Psuedo Reversible Symmetric Extension for Lifting-based Nonlinear-Phase Paraunitary Filter Banks

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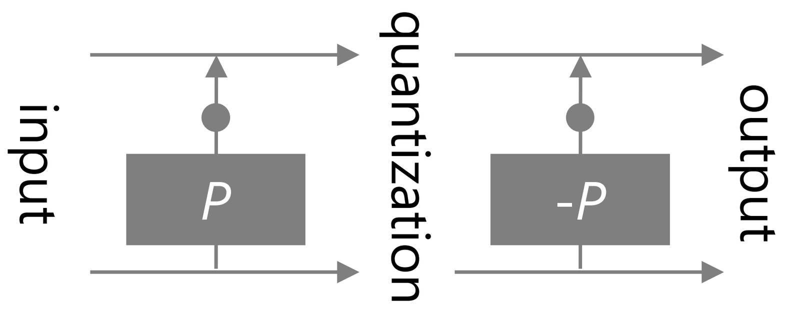
1. Purpose & Preparation

Our target is more efficient lossy-to-lossless (L2L) image coding.

- **lifting-based nonlinear-phase paraunitary filter banks (L-NLPPUFBs)** which are more efficient reversible transforms
- **pseudo reversible symmetric extension (P-RevSE)** which solves the image boundary problem on the reversible transforms

Lifting Structure

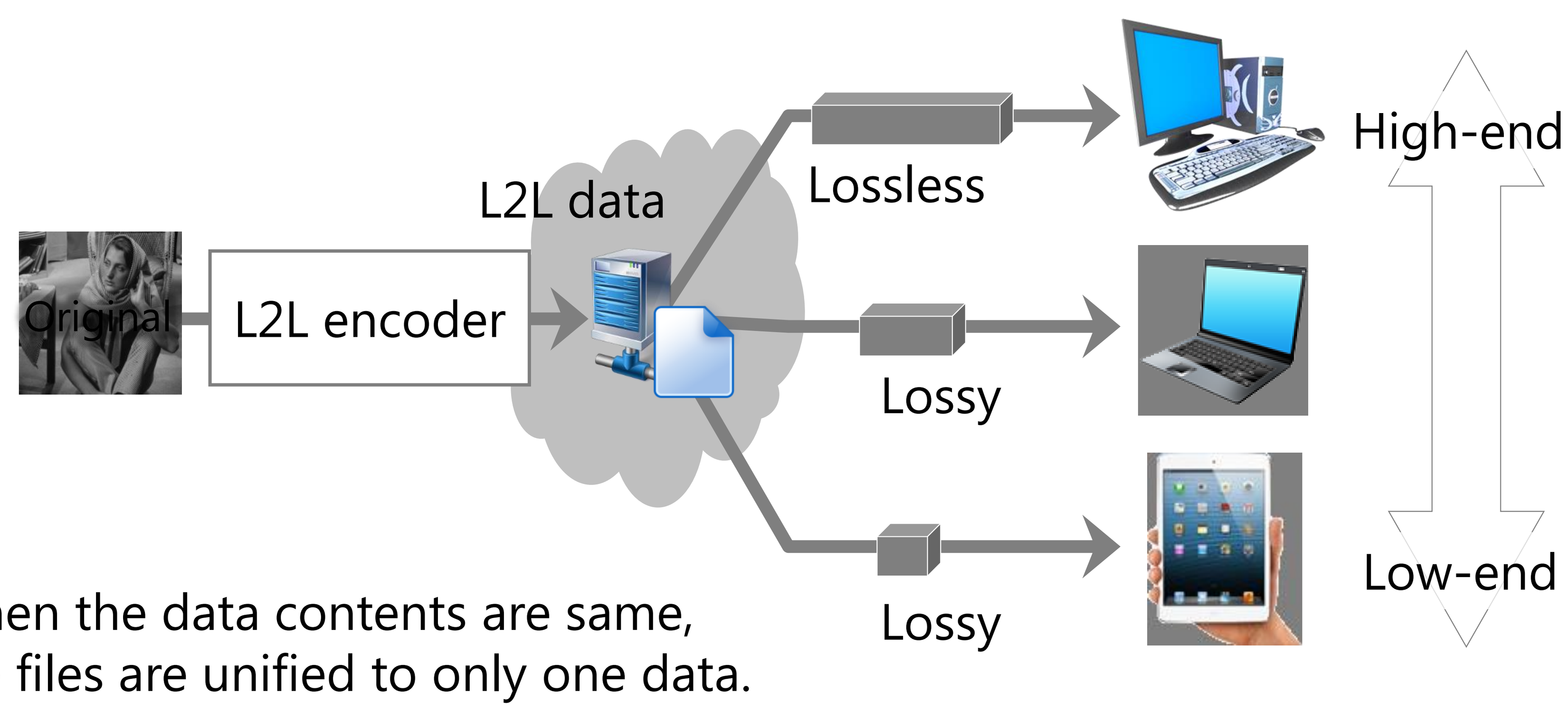


- Map **integer to integer** (signals)
- **Lossless** when quantization width = 1
- **Lossy** when quantization width > 1
- FBs can be factorized into lifting structures where the constraint is $\det(\mathbf{E}(z)) = \pm 1$
- Be also used for LT in JPEG XR

●: "rounding operation" which rounds a floating-point number to an integer

P: "lifting coefficient" which is a floating-point number

Lossy-to-lossless (L2L) Image Coding



When the data contents are same, the files are unified to only one data.

Lifting-based FBs can achieve L2L image coding.

2. P-RevSE for L-NLPPUFBs (Proposal)

Nonlinear-Phase Paraunitary Filter Banks (NLPPUFBs) [5]

- The lattice structure is as follows:

$$\mathbf{E}(z) = \left(\prod_{i=K-1}^{1} \mathbf{G}_i \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & z^{-1}\mathbf{I} \end{bmatrix} \right) \mathbf{G}_0$$

$\mathbf{G}_k$: an *MxM* arbitrary unitary matrix

$$\det\left(\mathbf{E}(z)|_{z=1}\right) = \pm 1$$

- **NOT limited by the linear-phase property**, i.e., they have high compression rates

ex) The lapped transform (LT) in JPEG XR has the linear-phase property

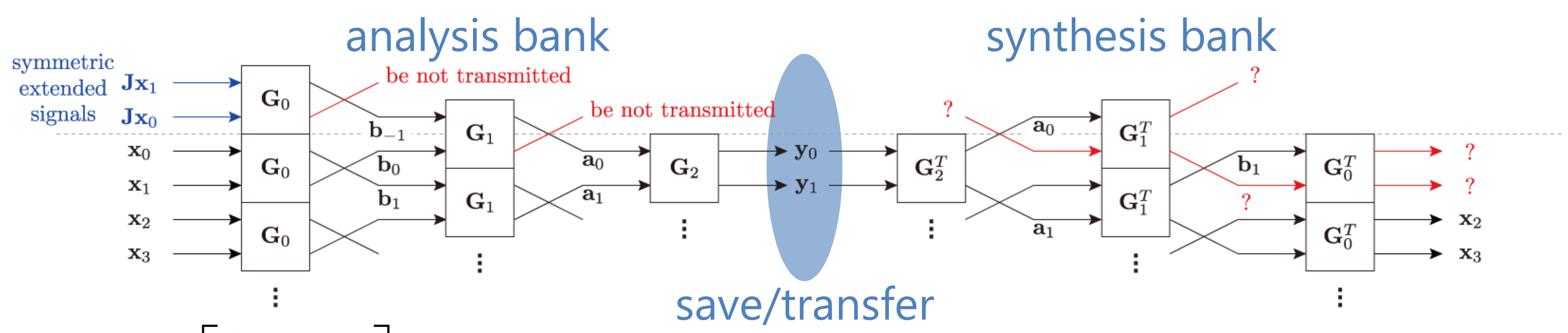
- **Can be easily factorized into lifting structures**

Symmetric Extension (SE) for NLPPUFBs [9]

In lapped transforms such as NLPPUFBs, * periodic extension (PE) is not smooth.

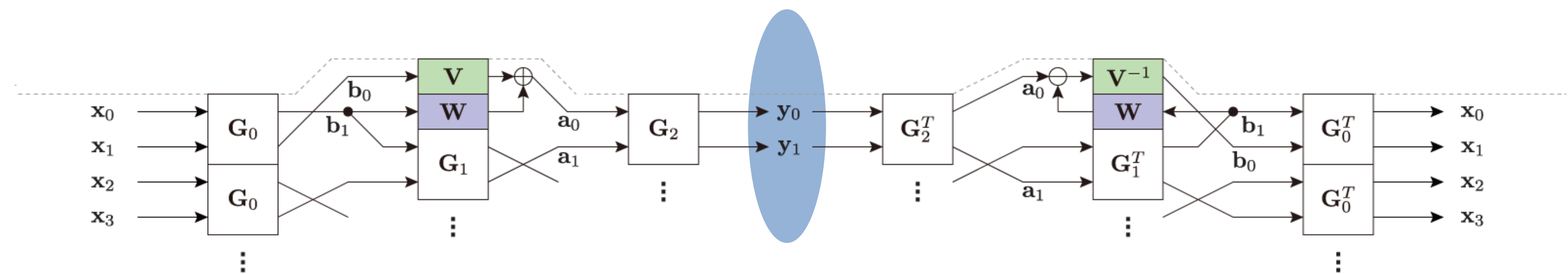
a smooth nonexpansive convolution should be used at the boundaries not to increase the number of samples and achieve more efficient coding.

Upper boundary processing of NLPPUFBs (*K*=3) are as follows:



When $\mathbf{G}_k = \begin{bmatrix} \mathbf{A}_k & \mathbf{B}_k \\ \mathbf{C}_k & \mathbf{D}_k \end{bmatrix}$ where each submatrix is *M*/2x*M*/2,

by solving a simultaneous matrix equation, we obtain the following forms:



$$\mathbf{V} = \mathbf{A}_1\left(\mathbf{B}_2\mathbf{J}\mathbf{C}_2^T + \mathbf{A}_2\mathbf{J}\mathbf{D}_2^T\right) + \mathbf{B}_1 \quad \text{where } \det(\mathbf{V}) \neq 0$$

$$\mathbf{W} = \mathbf{A}_1\left(\mathbf{B}_2\mathbf{J}\mathbf{A}_2^T + \mathbf{A}_2\mathbf{J}\mathbf{B}_2^T\right)$$

$$\mathbf{J} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & ⋰ & 1 & 0 \\ 0 & ⋰ & ⋰ & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}$$

Lower boundary case can be reconstructed in the same way as in the upper case.

Pseudo Reversible Symmetric Extension (P-RevSE) for Lifting-based NLPPUFBs (L-NLPPUFBs)

Even if NLPPUFBs can be easily factorized into lifting structures, the conventional SE cannot achieve reversible transforms.

↓

If $\mathbf{V}$ is also expressed as lifting structures, the SE can achieve reversible transforms.

A minimum condition to realize lifting factorization:

$$\det(\mathbf{V}) = \pm 1$$

According to the condition, we control the det. of the matrices in

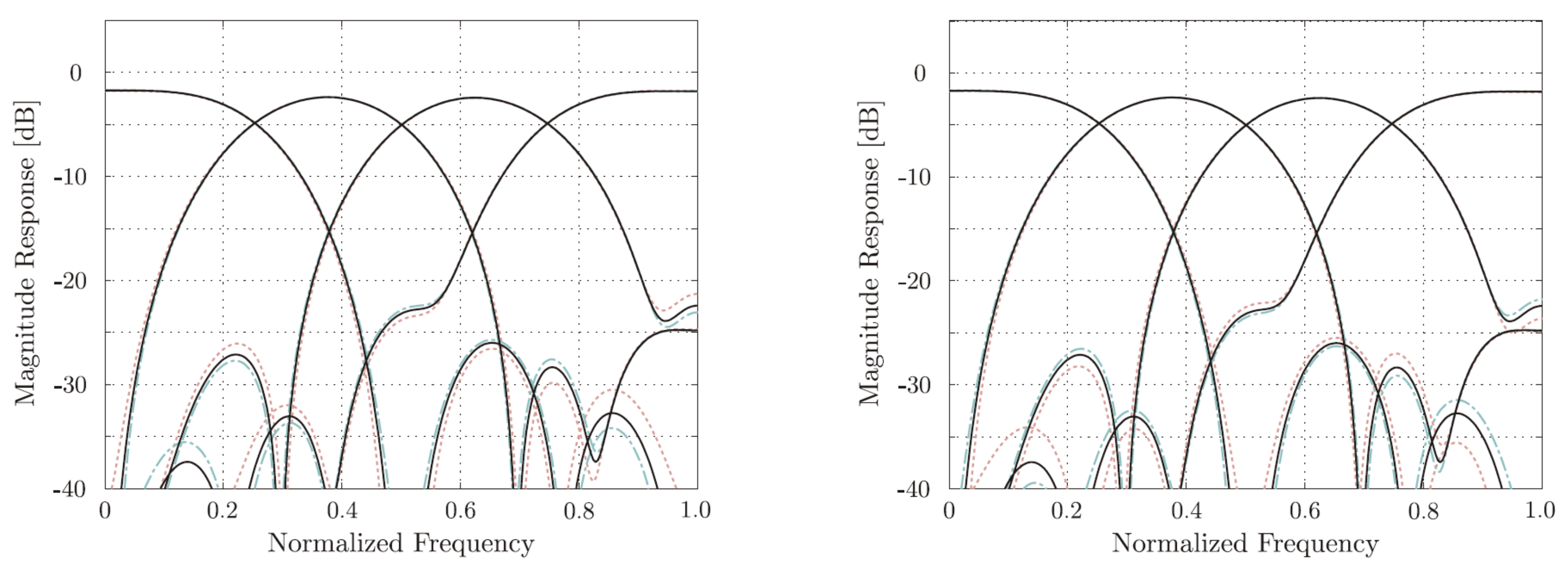
$$\tilde{\mathbf{V}} = \frac{\mathbf{V}}{\sqrt[M/2]{|\det(\mathbf{V})|}}$$

On the other hand, if $\tilde{\mathbf{U}}, \tilde{\mathbf{V}}$ are significantly different from $\mathbf{U}, \mathbf{V}$, smoothness at the boundary may be lost and may degrade compression efficiency.

To preserve the smoothness, we design the L-NLPPUFBs by considering the differences as

$$C_{det} = \left(|\det(\mathbf{V})| - 1\right)^2$$

We designed 3 types of 4x12 NLPPUFBs (*K*=3): not boundary (A), upper boundary (B), lower boundary (C)



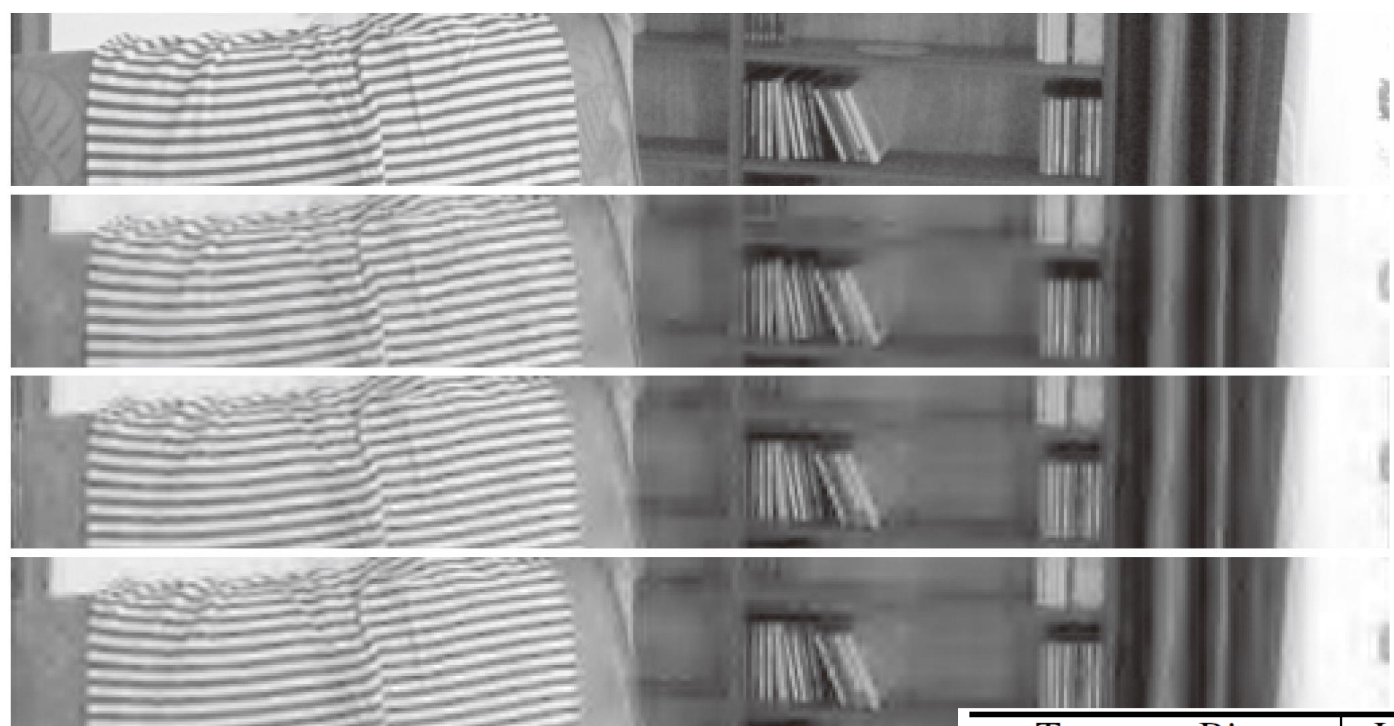
Frequency responses (black: type A, pink: type B, light blue: type C)

Coding gain of the resulting 4x12 L-NLPPUFBs

Boundary	Not	Upper	Lower
C_{cg}	8.3168	8.2852	8.3173

All types of NLPPUFBs have almost same property.

3. Experimental Results



Original image

LT (JPEG XR) with SE — no boundary error

FBs with PE — boundary error

FBs with P-RevSE — no boundary error

Particular area of *Room* (0.25[bpp]): top and bottom are NOT boundaries.

Lossy image coding results (PSNR [dB]) →

Test Images	Bitrate [bpp]	LT [2] RevSE	L-NLPPUFBs PE	L-NLPPUFBs P-RevSE
Barbara	0.25	26.569	27.436	**27.578**
	0.50	30.334	31.097	**31.234**
	1.00	34.952	35.601	**35.728**
Boat	0.25	27.261	28.129	**28.219**
	0.50	30.727	31.296	**31.371**
	1.00	34.213	34.758	**34.805**
Elaine	0.25	30.825	31.178	**31.381**
	0.50	32.502	32.876	**33.016**
	1.00	34.179	34.746	**34.894**
Lena	0.25	31.603	31.965	**32.251**
	0.50	35.024	35.300	**35.524**
	1.00	38.247	38.572	**38.708**
Pepper	0.25	31.026	31.078	**31.586**
	0.50	33.892	34.207	**34.328**
	1.00	35.428	36.200	**36.273**
Room	0.25	27.818	28.684	**29.048**
	0.50	32.715	32.838	**33.116**
	1.00	38.288	37.927	**38.407**

Lossless image coding results (lossless bitrate [bpp])

Test Images	LT [2] RevSE	L-NLPPUFBs PE	L-NLPPUFBs P-RevSE
Barbara	4.801	4.798	**4.775**
Boat	5.124	5.103	**5.093**
Elaine	5.166	5.132	**5.106**
Lena	**4.587**	4.615	**4.587**
Pepper	4.954	4.907	**4.897**
Room	**4.344**	4.452	4.427

[2] C. Tu et al., "Low-complexity hierarchical lapped transform for lossy-to-lossless image coding in JPEG" *SPIE*, 2008.
[5] X. Gao et al., "On factorization of *M*-channel paraunitary filterbanks," *IEEE TSP*, 2001.
[9] Y. Tanaka et al., "A non-expansive convolution for nonlinear-phase paraunitary filter banks and its application to image coding," *ACSSC*. 2005.

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