A NOVEL VARIATIONAL MODEL FOR RETINEX IN PRESENCE OF SEVERE NOISES

Lu Liu, Zhi-Feng Pang, Yuping Duan*

Background

Context: Retinex theory deals with compensation for illumination effects in images, which is usually an ill-posed problem. To set up the Retinex problem mathematically, we focus on decomposing a given image I into the reflection component R and the illumination component L, which satisfies

$$I(x,y) = R(x,y) \cdot L(x,y)$$

Problem: In real applications, intensity inhomogeneities and noises may simultaneously exist in given images. Although various algorithms have gained great success in dealing with Retinex problems. These methods paid little attention to the noises contained in the given images.

Our approach: To present a general variational Retinex model to effectively and robustly restore images corrupted by both noises and intensity inhomogeneities.

The Proposed Model

Image model: By applying the logarithmic transformation:

$$i(x,y) = r(x,y) + l(x,y)$$

- r is a function→ **reflection**
- l is a spatially smooth function→ **illumination**

➢ **Energy formulation:** $$\min_{u,r,l} \frac{1}{2}\|I-u\|_2^2 + \alpha\|\nabla u\|_1 + \beta\|\nabla r\|_1 + \gamma\|\nabla^2 l\|_1,$$
$$\text{s.t., } u = e^v,\ v = r + l.$$

➢ **The augmented Lagrangian functional:**

$$\mathcal{L}(u,v,\boldsymbol{x},\boldsymbol{y};\Lambda_1,\Lambda_2,\boldsymbol{\Lambda}_3) = G(u) + H(\boldsymbol{y}) + \langle \Lambda_1, u - e^v\rangle + \frac{\nu_1}{2}\|u-e^v\|_2^2 + \langle \Lambda_2, v - A\boldsymbol{x}\rangle + \frac{\nu_2}{2}\|v - A\boldsymbol{x}\|_2^2 + \langle \boldsymbol{\Lambda}_3, \boldsymbol{y} - L\boldsymbol{x}\rangle + \frac{\nu_3}{2}\|\boldsymbol{y} - L\boldsymbol{x}\|_2^2$$

Algorithm

Algorithm 1 The Proposed Model

Input: image I, parameters $\alpha, \beta, \gamma, \nu_i$ $(i = 1,2,3)$

Initialization: $v^0 \leftarrow \log I, \boldsymbol{x}^0 \leftarrow 0, \Lambda_1^0 \leftarrow 0, \Lambda_2^0 \leftarrow 0, \boldsymbol{\Lambda}_3^0 \leftarrow 0, k \leftarrow 0$

repeat

– With v^k and Λ_1^k, solve for u^{k+1} from
$$\min_u G(u) + \langle \Lambda_1, u\rangle + \frac{\nu_1}{2}\|u - e^v\|_2^2.$$

– With $\boldsymbol{x}^k$ and $\boldsymbol{\Lambda}_3^k$, solve for $\boldsymbol{y}^{k+1}$ from
$$\min_{\boldsymbol{y}} H(\boldsymbol{y}) + \langle \boldsymbol{\Lambda}_3, \boldsymbol{y}\rangle + \frac{\nu_3}{2}\|\boldsymbol{y} - L\boldsymbol{x}\|_2^2,$$

– With $v^k, \boldsymbol{y}^{k+1}, \Lambda_2^k$ and $\boldsymbol{\Lambda}_3^k$, solve for $\boldsymbol{x}^{k+1}$ from
$$\min_{\boldsymbol{x}} \langle \Lambda_2, -A\boldsymbol{x}\rangle + \frac{\nu_2}{2}\|v - A\boldsymbol{x}\|_2^2 + \langle \boldsymbol{\Lambda}_3, -L\boldsymbol{x}\rangle + \frac{\nu_3}{2}\|\boldsymbol{y} - L\boldsymbol{x}\|_2^2,$$

– With $u^{k+1}, \boldsymbol{x}^{k+1}, \Lambda_1^k$ and Λ_2^k, solve for v^{k+1} from
$$\min_v \langle \Lambda_1, -e^v\rangle + \frac{\nu_1}{2}\|u - e^v\|_2^2 + \langle \Lambda_2, v\rangle + \frac{\nu_2}{2}\|v - A\boldsymbol{x}\|_2^2,$$

– Update Lagrangian multipliers
$$\Lambda_1^{k+1} = \Lambda_1^k + \nu_1(u^k - e^{v^k});$$
$$\Lambda_2^{k+1} = \Lambda_2^k + \nu_2(v^k - A\boldsymbol{x}^k);$$
$$\boldsymbol{\Lambda}_3^{k+1} = \boldsymbol{\Lambda}_3^k + \nu_3(\boldsymbol{y}^k - L\boldsymbol{x}^k);$$

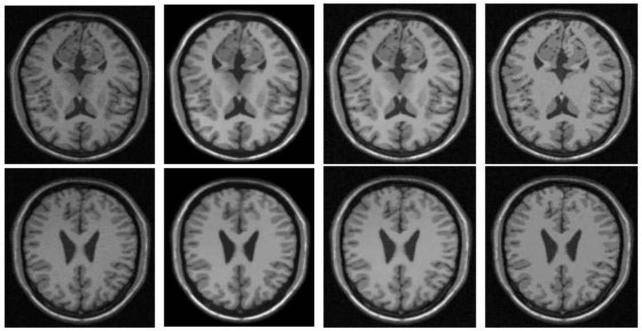
– $k \leftarrow k + 1.$

until k = 5e3

Output: r and l

Experiments

For different noises, the images denoising effect are better than other variational models

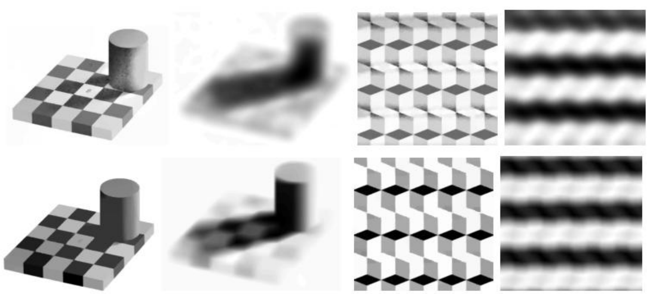


(a) Input (b) Our Model (c) HoTVL1 (d) L0MS

Table 1. PSNR and MSE of T1 brain images.

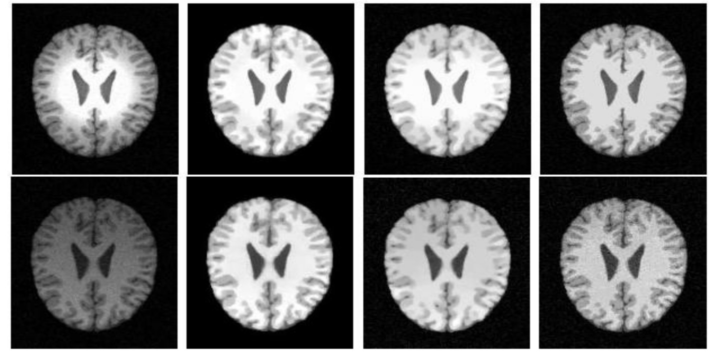
		Our model	HoTVL1	L0MS
Test image 1	PSNR	30.0261	27.5423	27.4783
	MSE	0.0010	0.0018	0.0018
Test image 2	PSNR	30.7099	28.8196	28.5584
	MSE	0.0008	0.0013	0.0014

For checkerboard illusion images, the effect of images decomposition and denoising are better than HoTVL1 model

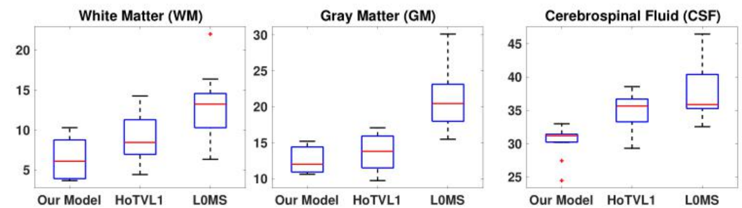


(a) r (b) l (c) r (d) l

For different intensity inhomogeneity, the correction results are better than other models



(a) Input (b) Our Model (c) HoTVL1 (d) L0MS



For impulsive noise, the results of image decomposition and denoising is good

(a) Input image (b) r by our model (c) l by our model

Conclusion

Our model is efficient, accurate and robust for Retinex problem, which is developed for images corrupted by both intensity inhomogeneities and noises. As one important application of our method is for medical image processing.

The center for Applied Mathematics recruits post doctoral, doctoral and postgraduate students in the direction of medical image processing, welcome students to consult, contact information: yuping.duan@tju.edu.cn.