Class-specific Poisson Image Denoising using Importance Sampling

Milad Niknejad, José M. Bioucas-Dias, Mário A. T. Figueiredo

Instituto de Telecomunicações, Instituto Superior Técnico,
University of Lisbon, Portugal

SpaRTaN
Sparse Representations and Compressed
Sensing Training Network

Overview

Class-specific image denoising

Often, the image to be denoised belongs to a known specific class,

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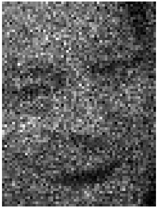

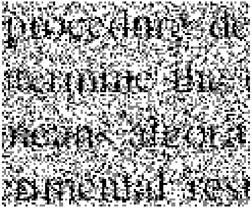

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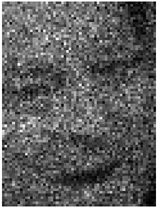
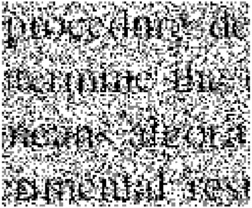

This knowledge should be exploited by the denoising method!

Class-specific image denoising

Assumption: A dataset of clean images of the same class is available.

Patch-based image denoising

Gaussian noise observation model:

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i$$

$\mathbf{x}_i$ is a patch of the original image; $\mathbf{y}_i$ is the corresponding noisy patch; $\mathbf{v}_i$ is i.i.d. Gaussian noise.

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Poisson noise observation model (the focus of this presentation):

$$\mathbf{y}_{i,j} \sim \mathcal{P}(\mathbf{x}_{i,j}).$$

$\mathbf{x}_{i,j}$ is the j^{th} pixel of $\mathbf{x}_i$.
$\mathcal{P}$ is a Poisson distribution with mean $\mathbf{x}_{i,j}$.

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Goal: recover the clean patch $\mathbf{x}_i$ from the noisy one $\mathbf{y}_i$

MMSE (*minimum mean squared error*) estimation

MMSE patch estimate ($p(\mathbf{y} = \mathbf{y}_i)$ is replaced by $p(\mathbf{y}_i)$):

$$\hat{\mathbf{x}}_i = \mathbb{E}[\mathbf{x}|\mathbf{y}_i] = \int_{\mathbb{R}^p} \mathbf{x}\; p(\mathbf{x}|\mathbf{y}_i)\; d\mathbf{x} = \int_{\mathbb{R}^p} \mathbf{x}\; \frac{p(\mathbf{y}_i|\mathbf{x})\; p(\mathbf{x})}{p(\mathbf{y}_i)}\; d\mathbf{x}$$

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- However, sampling from $p(\mathbf{x}|\mathbf{y}_i)$ is also intractable.
- Can we approximate $\hat{\mathbf{x}}_i$ by sampling from another distribution?

Self-Normalized Importance Sampling (SNIS)

Goal: to compute (or approximate)

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- As in plain Monte-Carlo: $\lim_{n\to\infty} \hat{\mathbb{E}}_n[f(\mathbf{z})] = \mathbb{E}[f(\mathbf{z})]$

Applying SNIS for MMSE patch estimation

Back to our problem:

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- For Poisson noise, the weights are easy to obtain ($\mathbf{y}_{i,j} \sim \mathcal{P}(\mathbf{x}_{i,j})$, i.i.d.)

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- It can be generalized to other noise models.

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1. Cluster the patches in the external dataset.
2. Assign each noisy patch to the closest cluster.
3. Use the corresponding clean patches as samples from the proposal distribution for SNIS.

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- The above integral is intractable, but we can use SNIS.

Proposed method

$$\mathbb{E}[\|\mathbf{x}-\mathbf{u}\|_2^2|\mathbf{y}_i,k] = \int_{\mathbb{R}_+^m} \|\mathbf{u}-\mathbf{x}\|_2^2 \; p(\mathbf{x}|\mathbf{y}_i,k) \; d\mathbf{x}.$$

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Using SNIS, the above can be approximated by

$$\hat{\mathbb{E}}_n[\|\mathbf{x}-\mathbf{u}\|_2^2|\mathbf{y}_i,k] = \frac{\sum_{j_k=1}^{n} \|\mathbf{u}-\mathbf{x}_{j_k}\|_2^2 \; w_{j_k}}{\sum_{j=1}^{n} w_{j_k}} \tag{1}$$

where the $\mathbf{x}_{j_k}$, for $j_k = 1, ..., n$ are samples from the distribution $p(\mathbf{x}|k)$.
$w_{j_k} = p(\mathbf{y}_i|\mathbf{x}_{j_k})$

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We Minimize by alternating minimization

- when $\mathbf{u} = \hat{\mathbf{x}}_i$ is fixed,

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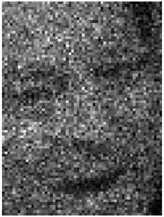
- Denoising: samples derived for each patch n_1 was set to 300.
- Overall: 900 patches are processed for each denoised patch (computational complexity is similar to an internal non-local denoising with the patches constrained in 30×30 window).

Experiment 1


Noisy image (Peak=10)

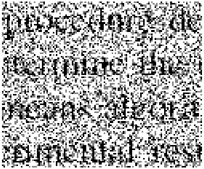

Non-local PCA (PSNR=22.60)


VST+BM3D (PSNR=24.79)

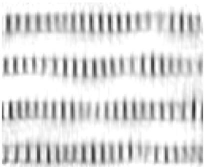

Poisson NL means (PSNR=24.55)


Proposed (PSNR=26.40)

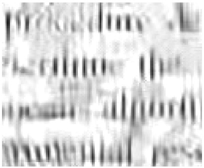
Experiment 2



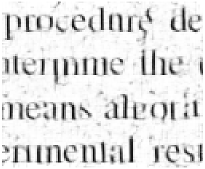
Noisy
(Peak=2)



Non-local PCA
(PSNR=14.95)



VST+BM3D
(PSNR=14.55)



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(PSNR=18.64)

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- The method can be generalized easily to other image restoration inverse problems.

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Thanks for your attention