COMPLEX NONSEPARABLE OVERSAMPLED LAPPED TRANSFORM FOR SPARSE REPRESENTATION OF MILLIMETER WAVE RADAR IMAGE

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Abstract

This work generalizes an existing framework of nonseparable oversampled lapped transforms (NSOLTs) to effectively represent complex-valued images.

- NSOLT is a redundant transform with linear-phase, compact-supported and perfect-reconstruction property.
- Generalized structure of NSOLTs is proposed to cover complex-valued atomic images.
- Effectiveness of the structure is verified by evaluating the sparse approximation performance.

Key words – Complex dictionary, NSOLT, Parseval tight frame, Millimeter wave radar, Sparse approximation

Introduction

- Sparsity-based image restoration
  - denoising, deblurring, super-resolution, inpainting, compressive sensing, etc.
- NSOLT [1]
  - Effective for image restoration problem
  - Example-based design (dictionary learning)
- Problem
  - Existing NSOLT can only deal with real-valued signal.
  - Insufficient degrees of freedom for complex-valued images.
- Objective
  - Generalize existing real-valued NSOLT ($\mathbb{R}$NSOLT) to complex number field.
  - Evaluate the effectiveness by sparse approximation performance of complex-valued images.

Complex Nonseparable Oversampled Lapped Transform

- Complex-valued redundant transform satisfying linear-phase, compact-supported and perfect-reconstruction property.
  - Type-I $\mathbb{C}$NSOLT includes Type-I $\mathbb{R}$NSOLT as a subset.
- Lattice structure of analysis bank is obtained by a cascade cunstruction as

$$\mathbf{E}(\mathbf{z}) = \mathbf{\Phi}\left(\prod_{k=1}^{K_\mathrm{v}} \mathbf{G}_k^{\{\mathrm{v}\}}(z_\mathrm{v})\right)\left(\prod_{k=1}^{K_\mathrm{h}} \mathbf{G}_k^{\{\mathrm{h}\}}(z_\mathrm{h})\right)\mathbf{E}_0(\mathbf{z}),$$

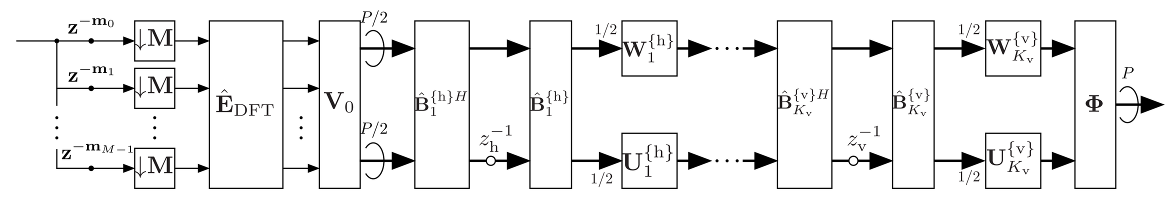


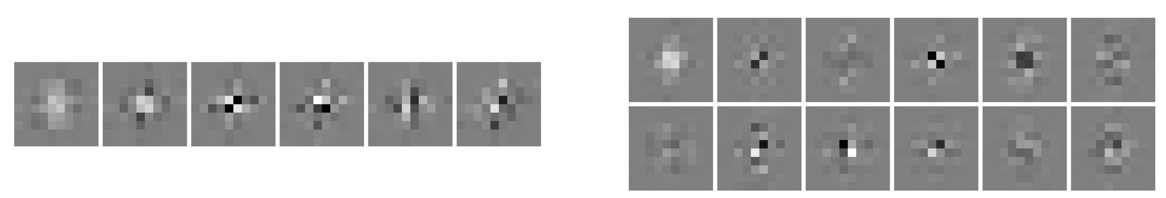
Figure 1:Lattice structure of analysis bank of 2D Type-I $\mathbb{C}$NSOLT

Performance Evaluation

- Design example
  - $\mathbb{R}$NSOLT and $\mathbb{C}$NSOLT

Table 1:Common settings of 2-D Type-I NSOLTs

Downsampling factor $\mathbf{M}$	2×2
Polyphase order $\bar{\mathbf{n}}$	$[2,2]^T$
#Channels P	6
#Tree levels τ	4
Redundancy R	$<\frac{5}{3}$



(a) (b)

Figure 3:Atomic images of (a) $\mathbb{R}$NSOLT, (b) $\mathbb{C}$NSOLT

Sparse Approximation

- Problem settings
  - Find a sparse vector $\hat{\mathbf{y}}$ that mimize:

$$\|\mathbf{x}-\mathbf{D}\hat{\mathbf{y}}\|_2^2 \text{ s.t. } \|\hat{\mathbf{y}}\|_0 \le K, \qquad (1)$$

  - K : Number of nonzero elements
- Iterative hard thresholding (IHT) algorithm
  - Heuristic algorithm for solving (1)

$$\mathbf{y}^{(i)} \leftarrow H_K\left(\mathbf{y}^{(i-1)} + \mathbf{D}^H\left(\mathbf{x}-\mathbf{D}\mathbf{y}^{(i-1)}\right)\right) \qquad (2)$$

  - $H_K(\cdot)$: Hard thresholding operator.

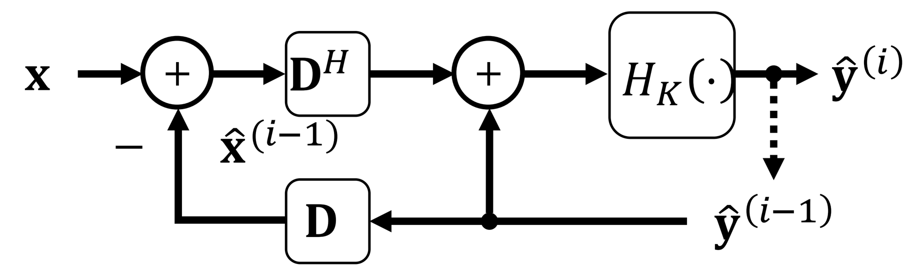


Figure 2:The block diagram of IHT.

Results

- Sparse approximate simulation
  - Orginal image $\mathbf{x}$: millimeter wave radar complex image in Fig. 4.
  - The intensity and hue show the magnitude and phase characteristics, respectively.

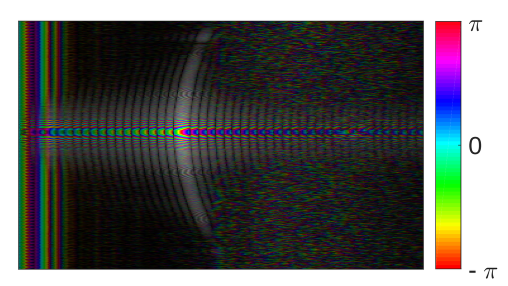
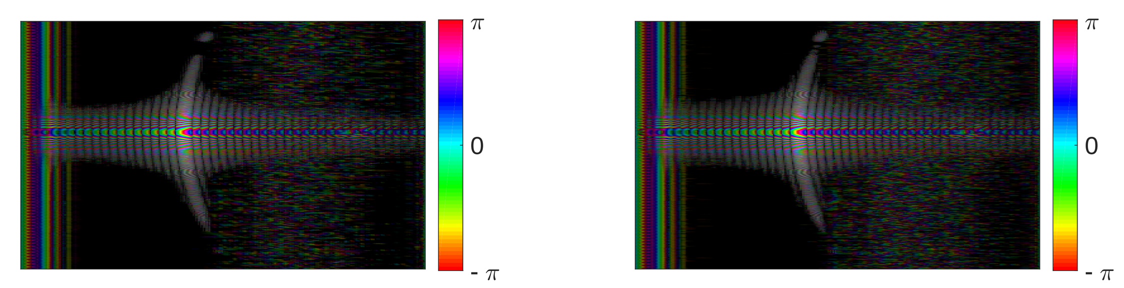


Figure 4:Original image of size $2,992\times 320$[pixels],



(a) (b)

Figure 5:Sparse approximation results width 2-D NSOLT dictionaries through IHT.

(a) $\mathbb{R}$NSOLT: MSE $= 2.36\times 10^{-5}$

(b) $\mathbb{C}$NSOLT: MSE $= 1.59\times 10^{-5}$

Conclusion

- Proposed the lattice structure of 2-D $\mathbb{C}$NSOLT
- Employed $\mathbb{C}$NSOLT as a sparse modeling method for complex-valued images
- Effectiveness of the proposed method for millimeter wave radar images was examined by sparse approximation performance.

References

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