SINGLE DEPTH IMAGE SUPER-RESOLUTION AND DENOISING BASED ON SPARSE GRAPHS VIA STRUCTURE TENSOR

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Abstract

The existing single depth image super-resolution (SR) methods suppose that the image to be interpolated is noise free. However, the supposition is invalid in practice because noise will be inevitably introduced in the depth image acquisition process. In this paper, we address the problem of image denoising and SR jointly based on designing sparse graphs that are useful for describing the geometric structures of data domains. In our method, we first cluster similar patches in a noisy depth image and compute an average patch. Different from the majority of the graph Fourier transform (GFT) that assumed an underlying 4-connected graph structure with vertical and horizontal edges only, we select more general sparse graph structures and edges weights based on the difference of the blocks' structure tensors. For the average patch, a graph template with edges orthogonal to the principal gradient is designed. Finally, the graph based transform (GBT) dictionary is learned from the derived correlation graph for signal representation.

PRELIMINARIES

Fig. 1. The depth map of Books and its color counterpart

1. Graph Fourier Transform

In this section, we briefly overview a few basic definitions for signal on graph. We define a weighted and undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ consists of a finite set of vertices $\mathcal{V}$ with cardinality $|\mathcal{V}| = N$, a set of edges connecting vertices, and a weighted adjacency matrix W. The unnormalized combinatorial graph Laplacian operator is defined as $L = D - W$, where D is the diagonal degree matrix whose ith diagonal element is equal to the sum of the weights of all the edges incident to vertex i.

2. Computing Optimal Graph

In our experiment, we also utilize the design methodology to learn more general sparse graphs. In order to achieve the goal, the first step is to identify blocks with dominant principal gradients. We accomplish this by examining the two eigenvalues of the computed structure tensor matrix:

$$\begin{bmatrix} \sum_{\mathbf{r}} w(\mathbf{r})(I_x(\mathbf{p}-\mathbf{r}))^2 & \sum_{\mathbf{r}} w(\mathbf{r}) I_x(\mathbf{p}-\mathbf{r}) I_y(\mathbf{p}-\mathbf{r}) \\ \sum_{\mathbf{r}} w(\mathbf{r}) I_x(\mathbf{p}-\mathbf{r}) I_y(\mathbf{p}-\mathbf{r}) & \sum_{\mathbf{r}} w(\mathbf{r})(I_y(\mathbf{p}-\mathbf{r}))^2 \end{bmatrix}$$

By performing eigen-decomposition on the $2D$ structure tensor $S_w(p)$, we can obtain eigenvalues λ_1 and λ_2, where $\lambda_1 \geq \lambda_2 \geq 0$ and the corresponding eigenvectors v_1 and v_2 that describe the gradient $\nabla = (I_x, I_y)$ of the patch. v_1 corresponding to the larger λ_1 is the principal gradient.

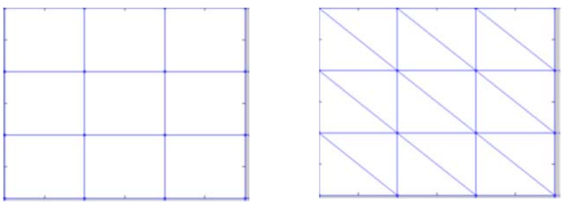

Fig. 2. Examples of the graph template

Proposed method

After discussing the construction of a graph, we formulate the depth map joint denoising / SR problem.

1. **GBT Dictionary Learning**

For a given patch, we first search for its K-nearest-neighbors (kNN) and compute an average patch, from which we produce a graph describing correlations among adjacent pixels. For the average patch, we use the structure tensor matrix to judge that if the patch has dominant principal gradients. The graph Laplacian matrix $L = D - W$. We denote U as the GBT dictionary and it is a matrix with eigenvectors of L as rows.

2. **Image Reconstruction**

$$\min_{U,\alpha} \|y - U\alpha\|_2 + \tau\|\alpha\|_0$$

In our experiment, we group a set of the similar patches together and optimize the joint sparity of the group using the same GBT dictionary U :

$$\min \sum_{i=1}^{N} \|\boldsymbol{y}_i - \boldsymbol{U}\boldsymbol{\alpha}_i\|_2 + \boldsymbol{\tau} \min \sum_{i=1}^{N} \|\boldsymbol{\alpha}_1\|_0$$

After calculating the GBT dictionaty U and the sparse coefficient for the average patch of the noisy and LR depth image, we can reconstruct the noiseless and HR patch by the following equation: $x = U\alpha$

Finally, all the patches are reconstructed and they are noiseless and HR patches.

EXPERIMENTAL RESULTS

In order to have a visual comparison and demonstrate the effectiveness of our proposed method, we show the results in Figs. 4 with different test images with two standard deviation.

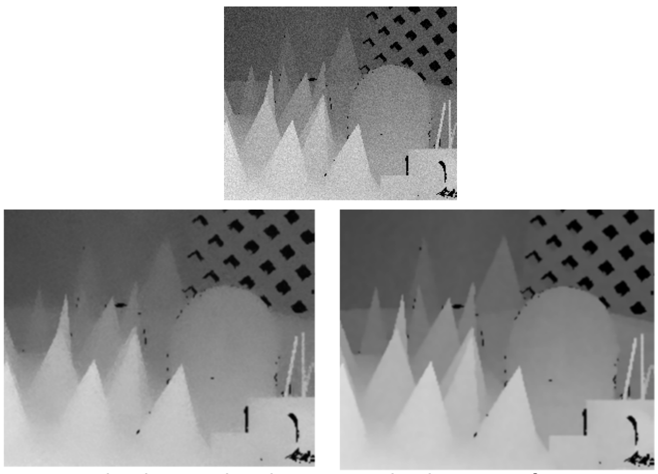

Fig. 3. The denoised and super-resolved images of Cones

CONCLUSION

In this paper, we propose a novel method for single depth image denoising / SR based on more general sparse graph structures that capture principal gradients in blocks. Unlike the conventional schemes that perform denoising first and interpolation later, the proposed method jointly solve the denoising / SR problem. More specially, different from the majority of the GFTs that assumed an underlying 4-connected graph structure, we select more general sparse graph based on the structure tensors. After designing the sparse graph, we can obtain the GBT dictionary for the single noisy and LR depth image reconstruction.