DESIGN OF BINARY LDPC CODES FOR SLEPIAN-WOLF CODING OF CORRELATED INFORMATION SOURCES

by

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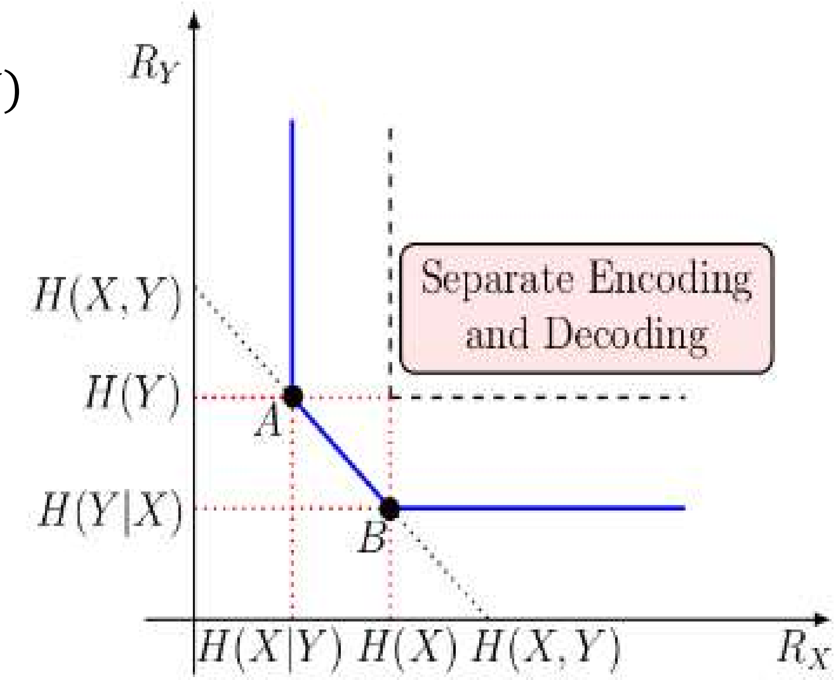
Introduction to Distributed Source Coding

- Distributed Source Coding refers to the compression of multiple statistically dependent sources that do not communicate with each other and therefore are encoded in a distributed manner
- Slepian and Wolf, in 1973 laid the foundation of distributed source coding when they proved the counter-intuitive result that separate encoding with joint decoding achieves the same compression rate as joint encoding does
- Low density parity check (LDPC) codes first introduced by Gallager, in the early 1960s are favoured over other code types because they have remarkably good performance when decoded using low complexity iterative decoding schemes.

Forms of Distributed Source Coding

- Lossless Distributed Source Coding or Slepian-Wolf Coding(SWC)
  - Rate bound given by Slepian-Wolf Theorem
  - The optimal rate region of two DMS $(X, Y) \sim p(x, y)$ is the set of rate pairs (R_X, R_Y) that:
    - $R_X \geq H(X|Y)$
    - $R_Y \geq H(Y|X)$
    - $R_X + R_Y \geq H(X, Y)$

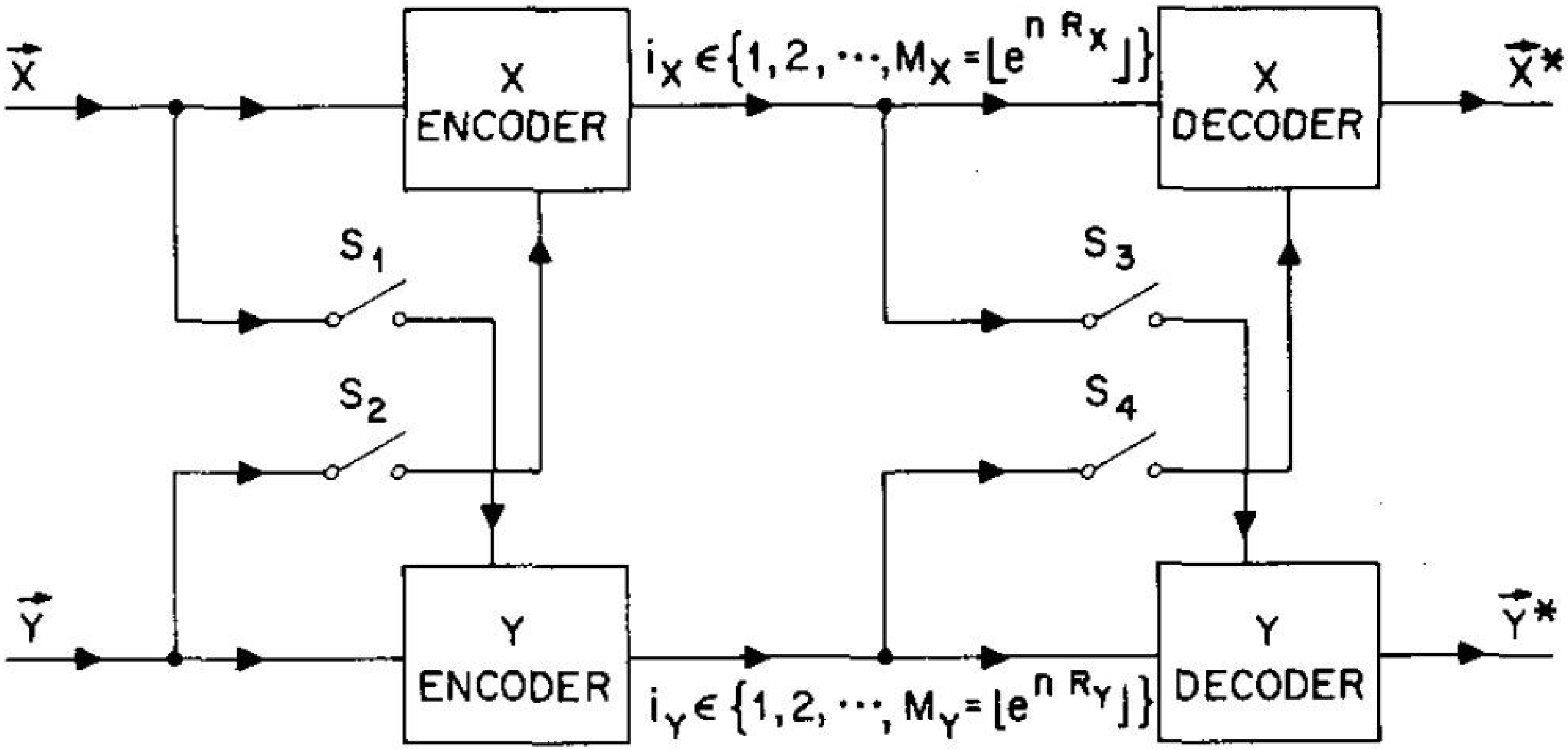


Forms of Distributed Source Coding

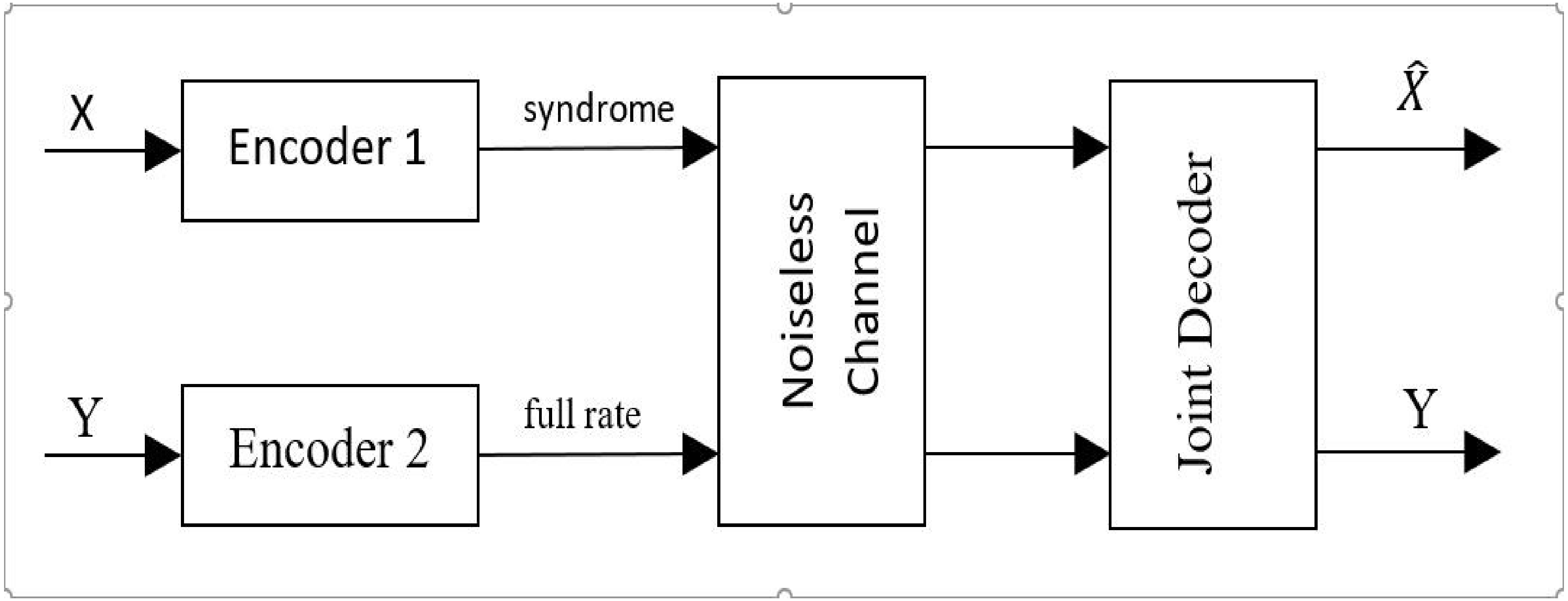
- Lossy Distributed Source Coding or Berger-Tung Coding(BTC)
  - Tight Rate bound still remain unknown
  - Wyner-Ziv Coding is a degraded form of BTC with decoder side information
  - Distributed Video Coding is an application of Wyner-Ziv Coding

Sixteen Cases of Correlated Source Coding

Most Interesting is (0011)



Asymmetric Slepian-Wolf Coding



$$Y = X + N$$

N is noise

Modification of Gallager's Equations

Even Parity-Check Code, $c_0 c_1 c_2 \quad i = 0,1,2$

- Considering the *k = 2, n = 3* even parity-check code
- Received probability is given by

$$p_i^c = P(c_i = c \mid r_i) = ae^{-\frac{(r_i - v^c)^2}{2\sigma^2}} \qquad (1)$$

$c_0 c_1 c_2$	$P(c \mid r)$
000	$p_0^0 p_1^0 p_2^0$
011	$p_0^0 p_1^1 p_2^1$
101	$p_0^1 p_1^0 p_2^1$
110	$p_0^1 p_1^1 p_2^0$

Modification of Gallager's Equations

Even Parity-Check Code, $c_0c_1c_2 \quad i = 0,1,2$

$c_0c_1c_2$	$P(c\|r)$
000	$p_0^0 p_1^0 p_2^0$
011	$p_0^0 p_1^1 p_2^1$
101	$p_0^1 p_1^0 p_2^1$
110	$p_0^1 p_1^1 p_2^0$

- The output probability of c_1 being 1 is:

$$p_1^{1o} = p_0^0 p_1^1 p_2^1 + p_0^1 p_1^1 p_2^0 = p_1^1 (p_0^0 p_2^1 + p_0^1 p_2^0) \qquad (2)$$

- The extrinsic information for bit i to be 1, p_i^{1e} is the sum of all products of the other bit probabilities for which the sum of the bits is odd

- The extrinsic information for bit i to be 0, p_i^{0e} is the sum of all products of the other bit probabilities for which the sum of the bits is even

Modification of Gallager's Equations

Even Parity-Check Code, $c_0 c_1 c_2 \quad i = 0,1,2$

- Solving for bit position $i = 1$

$$(p_0^0 + p_0^1)(p_2^0 + p_2^1) = 1 = p_0^0 p_2^0 + p_0^0 p_2^1 + p_0^1 p_2^0 + p_0^1 p_2^1 \qquad (3)$$

$$(p_0^0 - p_0^1)(p_2^0 - p_2^1) = (1 - 2p_0^1)(1 - 2p_2^1) = p_0^0 p_2^0 - p_0^0 p_2^1 - p_0^1 p_2^0 + p_0^1 p_2^1 \qquad (4)$$

- Adding (4) to (3) gives twice the sum of all even terms $2p_1^{0e}$ while subtracting gives the sum of all odd terms $2p_1^{1e}$

$$p_1^{0e} = \frac{1 + (1 - 2p_0^1)(1 - 2p_2^1)}{2} = p_0^0 p_2^0 + p_0^1 p_2^1 \qquad (5)$$

$$p_1^{1e} = \frac{1 - (1 - 2p_0^1)(1 - 2p_2^1)}{2} = p_0^1 p_2^0 + p_0^1 p_2^0 \qquad (6)$$

Modification of Gallager's Equations

Even Parity-Check Code, $c_0 c_1 c_2 \quad i = 0,1,2$

- Solving for bit position $i = 2$

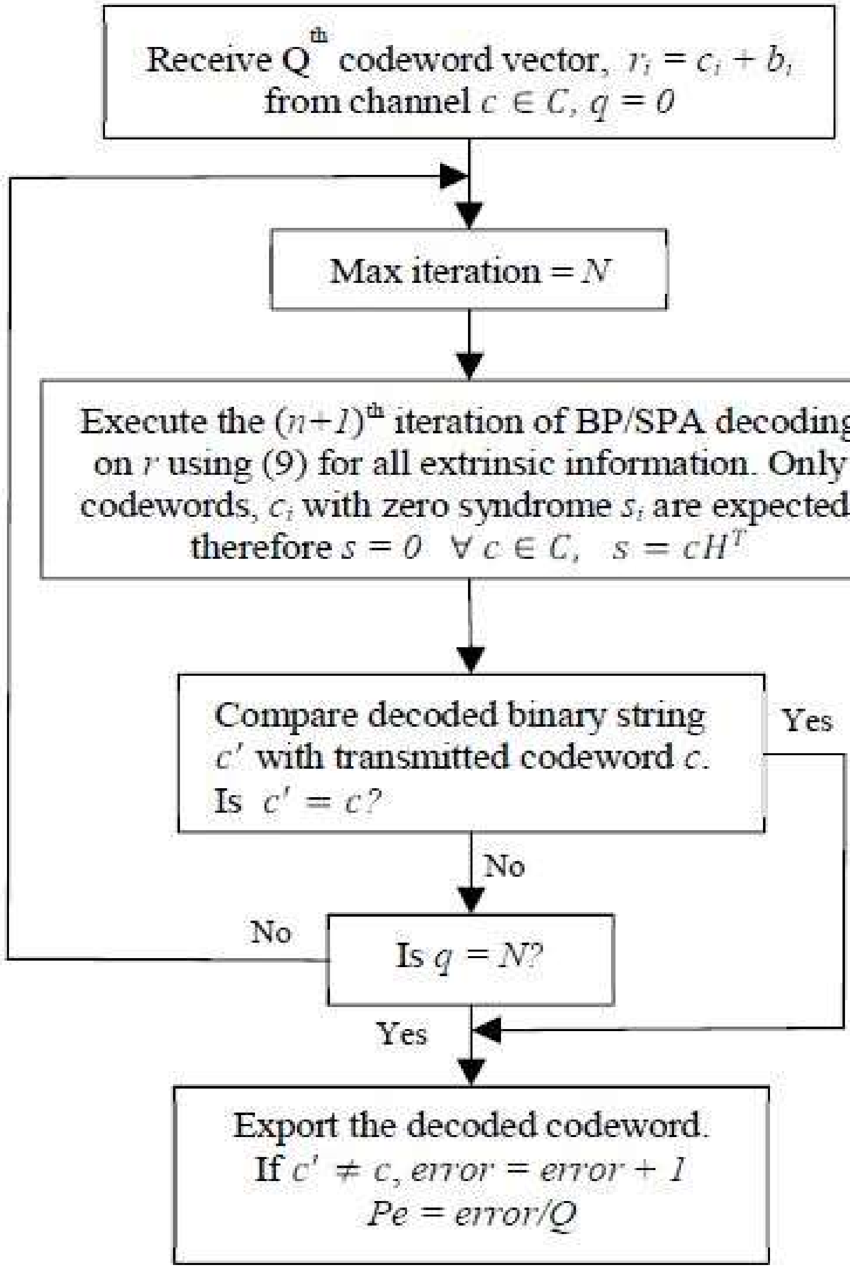
$$p_2^{1e} = \frac{1-(1-2p_0^1)(1-2p_1^1)}{2} \qquad (7)$$

- In General,

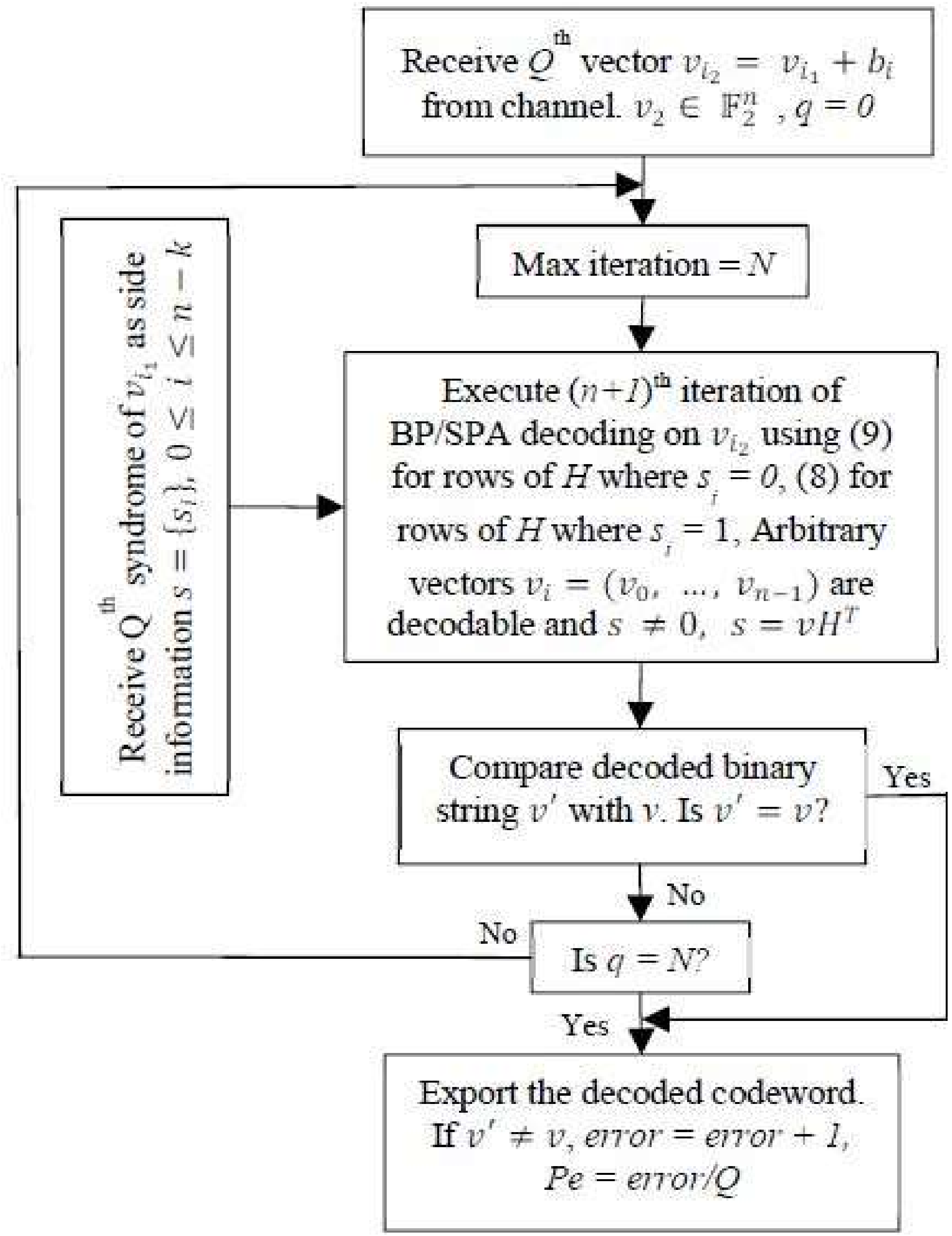
$$p_i^{0e} = \frac{1+\prod_{j\neq i}(1-2p_j^1)}{2} \qquad (8)$$

$$p_i^{1e} = \frac{1-\prod_{j\neq i}(1-2p_j^1)}{2} \qquad (9)$$

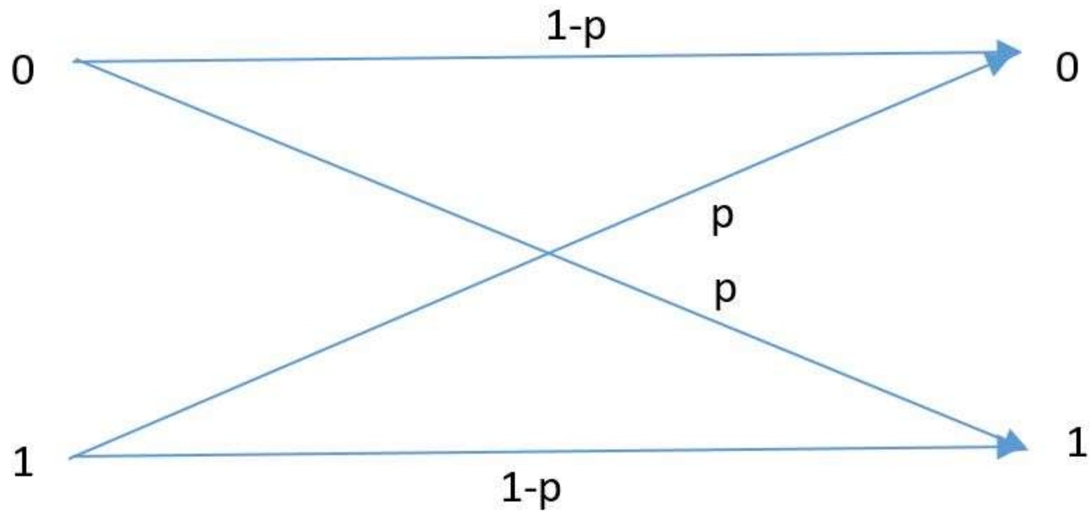
The Conventional Decoder



The Proposed Decoder



The Binary Symmetric Channel



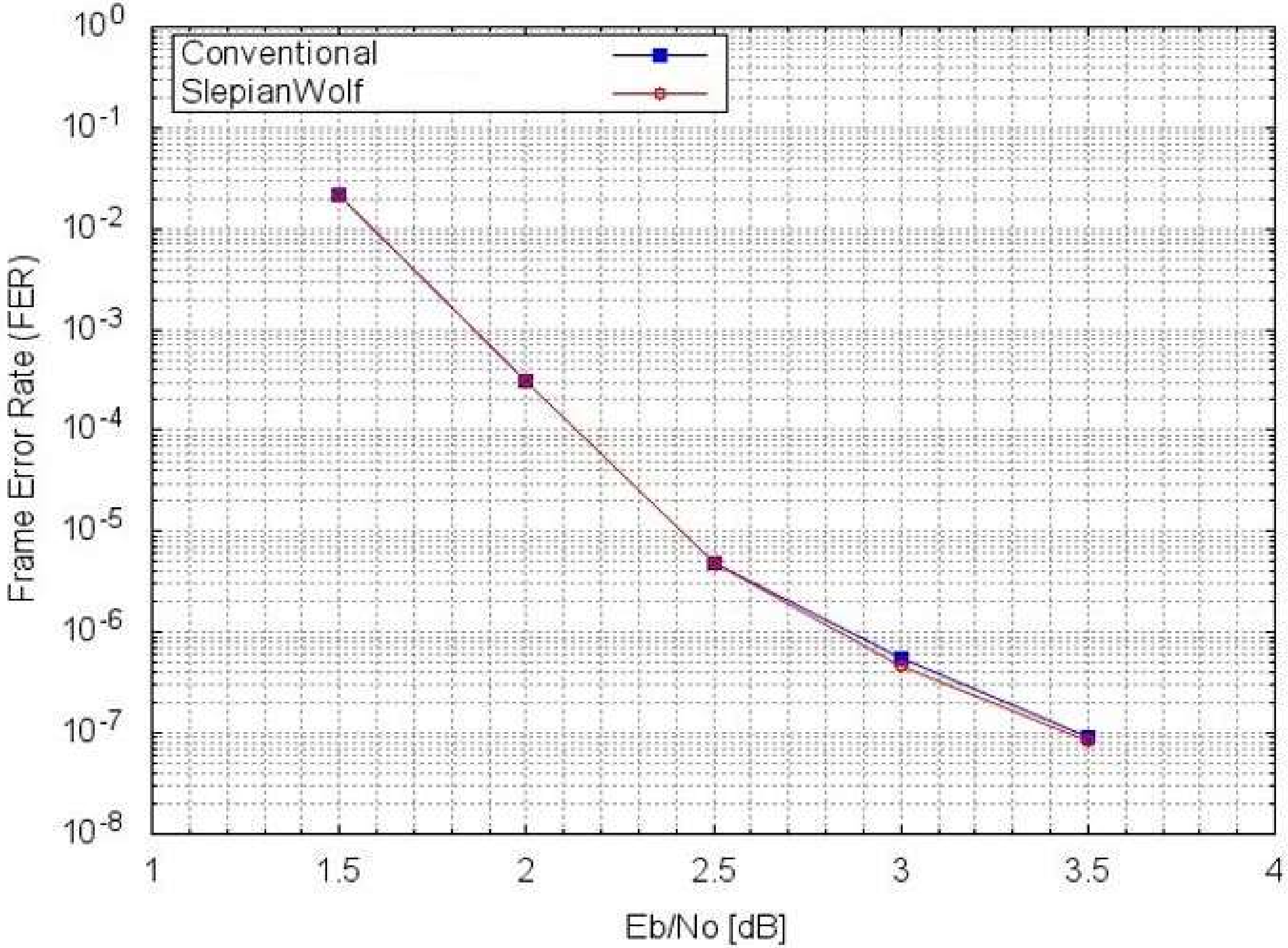
$p = Q(\sqrt{SNR})$

- The correlation between the sources is defined as follows:

$$Y = \begin{cases} X, & \text{with probability } 1-p, \\ X \oplus 1, & \text{with probability } p, \end{cases} \qquad (10)$$

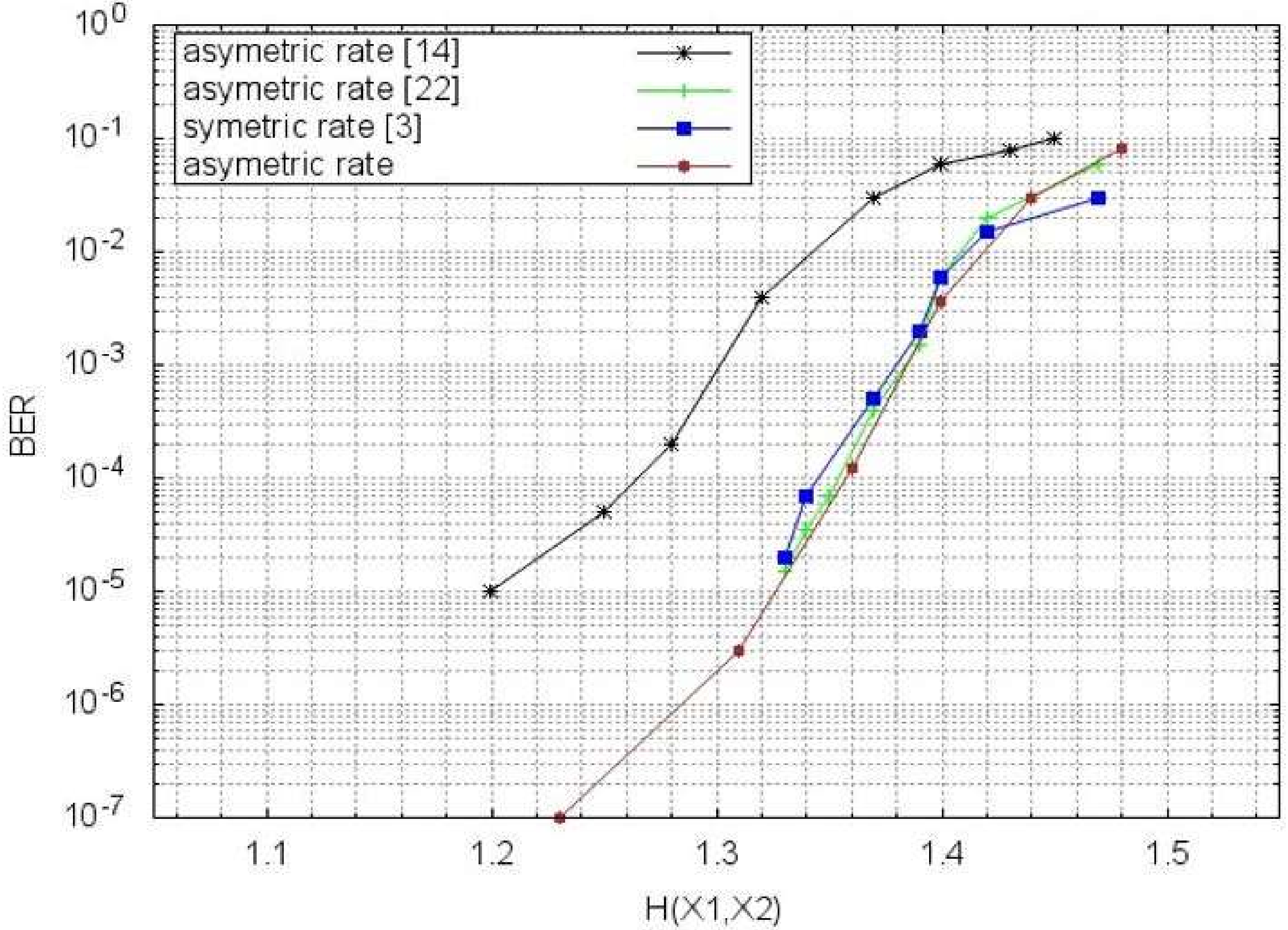
$$p_i^c = P(c_i = c \mid r_i) = ae^{-\frac{(r_i - v^c)^2}{2\sigma^2}} \qquad (1)$$

Results



Conventional
SlepianWolf
Frame Error Rate (FER)
Eb/No [dB]
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
1
1.5
2
2.5
3
3.5
4

Results



asymetric rate [14]
asymetric rate [22]
symetric rate [3]
asymetric rate
BER
H(X1,X2)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
1.1
1.2
1.3
1.4
1.5

Conclusion

- A very good compression was achieved with no apparent loss in performance compared to the conventional decoder
- Additional computational overhead due to the formation of syndromes offset by the absence of encoding operation all together in the proposed model
- The proposed model outperforms those presented in the previous works