COMPLEX COEFFICIENT REPRESENTATION FOR IIR BILATERAL FILTER

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Introduction

Edge-preserving filtering is essential tools for image processing and photo editing. There are much applications:
Denoising, Detail enhancement, Stylization, HDR, Haze remove, Stereo matching, Optical flow estimation, and etc.

Main issue of the edge-preserving filtering is processing cost. There are serval acceleration approaches.

- Approximation of FIR convolution (bilateral & non-local means)
    - decomposing to multiple Gaussian filters
        - Real-time O(1) bilateral filter
        - Raised cosine approximation
        - Compressive bilateral filter
- Filtering with Local linear assumption
    - filtering with stack of box filtering
        - guided image filter
- **Edge-preserving filtering with IIR filtering**
    - coefficients of IIR filtering for edge-preserving
        - Domain transform filter
        - Recursive bilateral filter

Advantage and limitations in IIR representation:

O Computationally efficient
X Filtering with only geodetic distance,
Euclid distance is not supported.
X Does not support separability

Contribution of this paper is:

- Representation of edge-preserving filtering with complex IIR filter
- Connection between the other approximation bilateral filtering with IIR filter

#Note that the proposed method is a representation of bilateral filtering, but not an optimal approximation of bilateral filtering.

Conventional and Proposed Method

FIR to 1st order IIR filter

$$J_p = \sum_{q=1}^{N} W_{q,p} I_q,$$

normalization case: $J_p = \frac{\sum_{q=1}^{N} W_{q,p} I_q}{\sum_{q=1}^{N} W_{q,p}}$

$$J_p = \sum_{q=1}^{p} W_{q,p} I_q + \sum_{q=p+1}^{N} W_{q,p} I_q = J_p^L + J_p^R.$$

assumption

$$W_{p,p} = 1, \quad W_{q,p} = W_{q,r} W_{r,p} \quad (q \le r \le p).$$

$$W_{qp} = W_{q,q+1} W_{q+1,q+2} \cdots W_{p-2,p-1} W_{p-1,p} = \prod_{j=q}^{p-1} W_{j,j+1}$$

1st IIR

$$J_p^L = \sum_{q=1}^{p} W_{q,p} I_q = \sum_{q=1}^{p-1} W_{q,p} I_q + W_{p,p} I_p = W_{p-1,p} J_{p-1}^L + I_p$$

$$= W_{p-2,p-1} W_{p-1,p} J_{p-2}^L + W_{p-1,p} I_{p-1} + I_p = \cdots$$

Definition of recursive BF (conv. IIR)

$$R_{n,n+1} := \exp\left(\frac{-|I_n - I_{n+1}|^2}{2\sigma_r^2}\right),$$ geodesic

$$S_{n,n+1} := \exp\left(\frac{-1}{\sigma_s}\right)$$

$\prod_n \exp(x_n) = \exp(\sum_n x_n)$

$$W_{q,p} := \exp\left(\frac{-|q-p|}{\sigma_s}\right) \exp\left(\frac{-\sum_{n=q}^{p-1} |I_n - I_{n+1}|^2}{2\sigma_r^2}\right).$$

Proposed range weight

$$R_{p,p+1} := \exp\left(\frac{-\mathrm{j}(I_p - I_{p+1})}{\sigma_r}\right)$$ Euclid

$$R_{q,p} := \exp\left(\sum_{n=q}^{p-1} \frac{-\mathrm{j}(I_n - I_{n+1})}{\sigma_r}\right) = \exp\left(\frac{-\mathrm{j}(I_q - I_p)}{\sigma_r}\right)$$

$$= \cos\left(\frac{I_q - I_p}{\sigma_r}\right) - \mathrm{j} \sin\left(\frac{I_q - I_p}{\sigma_r}\right).$$

Extension to n-th order IIR filter

$$y_p = \sum_{l=0}^{m-1} (a_l x_{i-l}) - \sum_{k=1}^{m} (b_k y_{i-k}),$$ usual n-th order IIR filter

$$J_p^L = \sum_{l=0}^{m-1} (a_l R_{p,p-l} I_{p-l}) - \sum_{k=1}^{m} (b_k R_{p,p-k} J_{p-k}^L).$$

Fourier Series Decomposition

$$R_{qp} = f\left(\frac{x}{\sigma}\right) = \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos\left(\frac{2n\pi}{\sigma} x\right) + \beta_n \sin\left(\frac{2n\pi}{\sigma} x\right)$$

$$x = I_q - I_p \;\; R_{qp}^c = \cos\left(\frac{I_q - I_p}{\sigma_r}\right), R_{q,p}^s = \sin\left(\frac{I_q - I_p}{\sigma_r}\right)$$

Raised-Cosine Approximation

$$\lim_{M \to \infty} \cos^M\left(\frac{x}{\sigma\sqrt{M}}\right) = \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$R_{qp} = \cos^M\left(\frac{x}{\sigma}\right) = \sum_{m=0}^{M} \binom{M}{m} \exp\left(\mathrm{j}(2m-M)\frac{x}{\sigma\sqrt{M}}\right)$$

$$\approx \exp\left(-\frac{x^2}{2\sigma^2}\right),$$

Filtering output: summation of IIR filtering

[redacted]

Multiple results of IIR filtering is required.

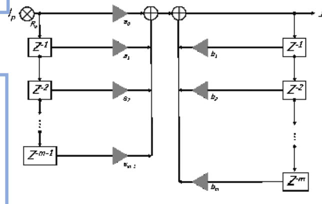
Relationship between IIR BF and constant-time BF

Constant-time BF can be represented in weighted summation of Gaussian filtering.

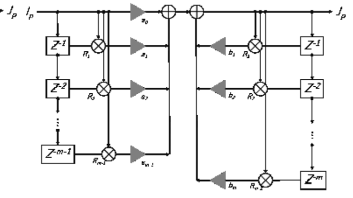
[redacted]

Gaussian filtering can ber approximated with a normal IIR system, FIR filtering or FFT.

Representation of output is similar.



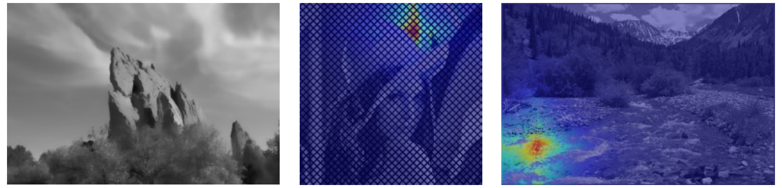
IIR system in conv.



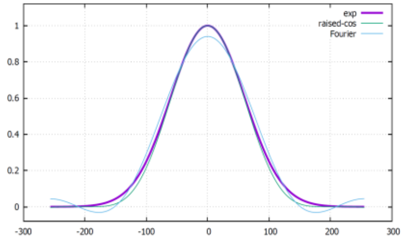
IIR system in prop.

Experimental Results

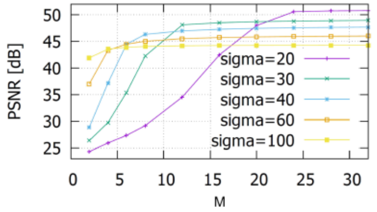
Recursive bilateral filter (output, kernel with mesh, kernel on textured)



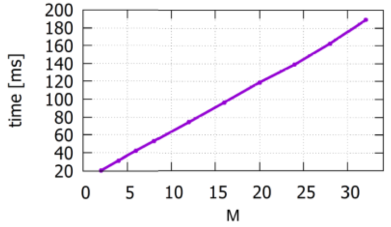
IIR bilateral filter (output, kernel with mesh, kernel on textured)



Comparison: Furrier and Raised-Cosine



Approximation of bilateral filter w.r.t M



Computational time

IIR BF

σs \ σr	10	30	50	70	90
3	39.21	35.41	35.02	35.19	35.53
9	35.95	30.64	30.10	30.61	31.52
15	35.00	29.12	28.46	29.06	30.17
21	34.52	28.25	27.45	28.05	29.23
27	34.22	27.65	26.74	27.30	28.47

Recursive BF

σs \ σr	10	30	50	70	90
3	22.40	39.56	43.38	42.80	42.54
9	19.73	35.28	43.22	42.74	42.71
15	18.89	33.94	42.88	42.46	42.64
21	18.37	33.29	42.65	42.37	42.61
27	17.98	32.86	42.47	42.24	42.43

Approximation of bilateral filter w.r.t sigma range and space

Acknowledgments

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