Blur Estimation for Photon-Limited Images

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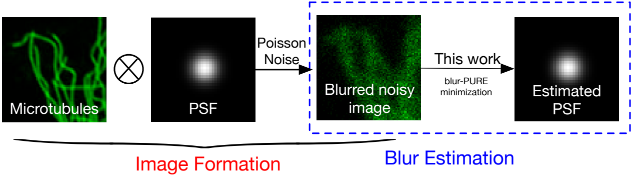

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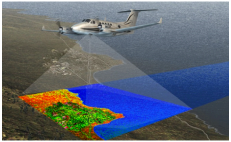
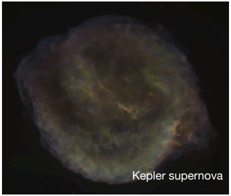
in *2017 Proc. IEEE Int. Conf. Img. Proc. (ICIP 2017)*

Photon Limitations

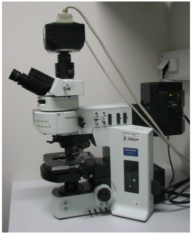
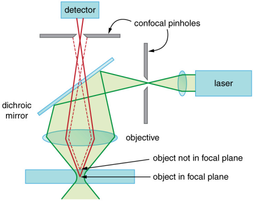

Night Vision



Spectral Imaging



Astronomy

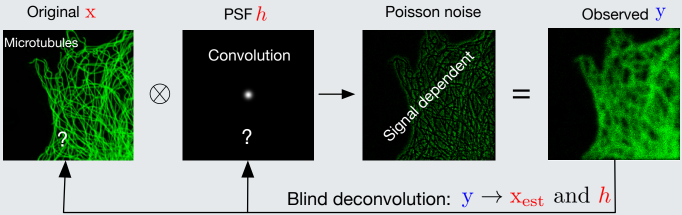



Fluorescence Microscopy

- **Blurring**
  - diffraction limits and aberrations
  - PSF can be parametrized (e.g. Gaussian)
- **Noise**
  - detectable statistical fluctuation
  - Poisson distribution

Problem Statement

Image acquisition model



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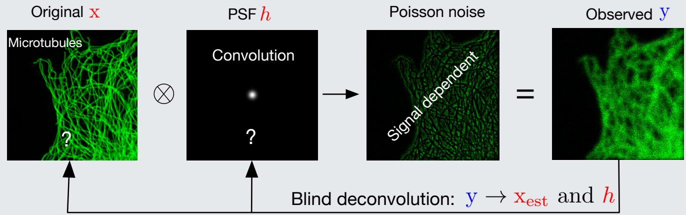
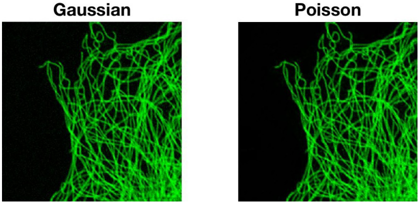


PSNR = 28.1 dB



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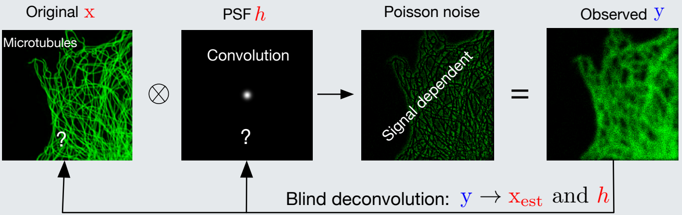

The observation model is given by

$$\mathrm{y} = \alpha\,\mathcal{P}\left(\frac{\mathbf{H}_0\mathrm{x}}{\alpha}\right)$$

- $\mathbf{H}_0$ $\leadsto$ matrix notation of the convolution of PSF h
- $\mathcal{P}(\cdot)$ $\leadsto$ the effect of Poisson noise
- α $\leadsto$ the scaling factor, which controls the strength of noise

blur-MSE Minimization

Oracle criterion

Starting with the general restoration problem,

$$\min_{\mathbf{H}} \underbrace{\frac{1}{N}\| \mathbf{W}_{\mathbf{H}}\mathrm{y} - \mathrm{x}\|^2}_{\text{Mean-squared error (MSE)}}.$$

* Wiener filter $\mathbf{W}_{\mathbf{H}}\mathrm{y} = \mathbf{H}^{\mathbf{T}}(\mathbf{H}\mathbf{H}^{\mathbf{T}} + \lambda\mathbf{P})^{-1}$ can be generalized by any processing $\mathbf{F}_{\mathbf{H}}(\mathrm{y})$.

** $\mathbf{H}$ is not limited to parametric PSF models, and even the convolution operation.

blur-MSE Minimization

Oracle criterion

The PSF estimation is formulated as the following optimization problem:

$$\min_{\mathbf{H}} \underbrace{\frac{1}{N}\|\mathbf{H}\mathbf{W}_{\mathbf{H}}\mathrm{y} - \mathbf{H}_0\mathrm{x}\|^2}_{\text{blur-MSE}}.$$

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The blur-MSE minimization over $\mathbf{H}$ yields $\mathbf{H}\mathbf{H}^{\mathbf{T}} = \mathbf{H}_0\mathbf{H}_0^{\mathbf{T}}$.

$\rightsquigarrow$ Find optimal PSF parameters that minimize the blur-MSE

Approximation by blur-PURE

Theorem of blur-PURE

Let $\mathbf{U} = \mathbf{H}\mathbf{W}_{\mathbf{H}}$, the random variable

$$\text{blur-PURE} = \frac{1}{N}||\mathbf{U}\mathrm{y}||^2 + \frac{1}{N}\|\mathrm{y}\|^2 - \frac{\alpha}{N}\mathbf{1}^{\mathrm{T}}\mathrm{y} - \frac{2}{N}\sum_{n=1}^{N}\mathrm{y}^{\mathrm{T}}\mathbf{U}(\mathrm{y} - \alpha\,\mathrm{e}_n),$$

is an unbiased estimate of the blur-MSE; i.e.,

$$\mathcal{E}\{\text{blur-PURE}\} = \frac{1}{N}\,\mathcal{E}\left\{||\mathbf{U}\mathrm{y} - \mathbf{H}_0\mathrm{x}||^2\right\}.$$

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blur-MSE Minimization $\rightsquigarrow$ blur-PURE Minimization

Gaussian kernel: a typical example

The Gaussian kernel is characterized by

$$\mathbf{h}_s(i,j;s) = C \cdot \exp(-\frac{i^2+j^2}{2s^2}), \text{ Ground truth } s_0.$$

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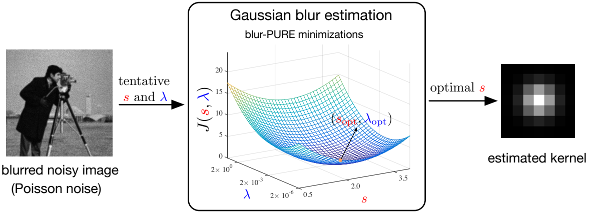
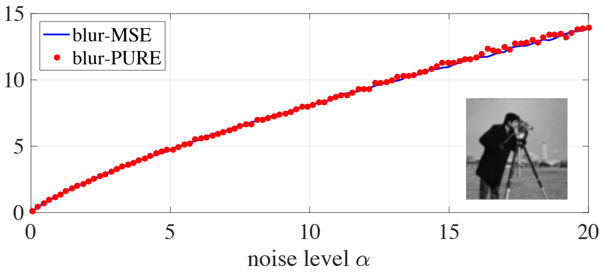
1. Gaussian blur size s
2. Parameter λ in the Wiener filter



The minimization problem: $(s_0, \lambda_0) = \underset{s,\lambda}{\operatorname{argmin}} \{\text{blur-PURE}\}$.

Closeness between blur-PURE and blur-MSE



Example: Cameraman image (256×256) degraded by Gaussian kernel $(s_0 = 2.0)$ and Poisson noise $(\alpha \in [0.05, 20])$. The maximum difference is 0.59.

Simulation results

Cameraman $s_0 = 1.5, \alpha = 10$

Lake $s_0 = 3.0, \alpha = 10$

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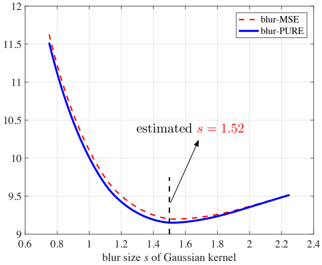
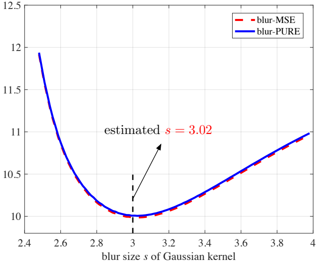




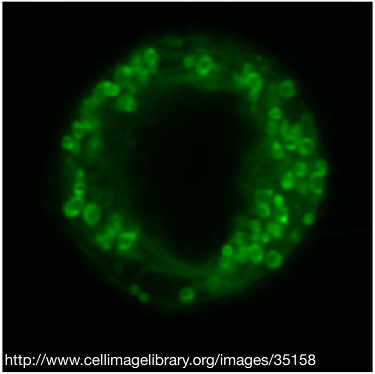
Compared with other approaches

Image	*Cameraman*				*Lake*			
true s_0	$s_0 = 1.5$		$s_0 = 3.0$		$s_0 = 1.5$		$s_0 = 3.0$	
noise level α	1	10	1	10	1	10	1	10
GCV[1]	1.80	2.24	3.72	3.23	1.81	2.05	3.53	3.04
APEX[2]	1.36	1.12	1.62	1.28	0.78	0.78	1.61	0.93
kurtosis[3]	1.55	1.83	2.92	2.35	2.05	2.25	3.29	3.56
DL1C [4]	2.10	2.25	3.47	4.43	2.23	1.98	4.12	2.75
blur-SURE[5]	1.75	2.78	3.43	**3.01**	1.91	2.11	3.51	3.79
blur-PURE	**1.49**	**1.52**	**2.98**	**3.02**	**1.53**	**1.57**	**3.05**	**3.02**

[1] Reeves and Mersereau, TIP'92, [2] Carasso, SIAM JAM'02,

[3] Li et al., IEEE GRS Lett. '07, [4] Chen et al. TIP'09, [5] Xue and Blu, TIP'15.

Blind deconvolution for fluorescence microscopy



Real image

Cell type

Live mitotic HeLa cell

Cellular Component

Mitotic membranes

Imaging Mode

Leica TCS SP5 Confocal Microscopy
63x/1.4 oil objective lens

Dimensions

$256\,\mathrm{px} \times 256\,\mathrm{px}$

Pixel size: 120nm

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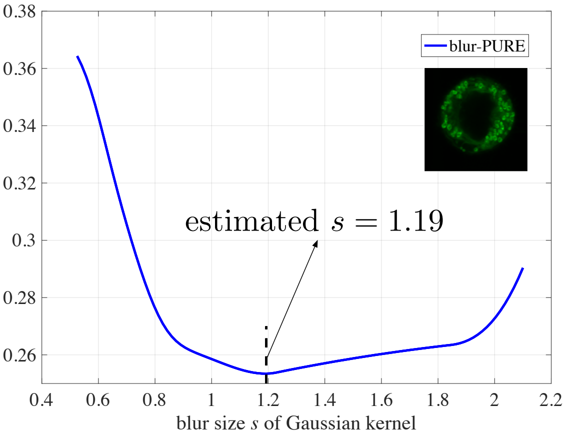

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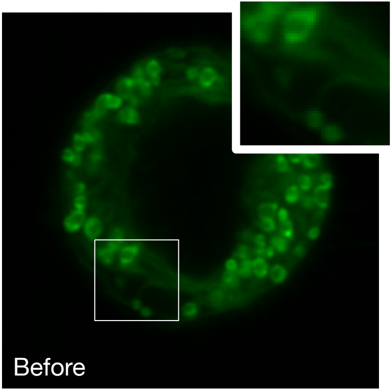
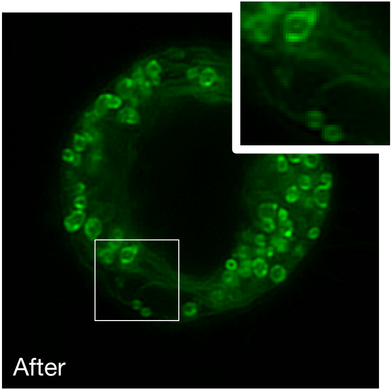
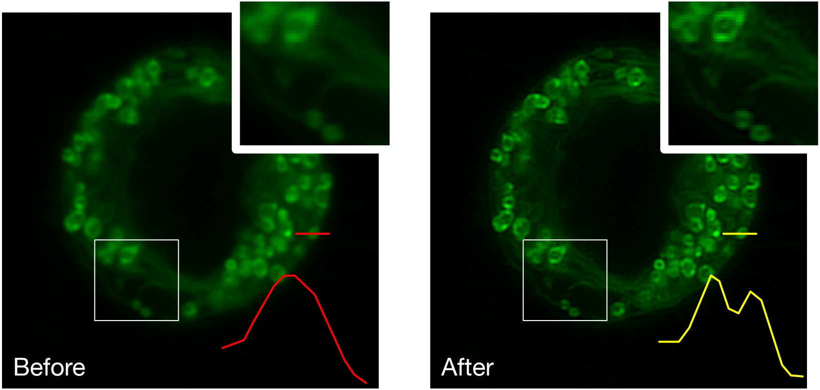




Blur Estimation + PURE-LET Deconvolution (*Li et al., ICIP'16, TIP'17*)

Blind deconvolution for fluorescence microscopy



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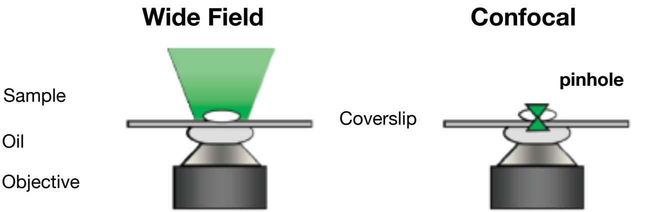
Extension to 3D wide-field microscopy

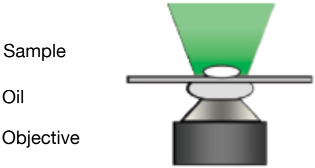


+ Fast, living cell imaging

- Slower acquisition

Extension to 3D wide-field microscopy

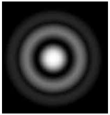
Wide Field



Confocal

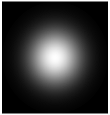
+ Fast, living cell imaging

- Slower acquisition



More than 10 parameters

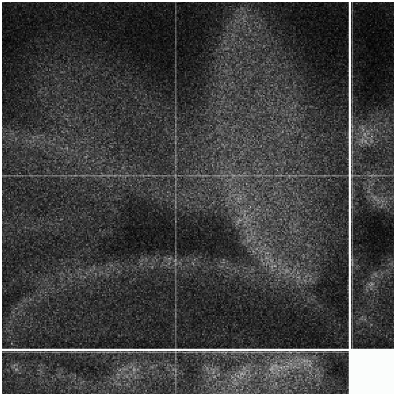
Gibson-Lanni
(Li et al., JOSA'17)



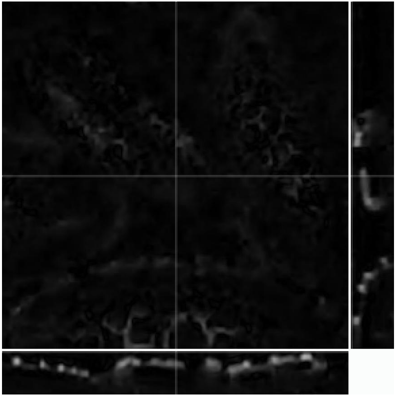
One parameter

Gaussian

3D deconvolution results



input (high noise)
Blurred noisy



blur-PURE + PURE-LET
(*Li et al. ISBI'17, TIP'17*)

Conclusion

Highlights

1. Poisson noise is important
   fewer photons = less data collection time, more temporal resolution
2. blur-PURE: a novel criterion for PSF estimation
3. combined with PURE-LET for efficient 3D deconvolution microscopy

On-going work:

- Incorporation of the motion information
- Application to other imaging modalities
- ImageJ/Icy plugins

More examples at http://www.ee.cuhk.edu.hk/~jzli.

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