Extended Conjugate Polar Fourier Transform in Convolution Network

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Abstract

This paper proposes an extended conjugate polar Fourier transform (ECPFT), to design iterated radial filter bank (RFB) and directional filter bank (DFB) convenient for accurate multiscale and multidirectional decomposition in discretization over a convolution network. With conjugated symmetric form, ECPFT would convert complex directional wavelets in original spatial domain to real ones in the inverse Fourier domain of ECPFT. Furthermore, it can contribute to changing the design of nonseparable RFB and DFB to decomposition in scales and directions with 1-D filter bank in each dimension separably in the ECPFT domain. The generated properties of conjugate symmetry and periodicity of 2π in both angular and radial dimensions, also guarantee convolution and sampling operations in both dimensions within the inverse Fourier domain of ECPFT. The fast algorithm for discretization is developed to reduce complexity.

ECPFT

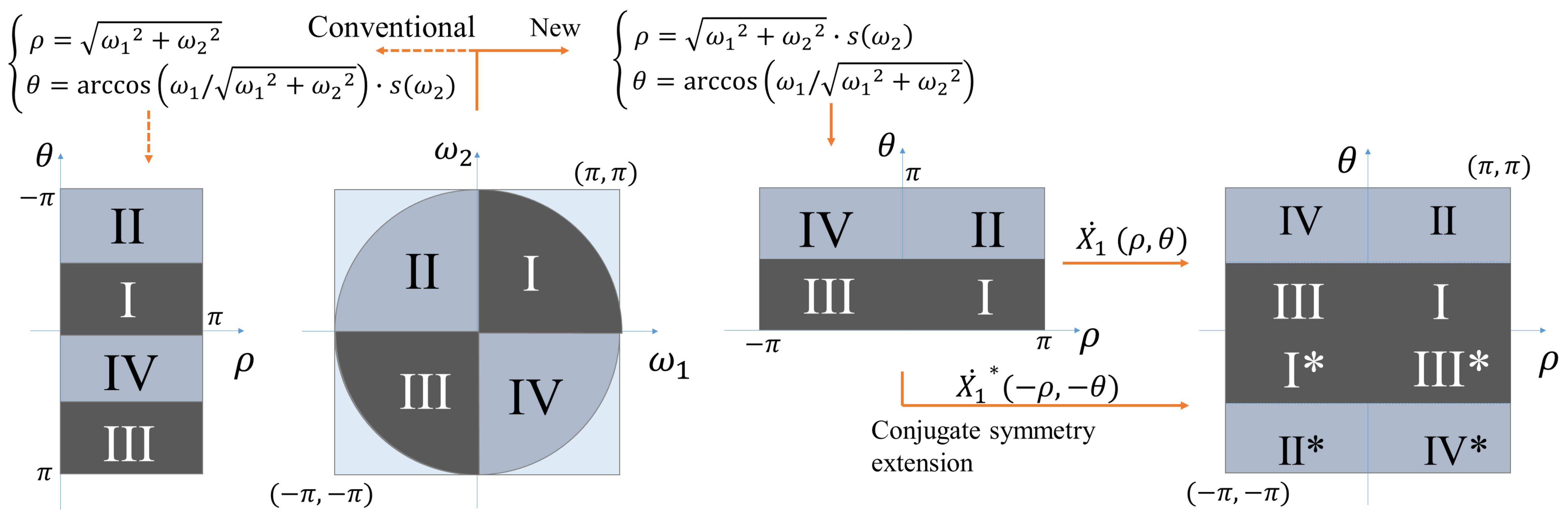

Fig. 1. The four sub-figures from left to right are: conventional polar Fourier transform (PFT) domain, Fourier transform (FT) domain, conjugated polar Fourier transform (CPFT) domain and extended conjugate polar Fourier transform (ECPFT) domain.

Fast Approximation Algorithm

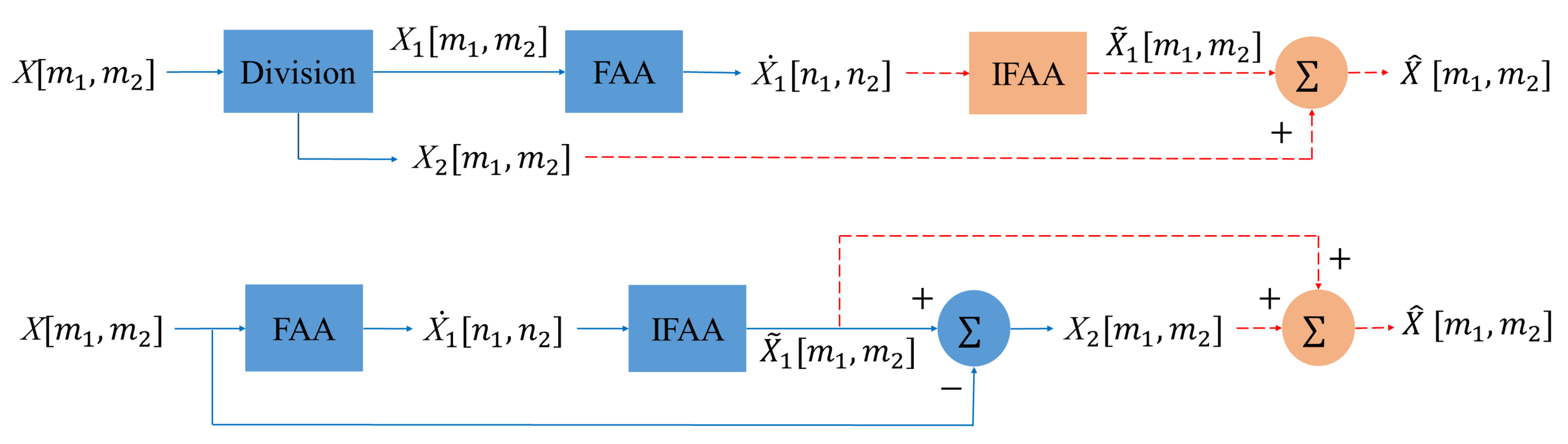

Fig. 2. Two analysis and synthesis diagrams of ECPFT. Top line is the direct discrete implementation of coordinate conversion. Bottom line is the scheme guarateeing perfect reconstrction.

RFB & DFB

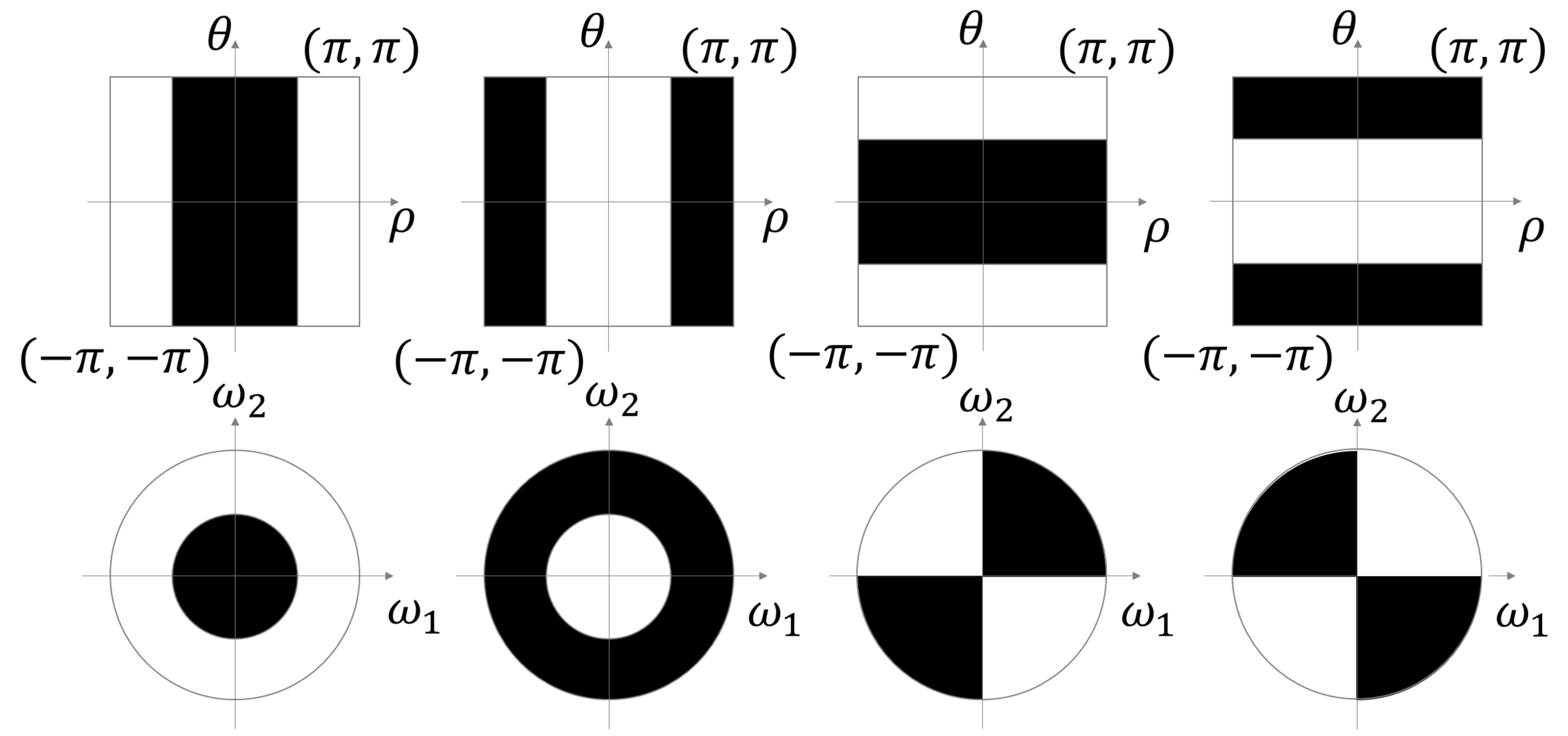

Fig. 3. Support configurations for the filters in the RFB and DFB. The top line is the frequency view of new polar coordinate and the bottom line corresponds to the frequency domain of the original coordinate. The first and second columns are the support configurations of radial filter pair $P_0(\beta), P_1(\beta)$; The third and fourth columns are the support configurations of directional filter pair $H_0(\beta), H_1(\beta)$.

Iterated Convolutional Network

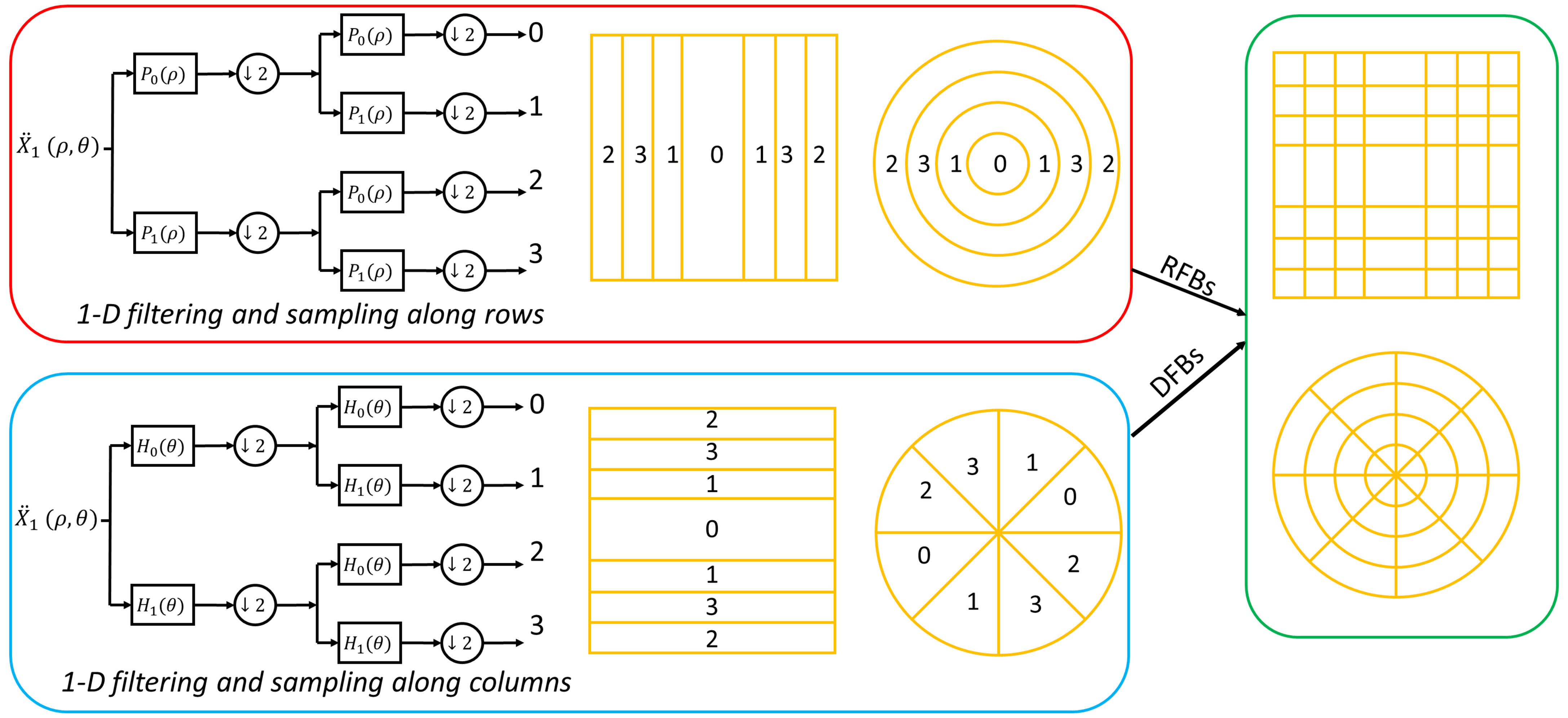

Fig. 4. An example of a full convolution network with RFBs and DFBs of depth $(l_1, l_2) = (2,2)$.

RFB and DFB are two types of filter banks as the foundation of the underlying convolution network. The support configurations for decomposition filter pairs are shown in Fig. 3, with their sampling matrix given in (11). RFB corresponds to multiscale decomposition and DFB the multidirection decomposition. Though described in 2-D form, it is in practice implemented by 1-D convolution and sampling along each dimension, respectively. For a better understanding, we take an example of a 2-level network in Fig. 5.

Experiments

- ***Fast approximation***

Table 1. Energy ratio and Running time of calculating $\ddot{X}_1[n_1,n_2]$ from both direct formula and fast approximation.

size	128	256	512	1024
energy ratio $\lVert x_2\rVert^2/\lVert x\rVert^2$	0.66%	0.66%	0.21%	0.09%
Running time of (4)(10)	0.77	8.70	92.08	1672.96
Running time of FAA	0.02	0.08	0.31	1.29

Since the input signal is divided into two parts x_1 and x_2, x_1 is first recovered from decomposition coefficients and then combined with x_2 to obtain the reconstructed signal. In discrete case, different orders have different effects.

Fig. 5. Reconstruction. Leftmost: the original image; Middle left: reconstructed by removing $X_2[m_1, m_2]$; Middle right and rightmost are reconstructed images with two coordinate conversion schemes in Fig. 2 (a) and (b), respectively.

- ***Nonlinear Approximation***

For the 12 test images, we select 3% most significant coefficients in each transform domain, and then compare the reconstructed images from these sets of coefficients.

Table 2. PSNR of reconstructed images with different transforms

PSNR(dB)	wavelet	contourlet	curvelet	proposed
barbara	17.68	22.28	21.97	**22.86**
cameraman	9.36	20.87	20.48	**20.93**
peppers	17.97	25.33	24.38	**26.70**
fingerprint	17.83	17.04	17.22	**17.93**
lena	17.87	25.39	24.95	**25.41**
baboon	15.57	21.97	21.24	**22.06**
bridge	16.00	21.65	20.62	**22.01**
lax	19.16	22.05	21.50	**22.21**
woman	19.43	25.04	24.17	**25.95**
crowd	19.68	23.08	22.22	**23.92**
milkdrop	22.26	27.41	27.37	**27.75**
plane	18.92	22.78	23.46	**23.72**