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A Game-Theoretic Approach for Communication-Free Voltage-VAR Optimization

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Outline

- Background & Motivation
- Game-theoretic Formulation
- Communication-free Algorithm
- Numerical Results
- Conclusions & Future work

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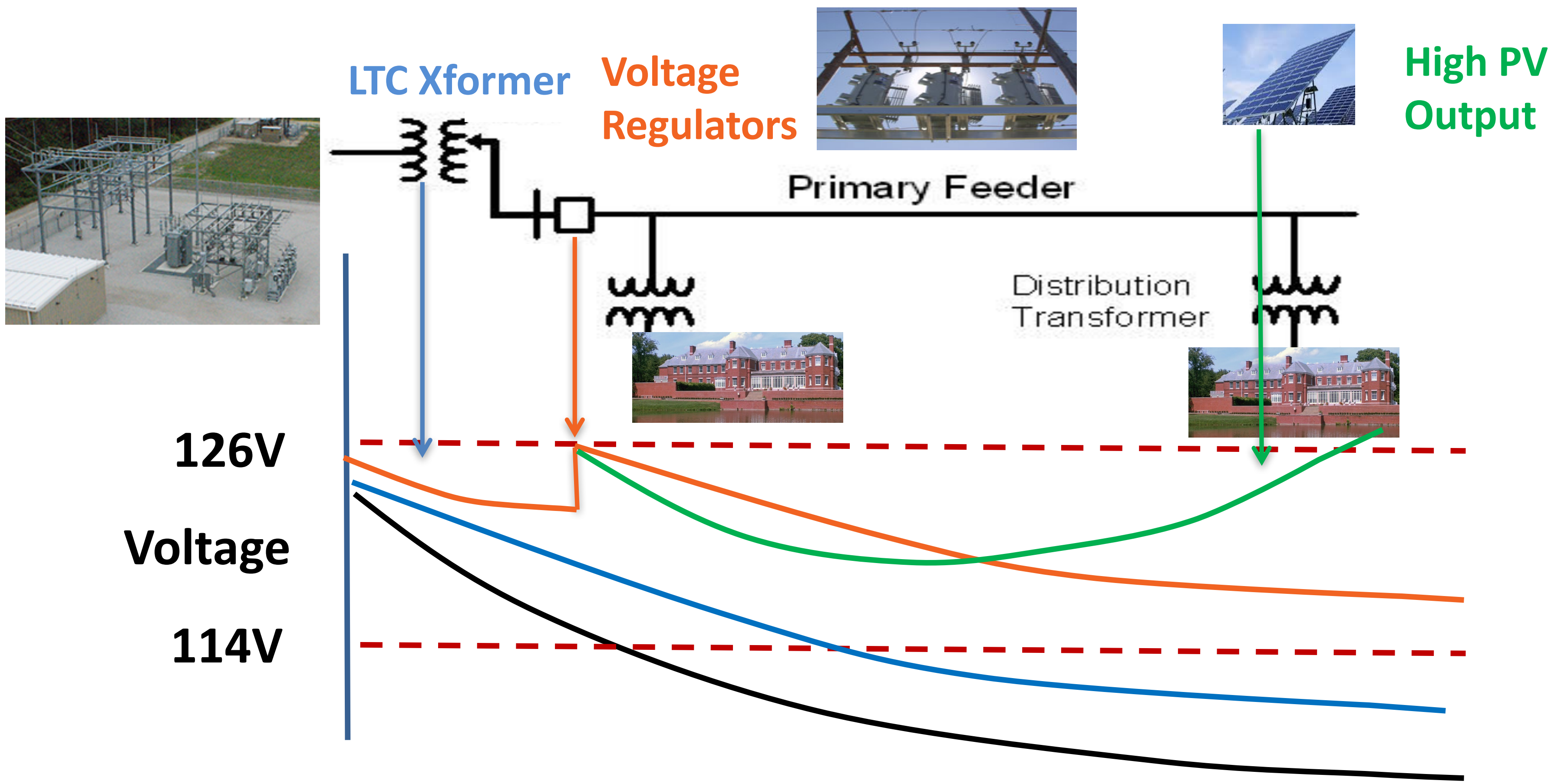
Voltage Control in Distribution Systems



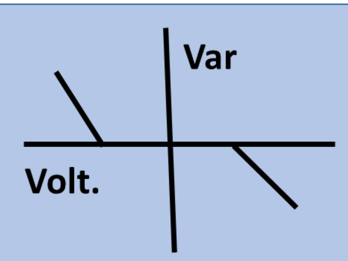
- Distributed energy resources (DERs) greatly challenge distribution system voltage control

Literature Review

- DERs also serve as reactive power (VAR) resources
- Facilitate fast voltage-VAR control
- Existing approaches
  - Optimization/optimal power flow (OPF)-based
  - Local feedback control
- OPF-based approaches
  - Centralized [Farivar-Low '12][Valverde et al '13]
  - Distributed [Dall'Anese-Zhu-Giannakis '13][Bolognani et al '15][Liu-Zhu '17][Liu-Shi-Zhu '17]
- Assumption: strongly-connected communication graph among DERs

Commu.-Optimality Tradeoff

- Commu. infrastructure is still under-deployed [DOE '15] in current distribution systems
- Local feedback control: [Farivar-Low '13][Zhu-Liu '15][Li-Qu '15][Zhu-Li '16][Kekatos et al '16]
- Suffers from loss of optimality
  - Min. weighted voltage mismatch [Far hu-Liu '15]
  - Not lead to the minimizer of the voltage control problem [Kekatos et al '16]



IEEE 1547.8 standard

Our Goal: Communication-free + Optimal?

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System Model

- Distribution network $(\mathcal{N}, \mathcal{E})$ with $\mathcal{N} := \{0, \cdots, N\}$ set of buses and $\mathcal{E} := \{(i, j), \forall i, j \in \mathcal{N}\}$ line segments
- *LinDistFlow* model: linearized power flow for distribution networks [Baran-Wu '89]

$$\mathbf{v} = \mathbf{X}\mathbf{q} + \bar{\mathbf{v}}$$

- Voltage-VAR optimization [Liu-Shi-Zhu '16, '17]

$$\min_{\mathbf{v},\mathbf{q}} \quad \frac{1}{2}\|\mathbf{v} - \boldsymbol{\mu}\|_2^2 + \frac{1}{2}\|\mathbf{q}\|_{\mathbf{C}}^2$$
$$s.t. \quad \mathbf{v} = \mathbf{X}\mathbf{q} + \bar{\mathbf{v}}$$
$$\mathbf{q} \in \mathcal{Q}$$

Network-wide operational cost

Desired voltage profile

VAR provision cost

Feasible VAR injection

- Cannot be solved exactly with no communications

Game-theoretic Formulation

- Information structure
  - Each bus-*j* accesses: voltage mismatch $(v_j - \mu_j)$, VAR input q_j , and its own cost $\frac{1}{2}\left[(v_j - \mu_j)^2 + c_j q_j^2\right]$
  - Operational cost is affected by other buses' q_{-j}
- Strategic game $\mathcal{G} := \langle \mathcal{K}, \{\mathcal{Q}_j\}_{j\in\mathcal{K}}, \{U_j\}_{j\in\mathcal{K}} \rangle$ with $\mathcal{K} := \mathcal{N}/\{0\}$
- Individual payoff: $$U_j(q_j, q_{-j}) = -\frac{1}{2}(v_j - \mu_j)^2 - \frac{1}{2}c_j q_j^2$$
$$= -\frac{1}{2}\left(\sum_{i=1}^{N} X_{ji} q_i + \bar{\mu}_j\right)^2 - \frac{1}{2}c_j q_j^2$$
- Equivalent to finding the *Pareto optimum:* maximize social welfare $$U(\mathbf{q}) = \sum_{j\in\mathcal{N}/\{0\}} U_j(q_j, q_{-j})$$

Game-theoretic Formulation

- Comparison of terms

Volt.-VAR-Opt.	Bus with DER	Operational cost	VAR injection	Feasible set	Network-wide cost
Game-theoretic formulation	Agent/player	Payoff	Action	Action set	Social welfare

- Solution concepts: Pareto optimum (PO), Nash equilibrium (NE)

$$U_j(q_j^*, q_{-j}^*) \geq U_j(q_j, q_{-j}^*), \forall q_j \in \mathcal{Q}_j, j \in \mathcal{K}$$

  – NE is generally inefficient in achieving system-level objective

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Payoff-based Learning (PBL)

- Learning process: dynamics solving for PO or NE
- Payoff-based learning
  - Choose $q_j(t)$ following a *strategy*

$$prob_j(t) = F_j\left(\left\{q_j\left(\tau\right), U_j\left(\mathbf{q}\left(\tau\right)\right)\right\}_{\tau=0,\cdots,t-1}\right)$$

  - Only observe $U_j\left(\mathbf{q}(t)\right)$
- Most PBL solve only for
  - NE [Goto et al ’12][Tatarenko ’16]
  - Potential games [Marden-Shamma ’12][Zhu et al ’13]
- For PO and generic payoffs under finite action sets: [Marden et al ’14]

Learning Dynamics

- Discretize action sets $\mathcal{Q}_j$
- Each agent-*j* maintains internal state $[\bar{q}_j, \bar{u}_j, m_j]$
  - $\bar{q}_j \in \mathcal{Q}_j$ the benchmark VAR injection
  - $\bar{u}_j$ the benchmark payoff
  - m_j the mood that takes values either content (C) or discontent (D)
- Two steps
  - (S1) VAR injection dynamics
  - (S2) State dynamics
- Parameters: exploration rate $\epsilon > 0$, constant $c \geq N$

Learning Dynamics

- (S1) VAR injection dynamics: determine strategy $prob_j^{q_j}$ by the state $[\bar{q}_j, \bar{u}_j, m_j]$
  - Content state $(m_j = C)$:

$$prob_j^{q_j} = \begin{cases} \frac{\epsilon^c}{|\mathcal{Q}_j|-1}, & \text{for} \quad q_j \neq \bar{q}_j \\ 1-\epsilon^c, & \text{for} \quad q_j = \bar{q}_j \end{cases}$$

  - Discontent state $(m_j = D)$:

$$prob_j^{q_j} = \frac{1}{|\mathcal{Q}_j|}, \forall q_j \in \mathcal{Q}_j$$

- (S2) state dynamics: update state $[\bar{q}_j, \bar{u}_j, m_j]$ by the payoff u_j and input q_j

Learning Dynamics

- (S2) state dynamics:
  - Content state $(m_j = C)$:
    - If $[q_j, u_j] = [\bar{q}_j, \bar{u}_j]$,

      $$[\bar{q}_j, \bar{u}_j, C] \xrightarrow{[\bar{q}_j, \bar{u}_j]} [\bar{q}_j, \bar{u}_j, C]$$

    - Else $[\bar{q}_j, \bar{u}_j, C] \xrightarrow{[q_j, u_j]} \begin{cases} [q_j, u_j, C], \text{with prob. } \epsilon^{1-u_j} \\ [q_j, u_j, D], \text{with prob. } 1-\epsilon^{1-u_j} \end{cases}$
  - Discontent state $(m_j = D)$:

    $$[\bar{q}_j, \bar{u}_j, D] \xrightarrow{[q_j, u_j]} \begin{cases} [q_j, u_j, C], \text{with prob. } \epsilon^{1-u_j} \\ [q_j, u_j, D], \text{with prob. } 1-\epsilon^{1-u_j} \end{cases}$$

- Commu.-free & model-free
- Used in wind farm turbine control [Marden-Pao '13]

Convergence

- The game $\mathcal{G}$ is interdependent

Definition 1. *(Interdependence.) An N-agent game $\mathcal{G}$ on a finite action space $\mathcal{Q}$ is interdependent if, for every $\mathbf{q} \in \mathcal{Q}$ and every proper subset of agents in $\mathcal{J} \subseteq \mathcal{N} / \{0\}$, there exists an agent $i \notin \mathcal{J}$ and a choice of actions $q'_{\mathcal{J}} \in \prod_{j \in \mathcal{J}} (\mathcal{Q}_j)$ such that $U_i \left(q'_{\mathcal{J}}, q_{-\mathcal{J}} \right) \neq U_i \left(q_{\mathcal{J}}, q_{-\mathcal{J}} \right)$.*

Lemma 1. *The voltage-VAR optimization game $\mathcal{G}$ has the interdependent structure.*

  – Idea: the distribution network $(\mathcal{N}, \mathcal{E})$ is connected

- Convergence in probability

Theorem 1. *Suppose all the buses choose their instantaneous VAR injection level following the updates in (S1)-(S2). For any given parameter $0 < \delta < 1$, if the exploration rate $\epsilon > 0$ is sufficiently small, then $\mathbf{q}(t) \in \arg\max_{\mathbf{q} \in \mathcal{Q}} U(\mathbf{q})$ will hold after for sufficiently large number of update periods with at least probability δ.*

Convergence

- Idea: the perturbed Markov process $[\bar{\mathbf{q}}, \bar{\mathbf{u}}, \mathbf{m}]$.
- The limit stationary distribution exists

$$\lim_{\epsilon \to 0} \mu^\epsilon = \mu^0$$

- The support of μ^0 , i.e., the states z that $\mu^0(z) > 0$, are the *stochastically stable (SS) states* the algorithm converges to
- The states $[\bar{\mathbf{q}}, \bar{\mathbf{u}}, \mathbf{m}]$ are SS iff $\bar{\mathbf{q}}$ minimizes the social welfare
- Convergence in probability with small ϵ

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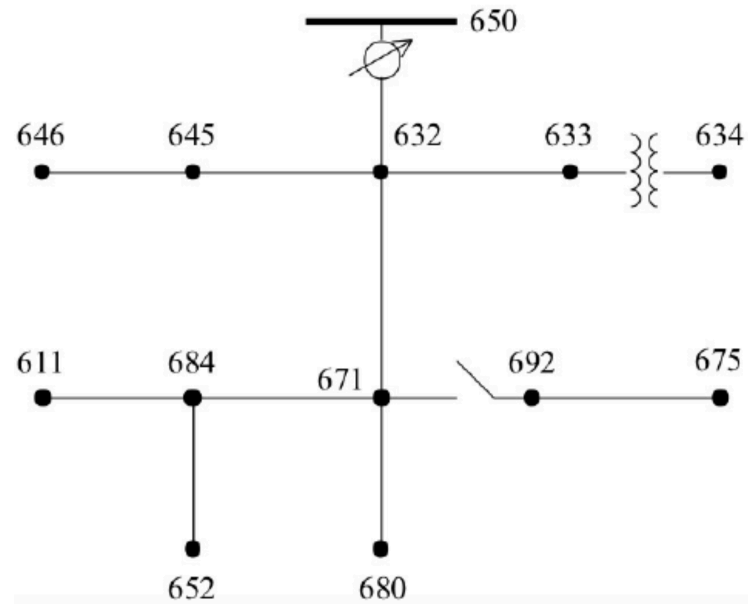
Numerical Results

- IEEE 13-bus test feeder
- Let $\epsilon = 1 \times 10^{-3}$ and $c = N + 1$
- Action set $\mathcal{Q}_j = [-0.6 : 0.2 : 0.6]$
- Sample average welfare $\bar{U}^t$

$$\bar{U}^t := \frac{1}{t} \sum_{\tau=0}^{t-1} U(\mathbf{q}(t))$$

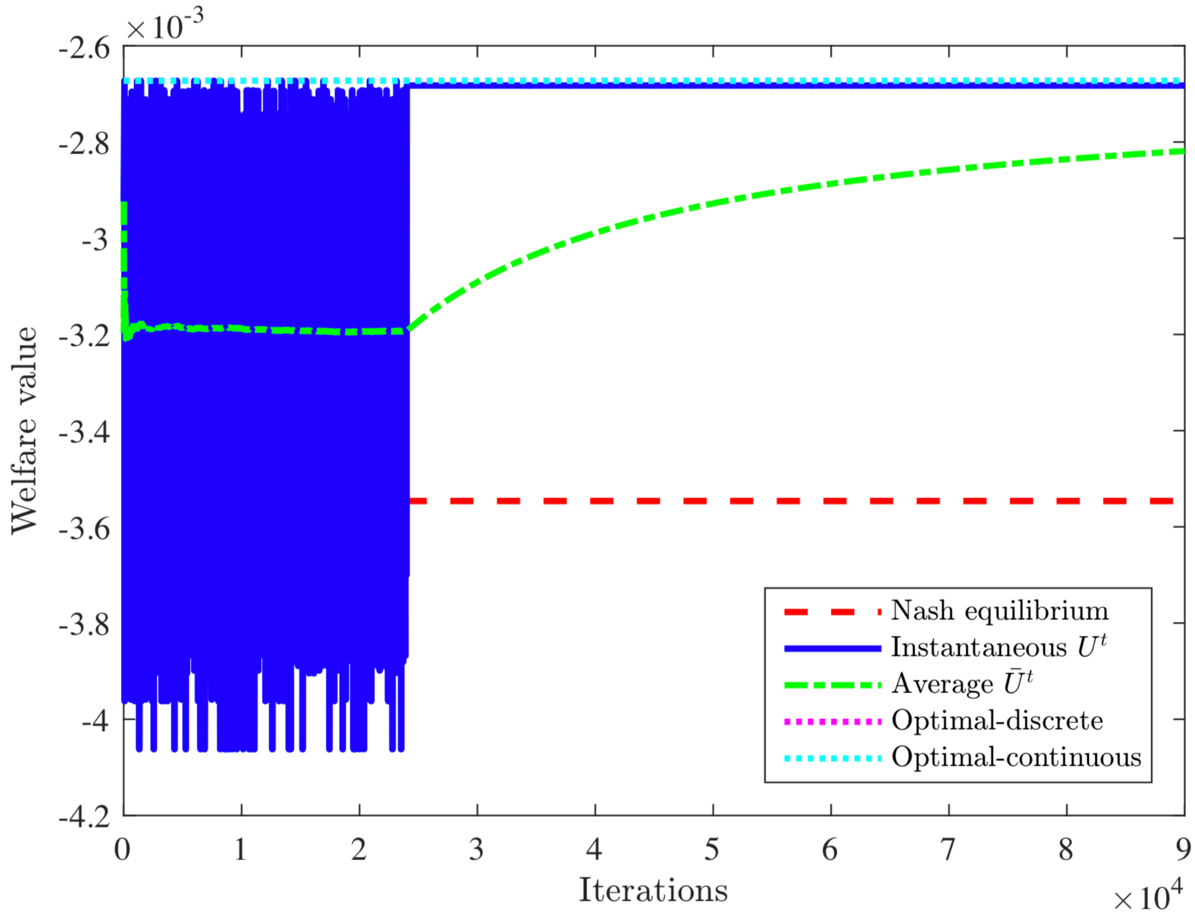


- Convergent VAR injection

Bus #	632	634	671(692)	611	675
Our Alg.	0.4150	0.3930	0.5980	0.5995	0.5990
NE	0.0211	0.0312	0.1450	0.1844	0.1612
Opt. Disc.	0.4000	0.4000	0.6000	0.6000	0.6000
Opt. Cont.	0.3991	0.4066	0.6000	0.6000	0.6000

Numerical Results

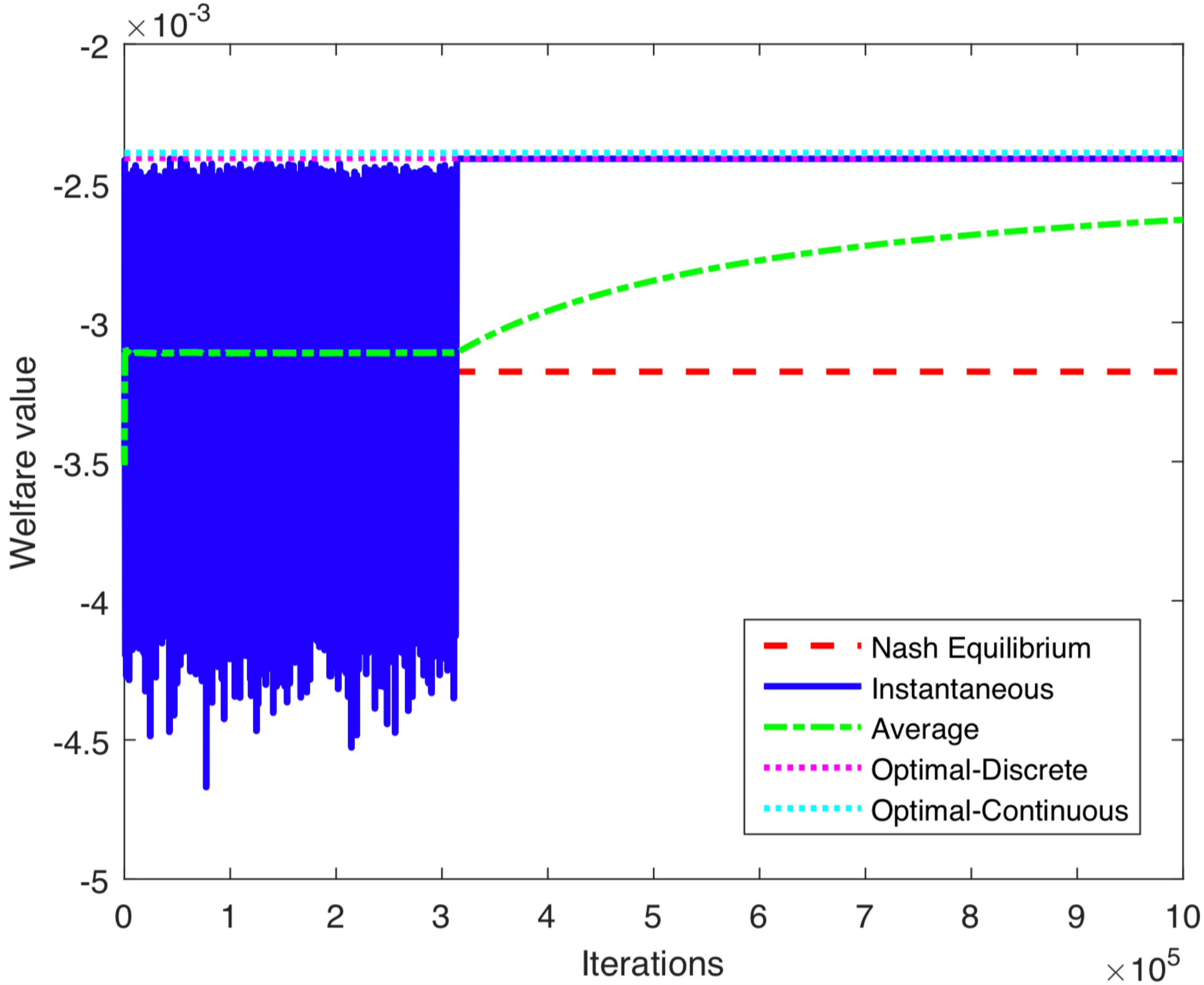
- Convergent social welfare



  - NE is about 33% less efficient
  - Discretized $\mathcal{Q}_j$ works fine

Numerical Results

- IEEE 37-bus test feeder



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Conclusions

- Develop a communication-free algorithm that achieves the network-wide optimal performance for voltage-VAR optimization
- Open up the possibility for leveraging tools from game-theoretic control to voltage regulation, especially under limited communications
- Future work
  - Explore more efficient payoff-based learning algorithms to handle dynamic settings
  - Understand the value of communications from a game-theoretic perspective

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Thank you!

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