MULTIDIMENSIONAL NONSEPARABLE OVERSAMPLED LAPPED TRANSFORMS: THEORY AND DESIGN

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Abstract

Multidimensional (MD) nonseparable oversampled lapped transforms (NSOLTs) are proposed.

- NSOLTs are MD **redundant** filter banks.
- The filters are **nonseparable, symmetric, real, overlapped** and **compact-supported**.
- They are based on **lattice structures** and **tree decomposition** is available.
- The **redundancy** is **flexibly controlled** by ♯ of Chs. P and downsampling ratio M.
- NSOLTs are capable of constructing **Parseval tight frames** in ANY ♯ of dimensions.
- **Example-based design** is available through the **dictionary learning** approach.
- Comparison with separable systems shows the significance in terms of non-linear Approx.

Index Terms - Multidimensional signal processing, oversampled filter banks, lattice structure, tight frame, dictionary learning, iterative hard thresholding.

I. Introduction & II. MD-OLPPRFB

MD Oversampled Linear-Phase Perfect Reconstruction Filter Banks are dealt with. Let $P = \sharp\mathrm{Chs}.$.

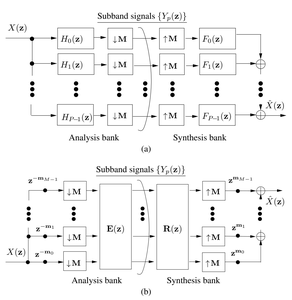


Figure 1: Equiv. Rep. of FBs

$\mathbf{M}$: Downsampl. Mtx
$\bar{\mathbf{n}}$: Polyphase order

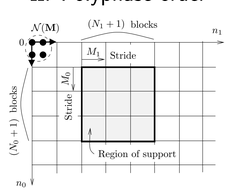


Figure 2: 2-D illustration on meanings of primal Params.

NOTE: Polyphase matrices $\mathbf{E}(\mathbf{z})$ and $\mathbf{R}(\mathbf{z})$ yield a compact matrix representation of convolution (linear) networks.

III. Lattice Structures

Type-I and Type-II lattice sturctures are proposed.

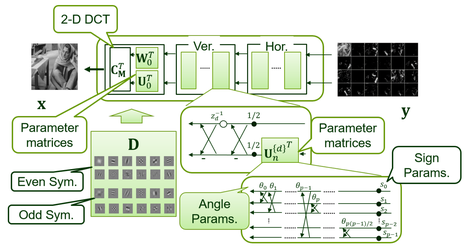


Figure 3: Lattice structure of a Type-I synthesis PU NSOLT. The impulse responses (atoms) are even- or odd-symmetric.

IV. Example-based Design

$$\{\hat{\mathbf{D}}, \hat{\mathbf{y}}\} = \arg\min_{\mathbf{D},\mathbf{y}} \|\mathbf{x}-\mathbf{D}\mathbf{y}\|_2^2, \quad \text{s.t. } \|\mathbf{y}\|_0 \le K$$

❶ Sparse Approximation

$$\hat{\mathbf{y}} = \arg\min_{\mathbf{y}} \|\mathbf{x}-\hat{\mathbf{D}}\mathbf{y}\|_2^2, \quad \text{s.t. } \|\mathbf{y}\|_0 \le K$$

❷ Dictionary Update

$$\hat{\Theta} = \arg\min_{\Theta} \|\mathbf{x}-\mathbf{D}_\Theta\hat{\mathbf{y}}\|_2^2$$

$$\hat{\mathbf{D}} = \mathbf{D}_{\hat{\Theta}}$$

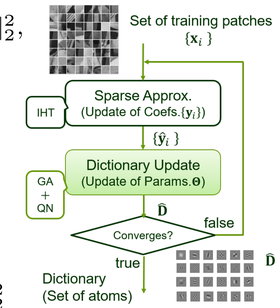
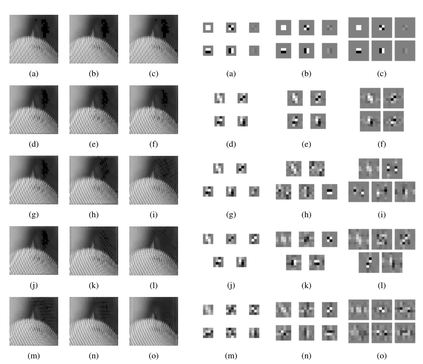


Figure 4: Dictionary learning

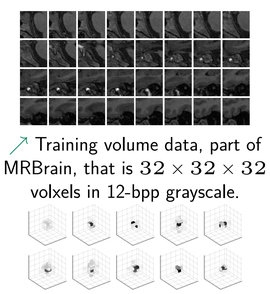
Design Examples

Training image, part of barbara, that is 64×64 pixels in 8-bpp grayscale. ↑

Sparse approximation results with 2-D NSOLT dictionaries through IHT for the training image, where $\mathbf{M} = \mathrm{diag}(M_0, M_1) = \mathrm{diag}(2,2)$; the other construction parameters are summarized in Tab. 1. →



← Impluse responses of synthesis filters designed through SPLOT and NSOLT frameworks for training image, where $\mathbf{M} = \mathrm{diag}(M_0, M_1) = \mathrm{diag}(2,2)$ and the other construction parameters are summarized in Tab. 1.



↗ Training volume data, part of MRBrain, that is $32 \times 32 \times 32$ volxels in 12-bpp grayscale.

↗ Impluse responses of synthesis filters designed through NSOLT framework for the training data.

V. Performance Evaluation

Table 1: PSNRs [dB] of sparse Approx. results with IHT, where "SP" means "Separable." p_s and p_a are ♯s of Sym. and Anti-sym. Chs., respectively.

Type	P		Redundancy	$\bar{\mathbf{n}} = (N_0, N_1)^T$		
	p_s	p_a	$R = P/M$	$(1,1)$	$(2,2)$	$(3,3)$
SP	3	3	3/2	(a) 23.50	(b) 23.50	(c) 23.50
I	2	2	1	(d) 24.71	(e) 24.86	(f) 25.38
II	2	3	5/4	(g) 24.71	(h) 25.27	(i) 28.22
II	3	2	5/4	(j) 24.71	(k) 26.69	(l) 28.89
I	3	3	3/2	(m) 26.90	(n) 29.69	(o) 29.75

VI. Conclusion

- The theory of the 1-D OLPPRFBs in [Gan et al., (2003)] is extended to the MD case.
- Parseval tight frame with non-separable symmetric atomic images can be constructed.
- NSOLTs yield a structured framework of MD convolution (linear) networks.
- Its applications include image/volumetric data restoration and pattern recognition.

Part of the References

[Muramatsu et al., (1999)] "A design method of multidimensional linear-phase paraunitary filter banks with a lattice structure," *IEEE Trans. Signal Process.*, vol. 47, no. 3, pp. 690-700, Mar. 1999.

[Gan et al., (2003)] "Oversampled linear-phase perfect reconstruction filterbanks: Theory lattice structure and parameterization," *IEEE Trans. Signal Process.*, vol. 51, no. 3, pp. 744-759, Mar. 2003.

Acknowledgements

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Related Topics Beyond the Paper

- SaivDr Package - File Exchange - MATLAB Central - MathWorks, `https://mathworks.com/matlabcentral/fileexchange/45084-saivdr-package`
- S. Muramatsu *et al.*, "Image restoration with 2-D non-separable oversampled lapped transforms," *Proc. of IEEE ICIP*, pp.1051-1055, Sep. 2013.
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