A GRAPH-BASED APPROACH FOR FEATURE EXTRACTION AND SEGMENTATION OF MULTIMODAL IMAGES

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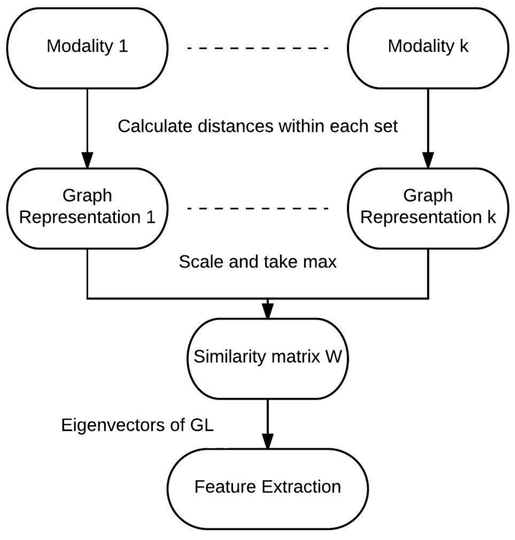
Introduction

With the increasing availability of data we often come upon problems that collect data from more than once source, or *modality*. To properly handle these problems, we need to compare data across the different modalities. The step presents a lot of difficulty, as it requires some understanding of the format of the data [5]. Our goal is to create a general algorithm for processing multiple modalities simultaneously.

In this paper, we assume our datasets are co-registered (each modality contains the same number of points, and they share a common indexing), as is often the case in image processing problems. Our method compares graph representations of each modality, extracts features in the form of eigenvectors of the graph Laplacian, then applies standard data-segmentation algorithms on these features to obtain a final classification.

1 Feature Extraction



Graph Representation

- Datasets $X^1, X^2, \ldots, X^k$, with $\left|X^1\right| = \cdots = \left|X^k\right| = m$.
- E^ℓ = *distance matrix*. $e^\ell_{ij} = \left\|x^\ell_i - x^\ell_j\right\|$.
- $\lambda_\ell = \text{stdev}\left(E^\ell\right)$, scaling factor.
- $X = (X^1, X^2, \ldots, X^k) \subseteq \mathbb{R}^{n\times(dim_1+\cdots+dim_k)}$ the concatenated dataset.
- W = *similarity matrix*. w_{ij} = similarity between x_i and x_j.

$$w_{ij} := \exp\left(-\max\left(e^\ell_{ij}/\lambda_\ell \mid 1 \le \ell \le k\right)\right). \tag{1}$$

With the $\max$ norm, two nodes are considered similar only if they are similar in every individual modality. Heuristically, this emphasizes the unique information that each dataset brings.

The Graph Laplacian (GL) and Eigendecomposition

Once we have created the weights, we define the *normalized graph Laplacian* (GL) [9].

$$L_{sym} = I - D^{-1/2}WD^{-1/2}, \tag{2}$$

Where D is the diagonal matrix consisting of degrees of nodes. We use the graph Laplacian in our energy minimizations below.

We use the GL to solve the *graph N-cut* problem. Given a partition of our into subsets $A_1, A_2, \ldots, A_m$, we define

$$\text{NCut}(A_1, \ldots, A_m) = \frac{1}{2}\sum_{i=1}^{m}\frac{W(A_i, A_i^c)}{vol(A_i)}. \tag{3}$$

Minimizing the NCut separates dissimilar nodes (the $W(A_i, A_i^c)$ term) and groups similar nodes (the $vol(A_i)$ term). Solving the graph min-cut problem is equivalent to finding an $n \times m$ indicator matrix u, where $u_{ij} = 1$ if $x_i \in A_j$, and $u_{ij} = 0$ otherwise. Note that

$$\text{NCut}(A_1, \ldots, A_m) = \text{Tr}\left(u^T L_{sym} u\right). \tag{4}$$

Minimizing this energy is computationally infeasible [4]. We relax the problem, allowing u to be an orthogonal matrix. We find

$$\text{argmin}_{u\in\mathbb{R}^{n\times m}}\text{Tr}\left(u^T L_{sym} u\right) \quad \text{where } u^T u = I. \tag{5}$$

This problem is solved by choosing the columns of u to be the m eigenvectors of L_{sym} corresponding to the m smallest eigenvalues. These eigenvectors are features extracted from the original dataset X.

Example: Data Fusion Contest 2015 [1]

Figure 1: RGB Data

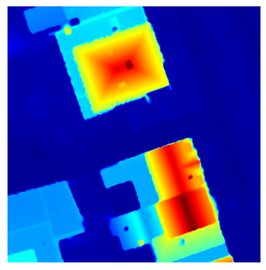


Figure 2: Lidar Data

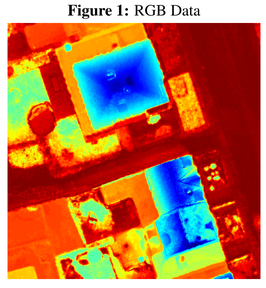


Figure 3: Eigenvector 1

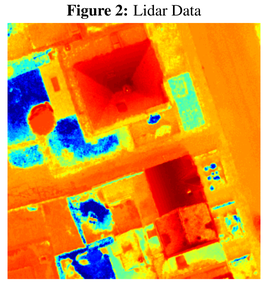


Figure 4: Eigenvector 2

Segmentation

We apply two different segmentation algorithms to these features. The first is *Spectral Clustering*, in which we directly apply k-means to the feature vectors. The second is *Graph MBO*, explained below.

Graph MBO

Here we minimize a Ginzburg-Landau energy with a semisupervised term [3, 7, 8]. Have u an $n \times m$ assignment matrix with

$$u_{ij} \ge 0 \;\forall i, j, \quad \sum_{j=1}^{m} u_{ij} = 1. \tag{6}$$

The final output of the algorithm will be a matrix u where each value is either 0 or 1. The energy we minimize is

$$E(u) = \epsilon \cdot \text{Tr}\left(u^T L_{sym} u\right) + \frac{1}{\epsilon}\sum_i W(u_i) + \sum_i \frac{\mu}{2}\lambda(x_i)\left\|u_i - \hat{u}_i\right\|^2_{L_2}. \tag{7}$$

The first term is the graph cut energy, similar to (3). The second term is the multiwell potential $W(u_i) = \prod_{k=1}^{m}\frac{1}{4}\left\|u_i - e_k\right\|^2_{L_1}$ where e_k is the k-th standard basis vector. The last term includes the fidelity, where $\hat{u}$ represents the semisupervised input,

$$\lambda(x_i) = \begin{cases} 1 & \text{if } x_i \text{ is part of fidelity input} \\ 0 & \text{else} \end{cases}, \tag{8}$$

We minimize this via iteritive diffusion and thresholding. If u^n represents the n-th iterate, then to calculate u^{n+1} we first diffuse

$$\frac{u^{n+\frac{1}{2}} - u^n}{dt} = -L_{sym}u^n - \mu\lambda(x)(u^n - \hat{u}). \tag{9}$$

Then threshold each row

$$u_i^{n+1} = e_r \quad \text{where } r = \text{argmax}_j u_{ij}^{n+\frac{1}{2}}. \tag{10}$$

The diffusion calculation can be done very efficiently by using the eigendecomposition of L_{sym} (the feature vectors described in (5)). If we write change coordinates to the eigenbasis, then the diffusion step reduces to solving for coefficients

$$a_k^{n+1} = (1 - dt\cdot\lambda_k)\cdot a_k^n - dt\cdot d_k^n. \tag{11}$$

where λ_k is the k-th eigenvalue of L_{sym}, in ascending order.

Results: Data Fusion Contest 2015 [1]

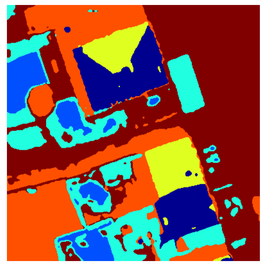


Figure 5: Spectral Clustering

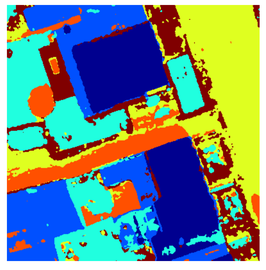


Figure 6: Graph MBO

Results: Umbrella Data [11]

Figure 7: RGB Data

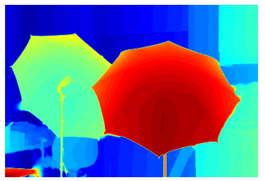


Figure 8: Lidar Data

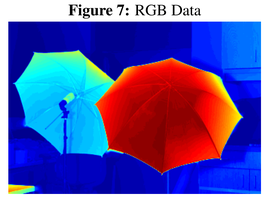


Figure 9: Eigenvector 1

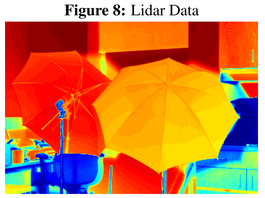


Figure 10: Eigenvector 2

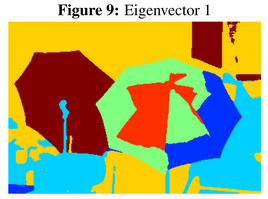


Figure 11: Spectral Clustering

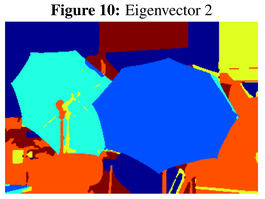


Figure 12: Graph MBO

Results: Jade Plant Data [11]

Figure 13: RGB Data

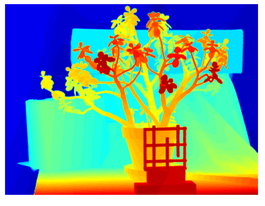


Figure 14: Lidar Data

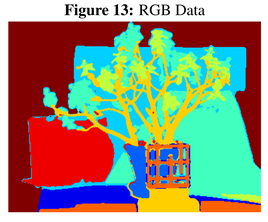


Figure 15: Spectral Clustering

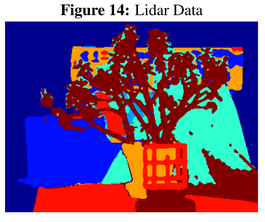


Figure 16: Graph MBO

Acknowledgments

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