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Learning Autoencoders with Low-Rank Weights

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INTRODUCTION

Autoencoder (AE) is a self supervised network, i.e. the output = input.

AE maps the input x to the latent space h by an encoder. This is given as:

$$h = \phi(Wx)$$

The Decoder reverse maps the latent space to the output = input.

$$x = W'h = W'\phi(Wx)$$

During training the problem is to learn the encoding and decoding weights .

This is done by minimizing the Euclidean cost.

$$\arg\min_{W,W'} \left\| X - W'\phi(WX) \right\|_F^2 \quad \text{here } X = [x_1 | \ldots | x_N]$$

Stacked autoencoders are formed by nesting one inside the other. Mathematically this is given by:

$$\arg\min_{W_1 \ldots W_L, W'_1 \ldots W'_L} \left\| X - g\left(f(X)\right) \right\|_F^2$$

$$\text{where } g = W_1'\phi\left(W_2'\ldots\phi(W_L'(f(X)))\right) \text{ and } f = \phi\left(W_L\phi\left(W_{L-1}\ldots\phi(W_1X)\right)\right)$$

Regularized Autoencoders

Generally expressed as:

$$\arg\min_{W,W'} \left\| X - W'\phi(WX) \right\|_F^2 + R(W, X)$$

- Regularizer can be a sparsity promoting feature
  - KL divergence
  - L0-norm
  - L1-norm
- Regularizer can be supervised, with class-sparsity promoting term like L2,1-norm
- It can be a contractive term like Euclidean norm of the Jacobian
- It can be sparsity on the weights

PROPOSED APPROACH

Prior on sparse weighted autoencoder is akin to Optimal Brain Damage proposed by LeCun.

Basically simulates killing all the neurons that are redundant.

In reality, human brain is a redundant system.

We propose to model this redundancy explicitly.

We model redundancy as linear dependency of the representation layer nodes. This leads to the fact that the encoder and decoder weights are linearly dependent. In other words, the encoder and decoder are rank deficient.

Mathematically Modeled as:

Non-smooth optimization; cannot be solved using backpropagation.

$$\arg\min_{W,W'} \left\| X - W'\phi(WX) \right\|_F^2 + \lambda\left(\left\|W'\right\|_{NN} + \left\|W\right\|_{NN}\right)$$

Majorization Minimization is used for solving the problem.

Update for Decoder

$$W'_k \leftarrow \arg\min_{W'} \left\| X - W'\phi(W_{k-1}X) \right\|_F^2 + \lambda\left\|W'\right\|_{NN}$$

Standard Nuclear Norm Minimization

Update for Encoder (Via MM)

$$B = \phi(W_iX) + \frac{1}{a}W'^T_k\left(X - W'_k\phi(W_iX)\right)$$

Leads to encoder update

$$W_k \leftarrow \arg\min_{W} \left\| B - \phi(WX) \right\|_F^2 + \lambda\left\|W\right\|_{NN}$$

$$\equiv \arg\min_{W} \left\| \phi^{-1}(B) - WX \right\|_F^2 + \lambda\left\|W\right\|_{NN}$$

Activation function is unitary.
Hence trivial to invert.

Our Algorithm

RESULTS

CLASSIFICATION EXPERIMENTS

Comparison on KNN

Dataset	SDAE	DropOut	DropConnect	KAE	CAE	DBN	SparseConnect	Proposed
MNIST	97.33	97.36	97.36	96.90	92.83	97.05	95.91	97.12
basic	95.25	94.94	95.02	91.64	90.92	95.37	92.49	95.37
basic-rot	84.83	84.53	84.31	80.24	78.56	84.71	81.01	84.83
bg-rand	86.42	85.99	85.87	85.89	85.61	86.36	84.87	86.77
bg-img	77.16	76.46	77.31	76.84	78.51	77.16	79.84	77.39
bg-img-rot	52.21	52.06	52.40	50.27	47.10	50.47	48.91	52.40
USPS	94.91	95.32	94.62	93.85	91.96	94.36	94.62	95.42
Bangla	95.22	94.97	95.77	93.27	92.49	94.08	95.77	96.20
Devanagari	88.79	87.18	90.10	87.26	87.16	88.23	90.10	89.68

Comparison on SRC

Dataset	SDAE	DropOut	DropConnect	KAE	CAE	DBN	SparseConnect	Proposed
MNIST	98.33	98.33	98.29	97.91	87.19	88.43	97.16	98.20
basic	96.91	96.93	96.97	95.07	95.03	87.49	95.43	96.97
basic-rot	90.04	89.97	90.16	88.85	88.63	79.47	87.76	90.23
bg-rand	91.03	91.00	91.09	83.59	82.25	79.67	86.17	91.61
bg-img	84.14	85.19	85.77	84.12	85.68	75.09	85.84	84.67
bg-img-rot	62.46	60.09	62.73	58.06	54.01	49.68	57.75	63.27
USPS	95.49	95.57	96.06	94.19	94.07	85.73	96.01	96.11
Bangla	92.11	96.51	93.70	93.57	93.20	95.11	96.78	97.24
Devanagari	90.06	93.00	91.05	91.19	91.69	59.50	93.16	93.05

Comparison on SVM

Dataset	SDAE	DropOut	DropConnect	KAE	CAE	DBN	SparseConnect	Proposed
MNIST	98.50	98.39	98.51	98.46	97.74	98.53	97.97	98.51
basic	96.96	96.97	96.96	97.02	96.61	97.07	96.23	97.13
basic-rot	89.43	89.16	89.31	88.75	72.54	89.05	89.21	89.55
bg-rand	91.28	91.17	91.52	90.07	85.20	89.59	90.70	91.52
bg-img	84.86	85.03	85.12	80.17	78.76	85.46	84.52	85.37
bg-img-rot	60.53	60.70	60.98	60.01	60.97	58.25	61.76	61.76
USPS	96.61	95.41	95.07	93.59	92.87	94.91	95.62	97.58
Bangla	95.09	95.43	93.33	92.06	92.69	94.51	96.81	93.50
Devanagari	94.11	89.83	88.98	90.52	90.99	93.64	95.18	96.07

DENOISING EXPERIMENTS

Original PSNR and SSIM	DAE	Sparse DAE	SparseConnect	BM 3D	Proposed
Variance=0.01 PSNR=19.97 SSIM=0.57	PSNR=21.96 SSIM=0.63	PSNR=22.94 SSIM=0.68	PSNR=23.90 SSIM=0.72	PSNR=33.90 SSIM=0.94	PSNR=26.41 SSIM=0.78
Variance=0.05 PSNR= 14.02 SSIM=0.33	PSNR=21.67 SSIM=0.62	PSNR=22.64 SSIM=0.67	PSNR=23.53 SSIM=0.70	PSNR=31.53 SSIM=0.86	PSNR=26.26 SSIM=0.73
Variance=0.1 PSNR= 10.49 SSIM=0.20	PSNR=21.31 SSIM=0.60	PSNR=22.25 SSIM=0.65	PSNR=23.01 SSIM=0.67	PSNR=27.53 SSIM=0.80	PSNR=25.86 SSIM=0.70

CONCLUSIONS

- Instead of deleting connections, modelling redundancy closely follows nature and in turn improves autoencoders.
- The optimization problem can be efficiently solved using Majorization Minimization. Solutions are orders of magnitude faster than backpropagation based techniques.
- The technique is generic and can be applied to both classification problems as well as for solving inverse problems like denoising.
- In future, we will incorporate this for supervised autoencoding tasks for example label consistent autoencoder and class-sparse autoencoder.

LITERATURE Reference

1 Y. LeCun, "Optimal Brain Damage", NIPS, 1990

2 M. Thom and G. Palm, "Sparse Activity and Sparse Connectivity in Supervised Learning", Journal of Machine Learning Research, Vol. 14, pp. 1091-1143, 2013.

3 K. Gupta and A. Majumdar, "Sparsely Connected Autoencoder", IEEE IJCNN, 2016.

4 N. Srivastava, G. Hinton, A. Krizhevsky, I. Sutskever and R. Salakhutdinov, "Dropout: A simple way to prevent neural networks from overfitting", Journal of Machine Learning Research, Vol. 15 (1), pp. 1929-1958, 2014.

5 L. Wan, M. Zeiler, S. Zhang, Y. LeCun and R. Fergus, Regularization of neural networks using dropconnect. ACM ICML, 2013.

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