \rho_1\varphi_{LSM}(\boldsymbol{w}) + \rho_2\varphi_{NLSM}(\boldsymbol{w}) \tag{3}$$

where $\varphi_{LSM}(\boldsymbol{w})$ and $\varphi_{NLSM}(\boldsymbol{w})$ indicate the image smoothness and nonlocal self-similarity prior information, respectively. They can be formulated as:

$$\varphi_{LSM}(\boldsymbol{w}) = \|\boldsymbol{w}\|_{FrTV} = \sum_{i=1}^{N}\sum_{j=1}^{N}\sqrt{\left[\left(\boldsymbol{w}_h^{\alpha}\right)_{i,j}\right]^2 + \left[\left(\boldsymbol{w}_v^{\alpha}\right)_{i,j}\right]^2} \tag{4}$$

$$\varphi_{NLSM}(\boldsymbol{w}) = \|\boldsymbol{\Theta}_{w}\|_0 = \sum_{l=1}^{P}\left\|\boldsymbol{D}_{w^l}\right\|_* = \sum_{l=1}^{P}\left\|\boldsymbol{G}_l(\boldsymbol{w})\right\|_* \tag{5}$$

$$\left[\boldsymbol{G}_l(\boldsymbol{w})\right]_{i,j} = \sum_{k=1}^{N^2}\left(\boldsymbol{R}_l^{\,j}\right)_{i,k}\left[vec(\boldsymbol{w})\right]_k = \sum_{ii=1}^{N}\sum_{jj=1}^{N}\left(\boldsymbol{R}_l^{\,j}\right)_{i,(jj-1)N+ii}\boldsymbol{w}_{ii,jj} \tag{6}$$

## Optimization algorithm

We employ the algorithmic framework of the ADM to solve the optimization problem (2) efficiently.

Firstly, the problem is reformulated into an equivalent problem by introducing some splitting variables as follows:

$$\begin{aligned} \widehat{\boldsymbol{w}} &= \min_{\boldsymbol{w}}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\boldsymbol{w}_{i,j} + \boldsymbol{f}_{i,j}e^{-\boldsymbol{w}_{i,j}}\right) + \xi\sum_{l=1}^{P}\|\boldsymbol{Z}_l\|_* + \lambda\|\boldsymbol{Z}_{P+1}\|_{FrTV} \\ s.t.\quad & \boldsymbol{Z}_l = \boldsymbol{G}_l(\boldsymbol{w}) \quad (l = 1,\cdots,P) \\ & \boldsymbol{Z}_{P+1} = \boldsymbol{w} \end{aligned} \tag{7}$$

The augmented Lagrangian function of problem (7) is given by:

$$\begin{aligned} L_A(\boldsymbol{Z}_l,\boldsymbol{w}) = &\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\boldsymbol{w}_{i,j} + \boldsymbol{f}_{i,j}e^{-\boldsymbol{w}_{i,j}}\right) + \xi\sum_{l=1}^{P}\|\boldsymbol{Z}_l\|_* + \lambda\|\boldsymbol{Z}_{P+1}\|_{FrTV} \\ &+ \sum_{l=1}^{P}\left[\left\langle\boldsymbol{Y}_l, \boldsymbol{Z}_l - \boldsymbol{G}_l(\boldsymbol{w})\right\rangle + \frac{\beta}{2}\left\|\boldsymbol{Z}_l - \boldsymbol{G}_l(\boldsymbol{w})\right\|_F^2\right] \\ &+ \left\langle\boldsymbol{Y}_{P+1}, \boldsymbol{Z}_{P+1} - \boldsymbol{w}\right\rangle + \frac{\beta}{2}\left\|\boldsymbol{Z}_{P+1} - \boldsymbol{w}\right\|_F^2 \end{aligned} \tag{8}$$

In each iteration, we optimize $\boldsymbol{Z}_l$ and $\boldsymbol{w}$ alternatively with other variables fixed, and the Lagrangian multipliers are updated by the following scheme:

$$\begin{cases} \boldsymbol{Y}_l \leftarrow \boldsymbol{Y}_l + \gamma\beta\left(\boldsymbol{Z}_l - \boldsymbol{G}_l(\boldsymbol{w})\right) & l = 1,\cdots P \\ \boldsymbol{Y}_{P+1} \leftarrow \boldsymbol{Y}_{P+1} + \gamma\beta\left(\boldsymbol{Z}_{P+1} - \boldsymbol{w}\right) \end{cases} \tag{9}$$

The detailed steps of solving the problem of SAR image despeckling are summarized in Algorithm I.

**Algorithm 1: ADM iterative algorithm**

Step 1: Input the noisy SAR image $\boldsymbol{f}$ and the parameters $\alpha,\beta,\xi,\lambda,\gamma,\boldsymbol{N}_0$.

Step 2: Set $k=0$, $\boldsymbol{w}^{(k)} = \log(\boldsymbol{f})$ and $\boldsymbol{Y}_l^{(k)},\boldsymbol{Z}_l^{(k)}\ (l=1,\cdots,P+1)$ as zero matrix

Step 3: While $k < \boldsymbol{N}_0$ do

  3.1 Block matching and compute $\boldsymbol{G}_l\left(\boldsymbol{w}^{(k)}\right)(l=1,\cdots,P)$

  3.2 Compute $\boldsymbol{w}_{i,j}^{(k+1)}$ using Newton iteration

  3.3 Optimize $\boldsymbol{Z}_l^{(k+1)}\ (l=1,\cdots,P+1)$ with other variables fixed

  3.4 Optimize $\boldsymbol{Y}_l^{(k+1)}\ (l=1,\cdots,P+1)$ with other variables fixed

  3.5 $k=k+1$

Step 4: Output $\boldsymbol{u} = \exp\left(\boldsymbol{w}^{(k)}\right)$

## Test images

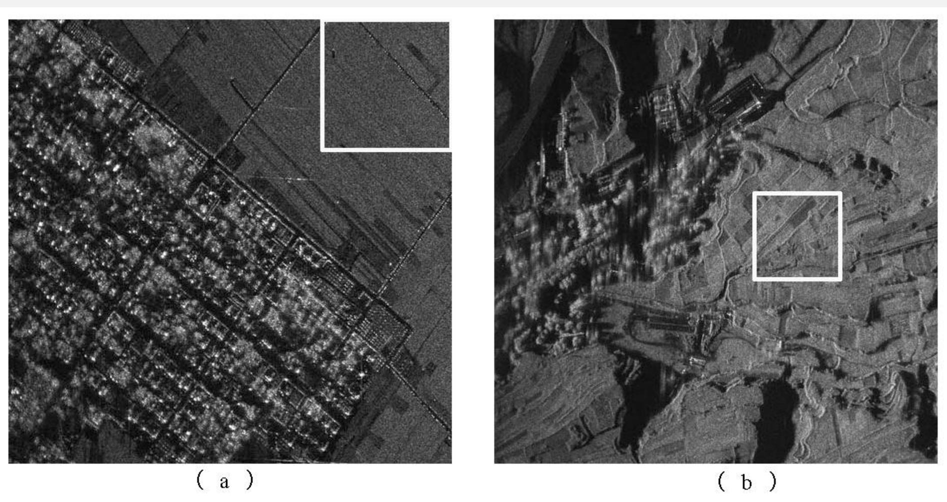


Figure 1. The real SAR images used in the experiment, the pixels in the white box are used for ENL estimation

## Experimental results

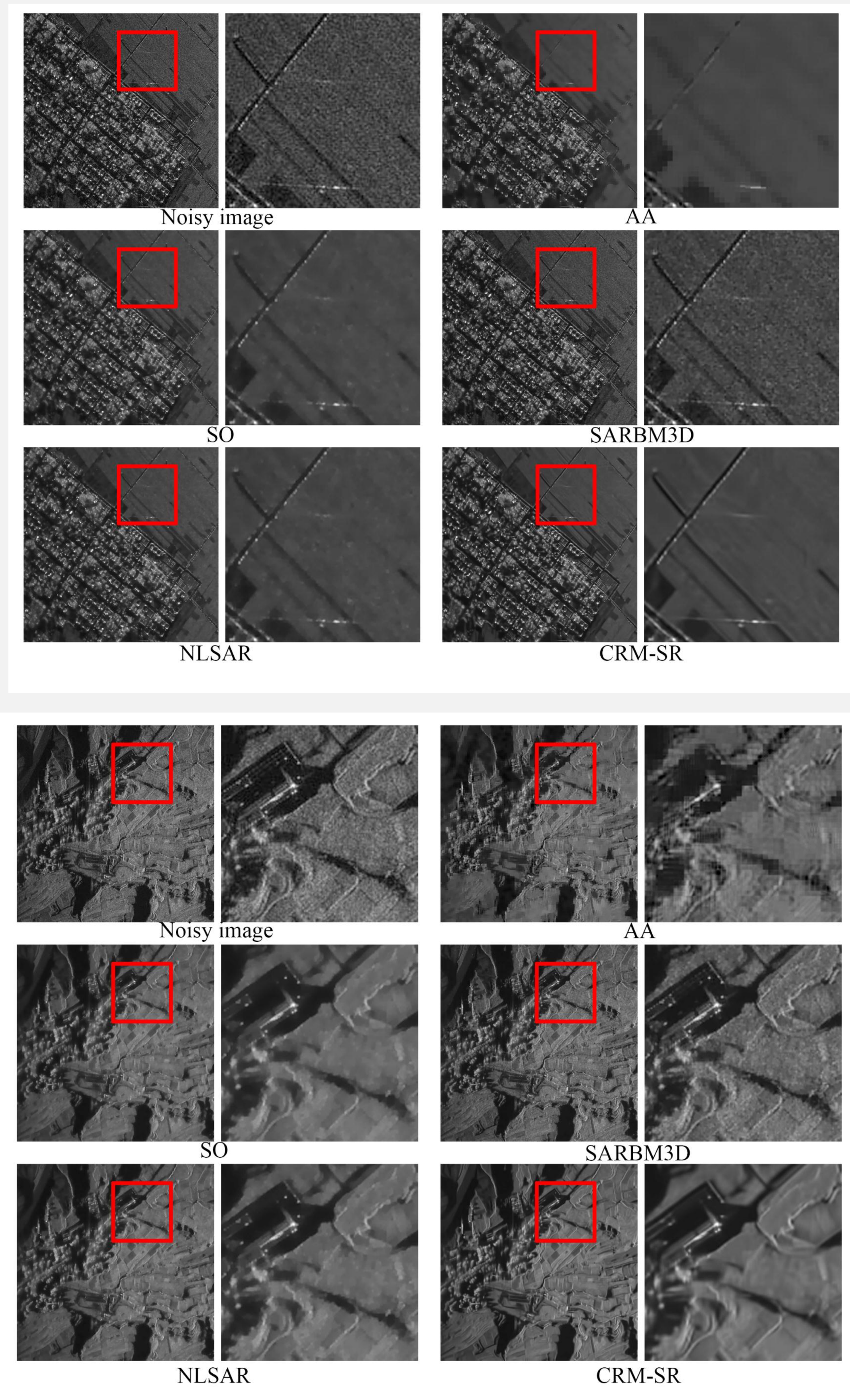


Figure 2. Visual quality comparison of the despeckling performances using different algorithms

## References


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