

# Iterative Fitting After Elastic Registration: An Efficient Strategy for Accurate Estimation of Parametric Deformations

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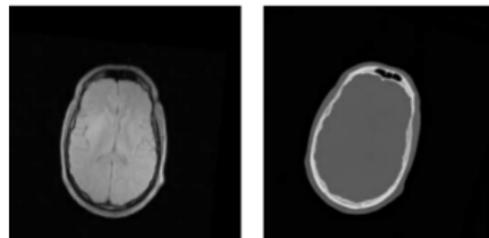
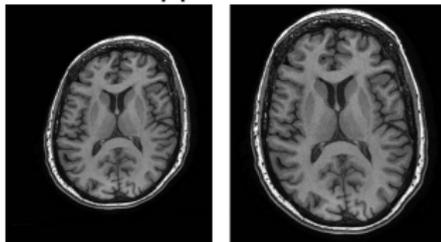
# Outline

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  - Problem Statement
  - Overview of Current Methods
- 2 Method
- 3 Experimental results
  - Synthetic images
  - Real images
  - Applications
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# Introduction

# Problem Statement

## ■ Medical applications



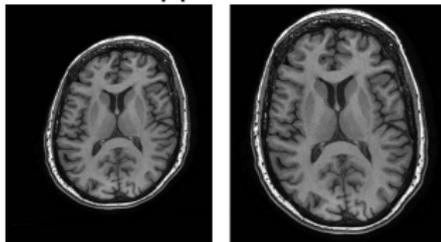
## ■ Remote sensing



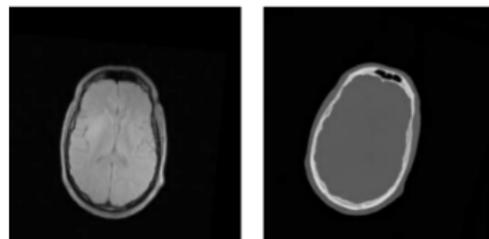
## ■ Others

# Problem Statement

## ■ Medical applications



monomodal



multimodal

## ■ Remote sensing



## ■ Others

# Problem Statement

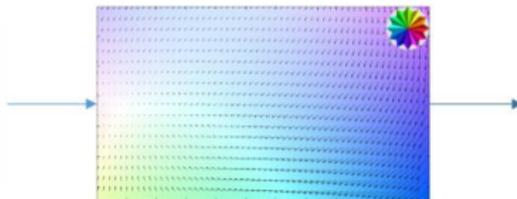
$$\blacksquare I_{\text{target}}(z) = I_{\text{source}}(z + u(z))$$

$$z = x + iy, u(z) = u_x(z) + iu_y(z),$$

technical details are too complex to cover in the book itself.

In teaching our courses, we have found it useful for the students to attempt a number of small implementation projects, which often build on one another, in order to get them used to working with real-world images and the challenges that these present. The students are then asked to choose an individual topic for each of their small-group final projects. (Sometimes these projects even turn into conference papers!) The exercises at the end of each chapter contain numerous suggestions for similar in-classroom projects, as well as more open-ended problems whose solutions are well suited to research topics. Whenever possible, I encourage students to try their algorithms on their own personal photographs, since this helps motivate them, often leads to creative variants on the problems, and better acquaints them with the variety and complexity of real-world imagery.

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$$I_{\text{source}}(z)$$

$$u(z)$$

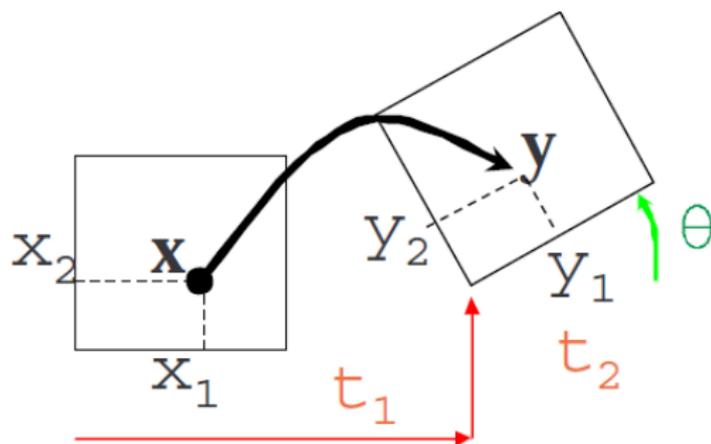
$$I_{\text{target}}(z)$$

# Overview of Current Methods

## ■ Global parametric registration

Calculate the parameters of the model.

Woods, 1992; Woods, 1998; Evangelidis, 2008

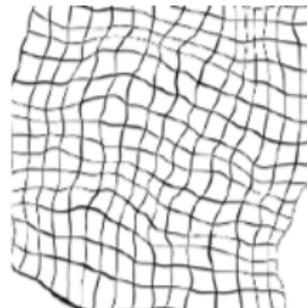
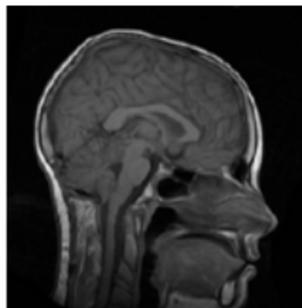
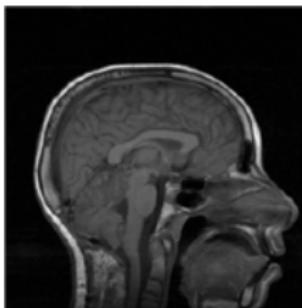


# Overview of Current Methods

## ■ Elastic registration

Estimate a displacement vector per pixel.

Arganda-Carreras, 2006; Bajcsy, 1989; Periaswamy, 2003; Klein, 2010; Periaswamy, 2006; Goshtasby, 1988; Kybic, 2003; Bruhn, 2005

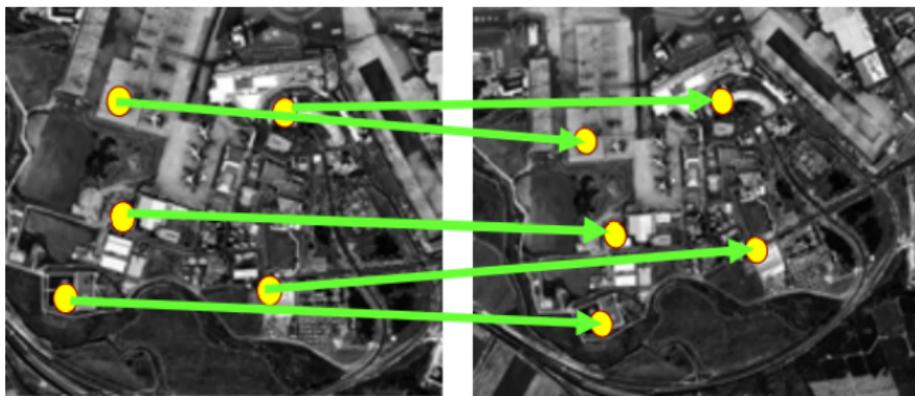


# Overview of Current Methods

## ■ Landmark/Feature-based Registration

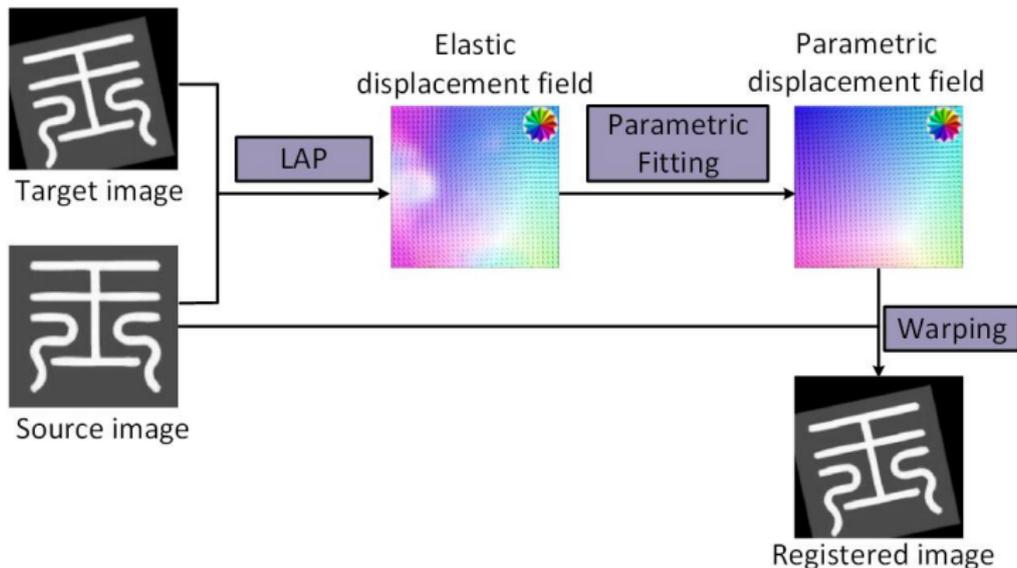
Establish the correspondence between the extracted features or landmarks from the two images.

Alhichri, 2003; Davatzikos, 1996; Li, 2009; Ma, 2015



# Method

# Parametric fitting of the elastic displacement field



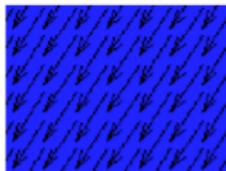
## LAP: Local All-Pass Filters

C. Gilliam and T. Blu, "Local all-pass filters for optical flow estimation," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 1533-1537, 2015.

# Parametric model

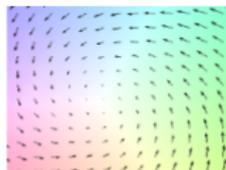
## Geometric Transformation

Translation

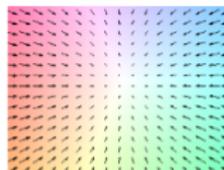


$$u(z) = c_1$$

Rotation



Scaling



$$u(z) = c_2 z$$

This paper



$$u(z) = c_1 + c_2 z + c_3 \bar{z} + c_4 z \bar{z} + c_5 z^2 + c_6 \bar{z}^2$$

# Parametric model fitting

Fitting model:  $u_{\text{fit}}(z) = c_1 + c_2z + c_3\bar{z} + c_4z\bar{z} + c_5z^2 + c_6\bar{z}^2$ ,

Fitting criterion:  $\min_c \sum_{z \in \Omega} |u_{\text{fit}}(z) - u_{\text{LAP}}(z)|^2$ ,

$c = [c_1, c_2, c_3, c_4, c_5, c_6]^T$  to be calculated.

↔ solution of a linear system of equations

★ Fast, efficient and flexible.

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# Choosing reliable pixels to fit parameters

Source image



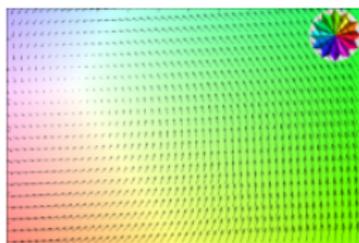
Target image



Common region

# Choosing reliable pixels to fit parameters

Source image

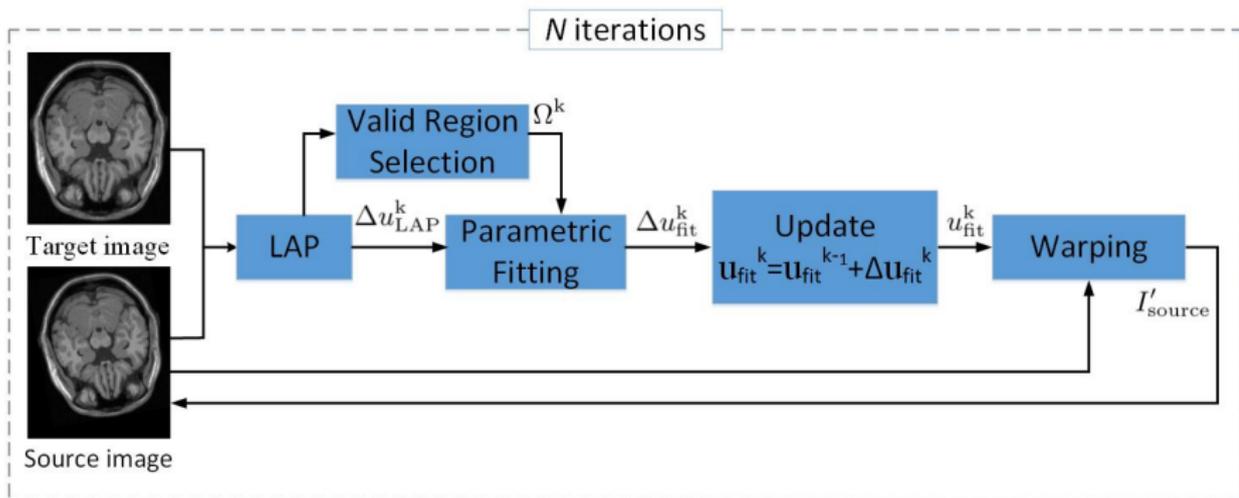


Target image



Valid region

# Flow chart of the proposed method



- $\Delta u_{LAP}^k$ : the displacement estimated by the Local All-Pass Filters (LAP) in the  $k$ th iteration.
- $u_{fit}^k$ : the displacement estimated by polynomial fitting in the  $k$ th iteration.
- $\Delta u_{fit}^k$ : the displacement increment estimated by polynomial fitting in the  $k$ th iteration.
- $\Omega^k$ : valid region in the  $k$ th iteration.

# Experimental results

# Comparison with the state-of-the-art

## Synthetic images

Table: Error comparison for the iterative fitting method and the state-of-the-art image registration methods.

		Noiseless Image			Noisy Image (15dB)			Gaussian Blurry Image			Missing Information		
		$E_{Med}$	$E_{Mean}$	Time	$E_{Med}$	$E_{Mean}$	Time	$E_{Med}$	$E_{Mean}$	Time	$E_{Med}$	$E_{Mean}$	Time
Parametric algorithms	Ours	<b>0</b>	<b>0</b>	18.45	<b>0.238</b>	<b>0.430</b>	19.08	<b>0.505</b>	<b>0.590</b>	32.22	<b>0</b>	<b>0</b>	38.58
	AECC	0.851	1.155	9.10	0.908	1.229	9.522	1.001	1.289	8.10	0.907	1.206	9.32
Elastic algorithms	LAP	0.006	1.189	<b>4.60</b>	1.799	4.076	<b>4.52</b>	3.118	3.415	<b>5.94</b>	0.011	2.131	<b>7.94</b>
	Demons	5.114	32.874	37.15	7.601	10.241	22.13	6.066	7.497	31.75	5.700	10.575	20.81
	MIRT	3.232	9.931	75.00	7.976	12.065	52.51	7.420	11.467	65.67	7.764	12.661	80.00
	bUnwarpJ	1.3402	1.4107	14.64	1.6763	1.8072	25.07	3.346	4.512	23.80	1.924	7.220	325.38

(1) Bold values indicate the best results. (2) The size of images is 388 by 584 pixels. (3) PSNR between the noisy image and original noiseless image is 15dB. (4) Results averaged over 5 different parametric deformation fields (maximum displacement is 16 pixels).

Absolute Error:  $E(j) = |u_{GT}(j) - u(j)|$ ,  $j \in D$ ,  $D$  is the common region

Median Absolute Error:  $E_{Med} = Median(E)$

Mean Absolute Error:  $E_{Mean} = Mean(E)$

## Reference:

AECC: G. D. Evangelidis and E. Z. Psarakis, 2008;

Demons: H. Lombaert, L. Grady, and X. Pennec et al., 2009;

MIRT: A. Myronenko and X. Song, 2010;

bUnwarpJ: I. Arganda-Carreras, C. O. S. Sorzano, and R. Marabini et al., 2006.

## Noiseless images

$I_{\text{source}}$  and  $I_{\text{target}}$

AECC

Demons

bUnwarpJ

LAP

Ours

## Noisy images (15dB)

$I_{\text{source}}$  and  $I_{\text{target}}$

AECC

Demons

bUnwarpJ

LAP

Ours

## Gaussian blurry images

$I_{\text{source}}$  and  $I_{\text{target}}$

AECC

Demons

bUnwarpJ

LAP

Ours

# Real images ( $400 \times 400$ )

$I_{\text{source}}$  and  $I_{\text{target}}$

MIRT 78.74s

Demons 13.90s

AECC 6.67s

LAP 5.39s

◀ ◻ ▶ Ours 21.32s ≡ ≡ ≡ ↺ ↻

# Real images ( $480 \times 640$ )

$I_{\text{source}}$  and  $I_{\text{target}}$

MIRT 82.32s

Demons 8.75s

AECC 6.67s

LAP 18.82s

Ours 91.67s

# Real images ( $480 \times 640$ )

$I_{\text{source}}$  and  $I_{\text{target}}$

MIRT 104.36s

Demons 178.97s

AECC 14.96s

LAP 24.34s

Ours 64.85s

# Real images ( $480 \times 640$ )

$I_{\text{source}}$  and  $I_{\text{target}}$

Ours 50.24s

# Video stabilization

Input

Output

# Conclusion

## Highlights

- 1 Accurate and fast.
- 2 Flexible and costless to add more parameters.
- 3 Robust to model mis-match (e.g. noise and blurring).
- 4 Robust to very large displacement.

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Thanks for your attention.

