CSMSDL: A Common Sequential Dictionary Learning Algorithm for Multi-Subject fMRI Data Sets Analysis

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Introduction

- Dictionary learning methods have been successfully applied to single subject fMRI data analysis.
- Dictionary learning methods do not naturally extend to multiple subjects.
- We propose CSMSDL to learn shared temporal dynamics from spatially concatenated fMRI datasets.
- Method is based on a modified Power Method.

Background

Given a set of signals $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N]$, our objective is to find a linear representation for the set of signals $\mathbf{Y}$.

$$\{\mathbf{D}, \mathbf{X}\} = \arg\min_{\mathbf{D},\mathbf{X}} \| \mathbf{Y} - \mathbf{D}\mathbf{X} \|_F^2$$

This problem is ill-posed, extra constraints are imposed on both $\mathbf{D}$ and $\mathbf{X}$, which are

- Columns of $\mathbf{X} \in \mathbb{R}^{K \times N}$ should be sparse.
- Columns of $\mathbf{D} \in \mathbb{R}^{n \times K}$ should have unit ℓ_2 norm.

The problem is solved by alternating two steps, i.e.

$$\hat{\mathbf{x}}_i = \arg\min_{x_i} \| \mathbf{y}_i - \mathbf{D}\mathbf{x}_i \|^2; \quad (1)$$
$$\textit{subject to } \| \mathbf{x}_i \|_0 \leq s \quad i = 1, ..., N.$$

and

$$\mathbf{D} = \arg\min_{D} \| \mathbf{Y} - \mathbf{D}\mathbf{X} \|_F^2 \quad (2)$$

In [1] (2) is solved sequentially by alternating

$$\mathbf{d}_k = \frac{\mathbf{E}_k \mathbf{x}_k^{row\top}}{||\mathbf{E}_k \mathbf{x}_k^{row\top}||_2}. \quad (3)$$

$$\mathbf{x}_k^{row} = \text{sgn}(\mathbf{d}_k^\top \mathbf{E}_k) \circ \left(|\mathbf{d}_k^\top \mathbf{E}_k| - \frac{\alpha}{2} \mathbf{1}_{(N)}^\top \right)_+ \quad (4)$$

Here $\mathbf{E}_k = \sum_{i=1, i\neq k}^{K} \mathbf{d}_i \mathbf{x}_i^{row}$.

CSMSDL

Starting with $\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_p]$ of size $n \times pN$, the proposed model is

$$[\mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_p] \simeq \mathbf{D}[\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_p] \quad (5)$$

The sparse codes are found by solving (1) followed by dictionary update stage which is obtained from

$$\{\mathbf{d}_k, \mathbf{x}_k^{row}\} = \arg\min_{\mathbf{d}_k, \mathbf{x}_k^{row}} \|\mathbf{E}_k - \mathbf{d}_k \mathbf{x}_k^{row}\|_F^2 + \alpha \|\mathbf{x}_k^{row}\|_1$$

where $\mathbf{E}_k = [\mathbf{E}_{k_1} \mathbf{E}_{k_2} ... \mathbf{E}_{k_p}]$ with $\mathbf{E}_{k_j} = \mathbf{Y}_j - \sum_{i=1, i\neq k}^{K} \mathbf{d}_i \mathbf{x}_{i_j}^{row}$. The full solution is shown below:

Algorithm Overview

Given: $\mathbf{Y}_1, \mathbf{Y}_2, ...,$ and $\mathbf{Y}_p$, $\mathbf{D}_{ini}$, $\mathbf{s}$, α, *iter* and *J*.
Set $\mathbf{D} = \mathbf{D}_{ini}$
For *i=1 to J*
1: *Sparse Coding Stage:*
Find sparse coefficients X, by approximately solving
$\hat{\mathbf{x}}_i = \arg\min_{x_i} \| \mathbf{y}_i - \mathbf{D}\mathbf{x}_i \|^2$;
subject to $\| \mathbf{x}_i \|_0 \leq s \quad i = 1, ..., pN$
2: Dictionary Update Stage:
For each column $k = 1, 2, ..., K$ in $\mathbf{D}$,
2.a: Compute the error matrices using
$\mathbf{E}_{k_j} = \mathbf{Y}_j - \sum_{i=1, i\neq k}^{K} \mathbf{d}_i \mathbf{x}_{i_j}^{row}$, $j = 1, ..., p$
2.b: Construct the error matrix $\mathbf{E}_k$ as
$\mathbf{E}_k = [\mathbf{E}_{k_1} \mathbf{E}_{k_2} ... \mathbf{E}_{k_p}]$
While $\|\mathbf{d}_k^{iter} - \mathbf{d}_k^{iter+1}\|_2^2 \geq \varepsilon$ iterate
2.c: Update the p block rows $\mathbf{x}_{k_j}^{row}$'s and its sparsity
$\mathbf{x}_{k_j}^{row} = \text{sgn}(\mathbf{d}_k^\top \mathbf{E}_{k_j}) . \left(|\mathbf{d}_k^\top \mathbf{E}_{k_j}| - \frac{\alpha}{2} \mathbf{1}_{(N)}^\top \right)_+$
2.d: Update the dictionary atom $\mathbf{d}_k$ using
$\mathbf{d}_k^{iter} = \frac{\sum_{j=1}^{p} \mathbf{E}_{k_j} \mathbf{x}_{k_j}^{row\top}}{\|\sum_{j=1}^{p} \mathbf{E}_{k_j} \mathbf{x}_{k_j}^{row\top}\|_2}$
iter = iter+1
end.
Output: $\mathbf{D}$,$\mathbf{X}$

Simulation Results

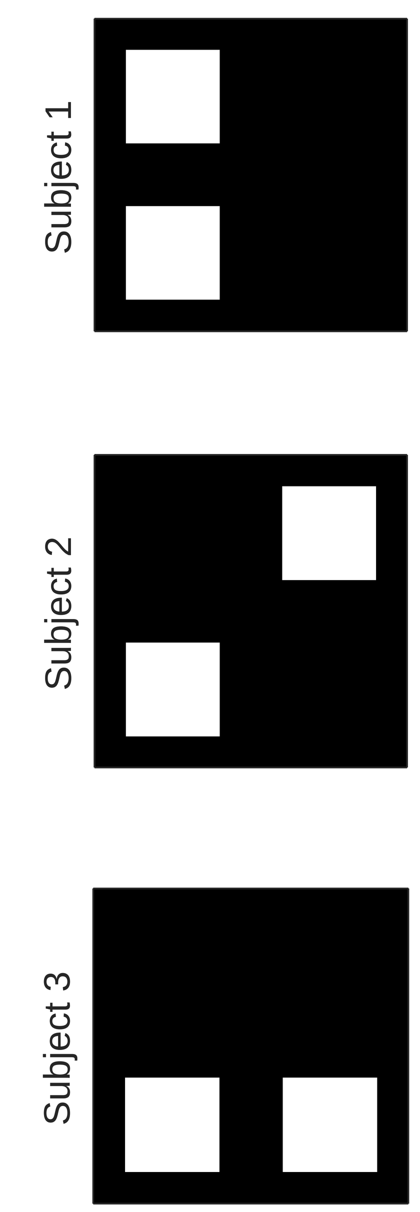

Figure 1: Subject setup.

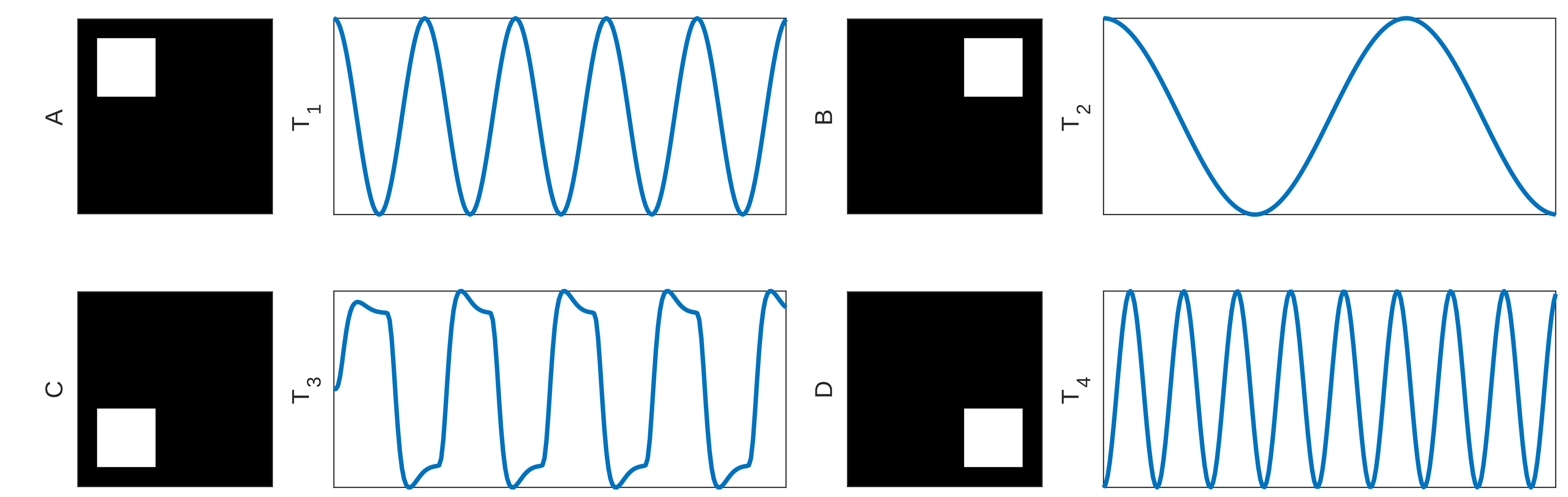

Figure 2: Activations patterns and the respective time-courses.

Table 1: Correlation of recovered time-series and spatial maps with ground truth averaged over 100 iterations

SNR dB		Sub_1	Sub_2	Sub_3
0	T_3	0.996	0.996	0.996
	C	0.978	0.995	0.996
-5	T_3	0.963	0.963	0.963
	C	0.971	0.984	0.985
-10	T_3	0.741	0.741	0.741
	C	0.866	0.869	0.872

Experimental fMRI Results

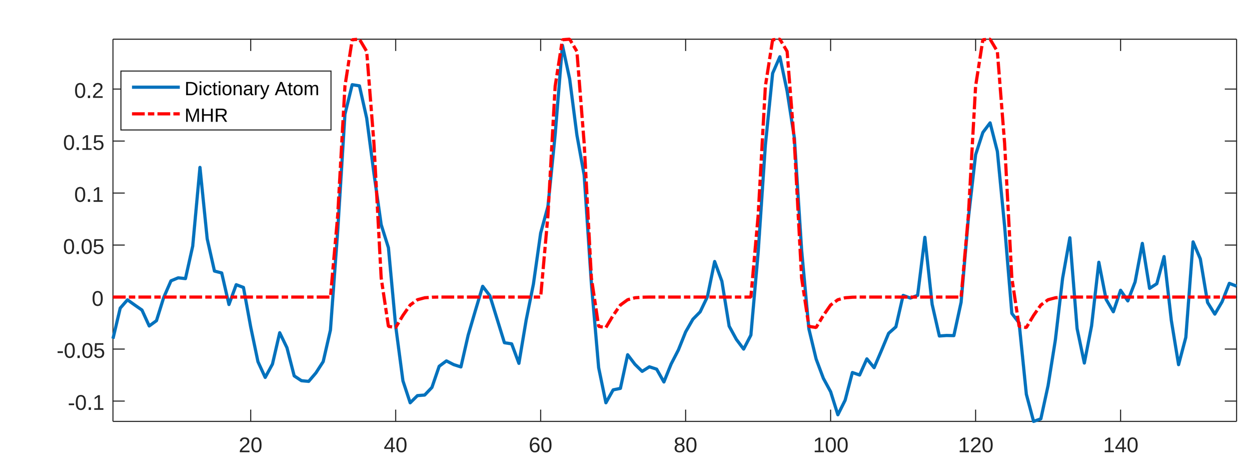

Figure 3: Most correlated dictionary atom w.r.t. MHR.

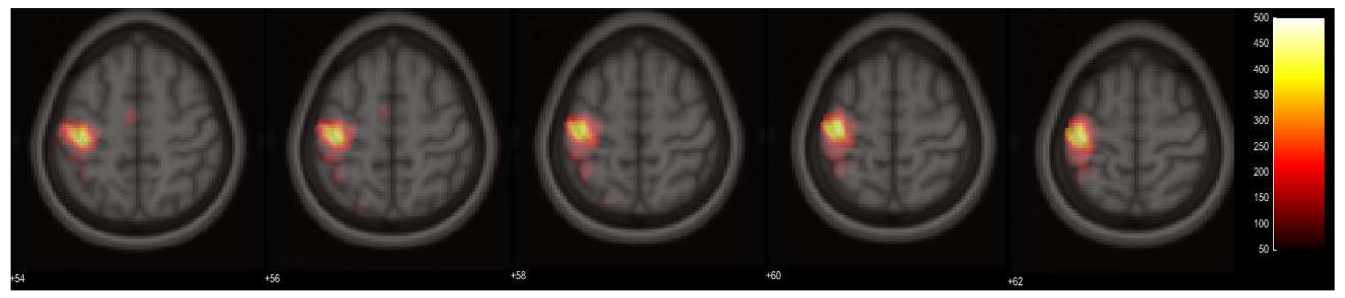

Figure 4: Average F-statistics activation maps *(5 contiguous slices)* for right finger tapping task stimulus random field correction $p < 0.0001$.

References

[1] A. K. Seghouane and M. Hanif. A sequential dictionary learning algorithm with enforced sparsity. *IEEE International Conference on Acoustic Speech and signal Processing, ICASSP*, pages 3876–3880, 2015.