# Class-specific Poisson Image Denoising using Importance Sampling

Milad Niknejad, José M. Bioucas-Dias, Mário A. T. Figueiredo

Instituto de Telecomunicações, Instituto Superior Técnico,
University of Lisbon, Portugal

SpaRTaN
Sparse Representations and Compressed Sensing Training Network

# Overview

1. Class-specific image denoising
2. Patch estimation using Monte-Carlo
3. Importance Sampling
4. Applying importance sampling for image denoising
5. Proposed method

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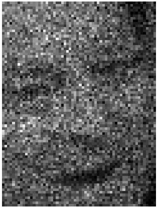

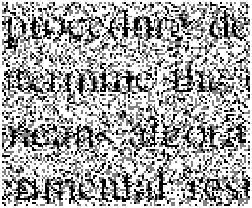

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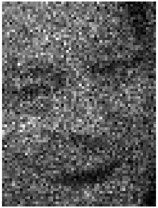

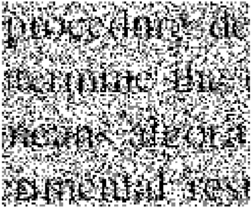

This knowledge should be exploited by the denoising method!

# Class-specific image denoising

Assumption: A dataset of clean images of the same class is available.

# Patch-based image denoising

Gaussian noise observation model:

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i$$

$\mathbf{x}_i$ is a patch of the original image; $\mathbf{y}_i$ is the corresponding noisy patch; $\mathbf{v}_i$ is i.i.d. Gaussian noise.

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Poisson noise observation model (the focus of this presentation):

$$\mathbf{y}_{i,j} \sim \mathcal{P}(\mathbf{x}_{i,j}).$$

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**Goal**: recover the clean patch $\mathbf{x}_i$ from the noisy one $\mathbf{y}_i$

# MMSE (*minimum mean squared error*) estimation

MMSE patch estimate ($p(\mathbf{y} = \mathbf{y}_i)$ is replaced by $p(\mathbf{y}_i)$):

$$\hat{\mathbf{x}}_i = \mathbb{E}[\mathbf{x}|\mathbf{y}_i] = \int_{\mathbb{R}^p} \mathbf{x}\, p(\mathbf{x}|\mathbf{y}_i)\, d\mathbf{x} = \int_{\mathbb{R}^p} \mathbf{x}\, \frac{p(\mathbf{y}_i|\mathbf{x})\, p(\mathbf{x})}{p(\mathbf{y}_i)}\, d\mathbf{x}$$

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- However, sampling from $p(\mathbf{x}|\mathbf{y}_i)$ is also intractable.
- Can we approximate $\hat{\mathbf{x}}_i$ by sampling from another distribution?

# Self-Normalized Importance Sampling (SNIS)

Goal: to compute (or approximate)

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$$\hat{\mathbb{E}}_n[f(\mathbf{z})] = \frac{\sum_{j=1}^{n} f(\mathbf{z}_j) w(\mathbf{z}_j)}{\sum_{j=1}^{n} w(\mathbf{z}_j)}, \qquad w(\mathbf{z}_j) = \frac{\tilde{p}(\mathbf{z}_j)}{\tilde{q}(\mathbf{z}_j)}.$$

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- As in plain Monte-Carlo: $\lim_{n\to\infty} \hat{\mathbb{E}}_n[f(\mathbf{z})] = \mathbb{E}[f(\mathbf{z})]$

# Applying SNIS for MMSE patch estimation

Back to our problem:

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- Why? $\tilde{p}(\mathbf{z}) = p(\mathbf{y}_i|\mathbf{x})\, p(\mathbf{x})$, $c = 1/p(\mathbf{y}_i)$ and $\tilde{q}(\mathbf{z}) = p(\mathbf{x})$.

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- It can be generalized to other noise models.

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3. Use the corresponding clean patches as samples from the proposal distribution for SNIS.

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- The above integral is intractable, but we can use SNIS.

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$$\mathbb{E}[\|\mathbf{x}-\mathbf{u}\|_2^2|\mathbf{y}_i,k] = \int_{\mathbb{R}_+^m} \|\mathbf{u}-\mathbf{x}\|_2^2 \; p(\mathbf{x}|\mathbf{y}_i,k) \; d\mathbf{x}.$$

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Using SNIS, the above can be approximated by

$$\hat{\mathbb{E}}_n[\|\mathbf{x}-\mathbf{u}\|_2^2|\mathbf{y}_i,k] = \frac{\sum_{j=1}^{n} \|\mathbf{u}-\mathbf{x}_j\|_2^2 \; p(\mathbf{y}_i|\mathbf{x}_{j_k})}{\sum_{j=1}^{n} p(\mathbf{y}_i|\mathbf{x}_{j_k})} \tag{1}$$

where the $\mathbf{x}_{j_k}$, for $j=1,...,n$ are samples from the distribution $p(\mathbf{x}|k)$.

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We minimize the approximation (1) by alternating minimization.

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- when $\mathbf{u}$ is fixed,

$$\hat{k}_i(\mathbf{u}) = \arg\min_k \frac{\sum_{j=1}^{n_2} \|\mathbf{u} - \mathbf{x}_j\|_2^2 \, p(\mathbf{y}_i | \mathbf{x}_{j_k})}{\sum_{j=1}^{n_2} p(\mathbf{y}_i | \mathbf{x}_{j_k})}. \tag{2}$$

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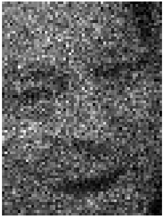
- Denoising: samples derived for each patch $n_1$ was set to 300.
- Overall: 900 patches are processed for each denoised patch (computational complexity is similar to an internal non-local denoising with the patches constrained in $30 \times 30$ window).

# Experiment 1



Noisy image
(Peak=10)

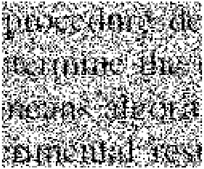
Non-local PCA
(PSNR=22.60)

VST+BM3D
(PSNR=24.79)

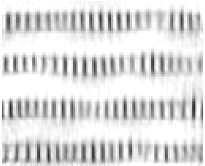
Poisson NL means
(PSNR=24.55)

Proposed
(PSNR=26.40)

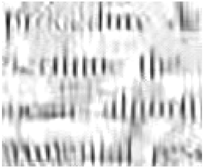
# Experiment 2



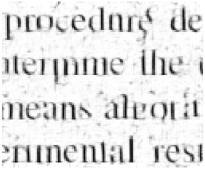
Noisy (Peak=2)



Non-local PCA (PSNR=14.95)



VST+BM3D (PSNR=14.55)



Proposed (PSNR=18.64)

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- Any clustering method can be used
- Each cluster can be seen as samples of unknown proposal distribution
- The method can be generalized easily to other image restoration inverse problems.

# References


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Thanks for your attention