NON-RIGID Image Deformation Algorithm based on MRLS-TPS

Huabing Zhou[1], Yuyu Kuang[1], Zhenghong Yu[2], Shiqiang Ren[1], Anna Dai[1], Yanduo Zhang[1], Tao Lu[1], Jiayi Ma[3]

[1]Wuhan Institute of Technology, Wuhan, China.

[2]Guangdong Polytechnic of Science and Technology, Zhuhai, China, [3]Wuhan University, Wuhan, China.

Introduction

- Image Deformation
  - Given a set of observed input-output pairs, image deformation aims to render new visual effects via geometric transformation and pixel interpolation.
  - The deformation is typically controlled by the user-selected handles, the image deforms in an intuitive fashion as the user modifies the position of these handles.
- Rigid Image Deformation
  - Deals with the situation when deformation have the property of piecewise rigid.
- Construction
  - We propose a novel closed-form transformation estimation method based on moving regularized least squares optimization with thin-plate spline (MRLS-TPS) for non-rigid image deformation.
  - Firstly, we introduce the TPS function and regularization technology to the deformation problem, which can help to get more realistic deformation.
  - Secondly, we provide a fast implementation, which enables our method to handle large-scale datasets.

Problem Formulation

- The mathematical formulation is based on Moving Least Squares (MLS). For any point $\mathbf{p}$ in the image, MLS solves for a rigid-body transformation $\mathbf{f}_{\mathbf{p}}(\mathbf{x})$ that minimizes a weighted least squares error functional

$$\sum_{i=1}^{n} w_i(\mathbf{p})\|\mathbf{f}_{\mathbf{p}}(\mathbf{x}_i) - \mathbf{y}_i\|^2, \qquad (1)$$

where $w_i(\mathbf{p})$ is a non-negative weight function defined as

$$w_i(\mathbf{p}) = \|\mathbf{p} - \mathbf{x}_i\|^{-2\alpha} \qquad (2)$$

with α controlling the weight of each control point and $\|\cdot\|$ being the Euclidean distance metric.

- Given an arbitrary point $\mathbf{p}$ in the image, we solve for the optimal transformation function $\mathbf{f}$ that minimizes a weighted regularized least squares error functional

$$\sum_{i=1}^{n} w_i(\mathbf{p})\|\mathbf{f}_{\mathbf{p}}(\mathbf{x}_i) - \mathbf{y}_i\|^2 + \lambda\phi(\mathbf{f}_{\mathbf{p}}). \qquad (3)$$

The functional ϕ has the form

$$\phi(\mathbf{f}_{\mathbf{p}}) = \|\mathbf{f}_{\mathbf{p}}\|^2_{\mathcal{H}}, \qquad (4)$$

- The TPS model can be decomposed into a global affine transformation and a local bending function, being controlled by affine matrix and bending function respectively:

$$\mathbf{f}(\mathbf{p}) = \mathbf{A}\overline{\mathbf{p}} + \mathbf{g}_{\mathbf{p}}(\mathbf{p}), \qquad (5)$$

$$\mathbf{g}_{\mathbf{p}}(\mathbf{p}) = \sum_{i=1}^{n}\mathbf{K}(\mathbf{p}, \mathbf{p}_i)\overline{\mathbf{c}}_i, \qquad (6)$$

- The radial basis function $\mathbf{K}$ is TPS kernel.

$$\mathbf{K}(\mathbf{p}, \mathbf{p}_i) = \|\mathbf{p} - \mathbf{p}_i\|^2 \log(\|\mathbf{p} - \mathbf{p}_i\|). \qquad (7)$$

- The weights w_i are dependent on the evaluation point $\mathbf{p}$, and the regularization technique with TPS kernel is used to impose smoothness.

A Closed-form Solution

- By substituting Eq. (5) and Eq. (6) into Eq. (1), selecting regularization parameter λ , the non-linear mapping $\mathbf{f}$ can be obtained by minimizing the following TPS energy function:

$$E(\mathbf{A}, \mathbf{C}) = \|\mathbf{W}^{\frac{1}{2}}(\overline{\mathbf{Y}} - \overline{\mathbf{X}}\mathbf{A}^T - \mathbf{K}\mathbf{C})\|^2 + \lambda \mathrm{tr}(\mathbf{C}^T\mathbf{K}\mathbf{C}), \qquad (8)$$

- First of all, we use QR decomposition on matrix $\overline{X}$ to compute the TPS parameters $\mathbf{A}$ and $\mathbf{C}$:

$$\overline{\mathbf{X}} = [\mathbf{Q}_1|\mathbf{Q}_2]\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix}, \qquad (9)$$

- Substituting QR decomposition into equation (8), we can obtain

$$\begin{aligned} E(\mathbf{A}, \widetilde{\mathbf{C}}) = &\| \mathbf{Q}_2^T(\widetilde{\mathbf{Y}} - \mathbf{W}^{1/2}\mathbf{K}\mathbf{Q}_2\widetilde{\mathbf{C}}) \|^2 \\ &+ \| \mathbf{Q}_1^T\widetilde{\mathbf{Y}} - \mathbf{R}\mathbf{A}^T - \mathbf{Q}_1^T\mathbf{W}^{1/2}\mathbf{K}\mathbf{Q}_2\widetilde{\mathbf{C}} \|^2 \\ &+ \lambda \mathrm{tr}(\widetilde{\mathbf{C}}^T\mathbf{Q}_2^T\mathbf{K}\mathbf{Q}_2\widetilde{\mathbf{C}}), \end{aligned} \qquad (10)$$

- By minimizing the energy equation (10) about $\widetilde{\mathbf{C}}$ and $\mathbf{A}$, we could get

$$\mathbf{C} = \mathbf{Q}_2\widetilde{\mathbf{C}} = \mathbf{Q}_2(\mathbf{S}^T\mathbf{S} + \lambda\mathbf{T} + \varepsilon\mathbf{I})^{-1}\mathbf{S}^T\mathbf{Q}_2^T\widetilde{\mathbf{Y}}, \qquad (11)$$

$$\mathbf{A} = (\widetilde{\mathbf{Y}} - \mathbf{W}^{1/2}\mathbf{K}\mathbf{C})^T\mathbf{Q}_1\mathbf{R}^{-T}, \qquad (12)$$

- Substituting Eq. (11) and Eq. (12) into Eq. (5) and Eq. (6), we get the closed-form solution of $\mathbf{f}$:

$$\mathbf{f}(\mathbf{p}) = \mathbf{A}\overline{\mathbf{p}} + (\mathbf{K}_{\mathbf{p}}\mathbf{C})^T \qquad (13)$$

Fast Implementation Computational Complexity

- $\mathbf{X}$ is generally fixed for the QR decomposition of the point set. Therefore, a lot of steps can be calculated in advance in formula (11), matrix $\mathbf{C}$ and $\mathbf{A}$ can be rewritten as

$$\mathbf{f}(\mathbf{p}) = (\mathbf{Y}^T - \mathbf{T}^T\mathbf{H}\mathbf{W}^{1/2})\mathbf{B}\overline{\mathbf{p}} + \mathbf{Y}^T\mathbf{M}. \qquad (14)$$

$$\mathbf{C} = \mathbf{Q}_2\widetilde{\mathbf{C}} = \mathbf{Q}_2(\mathbf{S}^T\mathbf{S} + \lambda\mathbf{T} + \varepsilon\mathbf{I})^{-1}\mathbf{S}^T\mathbf{Q}_2^T\widetilde{\mathbf{Y}} \qquad (15)$$

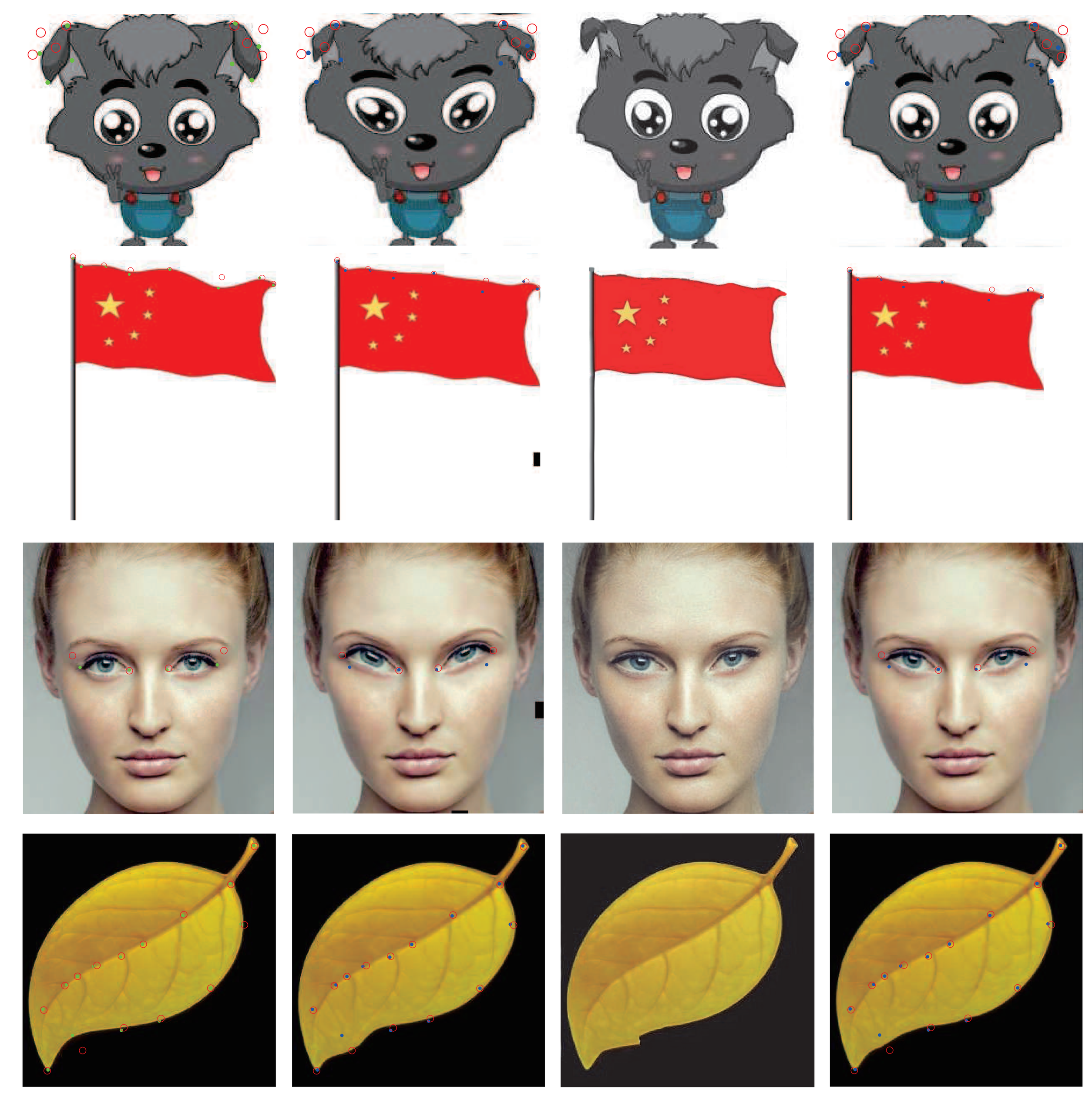
- The most time consuming step of our proposed algorithm is to solve the transformation $\mathbf{f}_{\mathbf{p}}$ using linear system (11), which requires $O(M^3)$ time complexity and may pose a serious problem for large values of M.
- Setting $\mathbf{E} = \mathbf{Q}_2(\mathbf{S}^T\mathbf{S} + \lambda\mathbf{T} + \varepsilon\mathbf{I})^{-T} - \mathbf{Q}_2^T$, combining solution (11) with formula (12), we can get $\mathbf{B} = \mathbf{Q}_1\mathbf{R}^{-T}$, $\mathbf{H} = \mathbf{E}\mathbf{K}$, $\mathbf{M} = \mathbf{E}(\mathbf{K}_{\mathbf{P}})^T$.

Parameter Setting

- There are mainly two parameters in our method: α and λ.
  - Parameter α controls the weight of each control point.
  - Parameter λ controls the complexity of the transformation.
  - We set $\alpha = 2$ and $\lambda = 8$.

Experimental Results

- Experimental Setup
  - The experiments have been conducted on a laptop with 2.5-GHz Intel Core CPU, 8-GB memory, and MATLAB code.
  - Some state-of-the art deformation methods such as MLS and the commercial software Adobe PhotoShop CS 6 are well-established for comparision.
  - All parameters are fixed in the experiments.
- Results on wide baseline images



green and **blue** points = initial control points, **red** circles = deformed control points.

- Efficiency test

Table 1: Runtime of MLS and MRLS-TPS.

	Wolf	Flag	Face	Leaf
#ctrl pt	8	8	8	15
grid	30×20	30×48	30×33	30×30
MLS (s)	0.7864	0.5832	0.5745	0.5624
MRLS-TPS (s)	0.4821	0.3351	0.3449	0.3458

- Conclusion
  - MRLS-TPS is very fast and can be performed in real-time.
  - The bending energy minimized by TPS has a specific physical explanation, being beneficial in the case of image deformation with non-rigid motions.