
Unsupervised Frequency Clustering for Null Space Estimation in Wideband Spectrum Sharing Networks

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◆ Introduction

- Wideband Null Steering for Interference Control in Spectrum Sharing Networks.

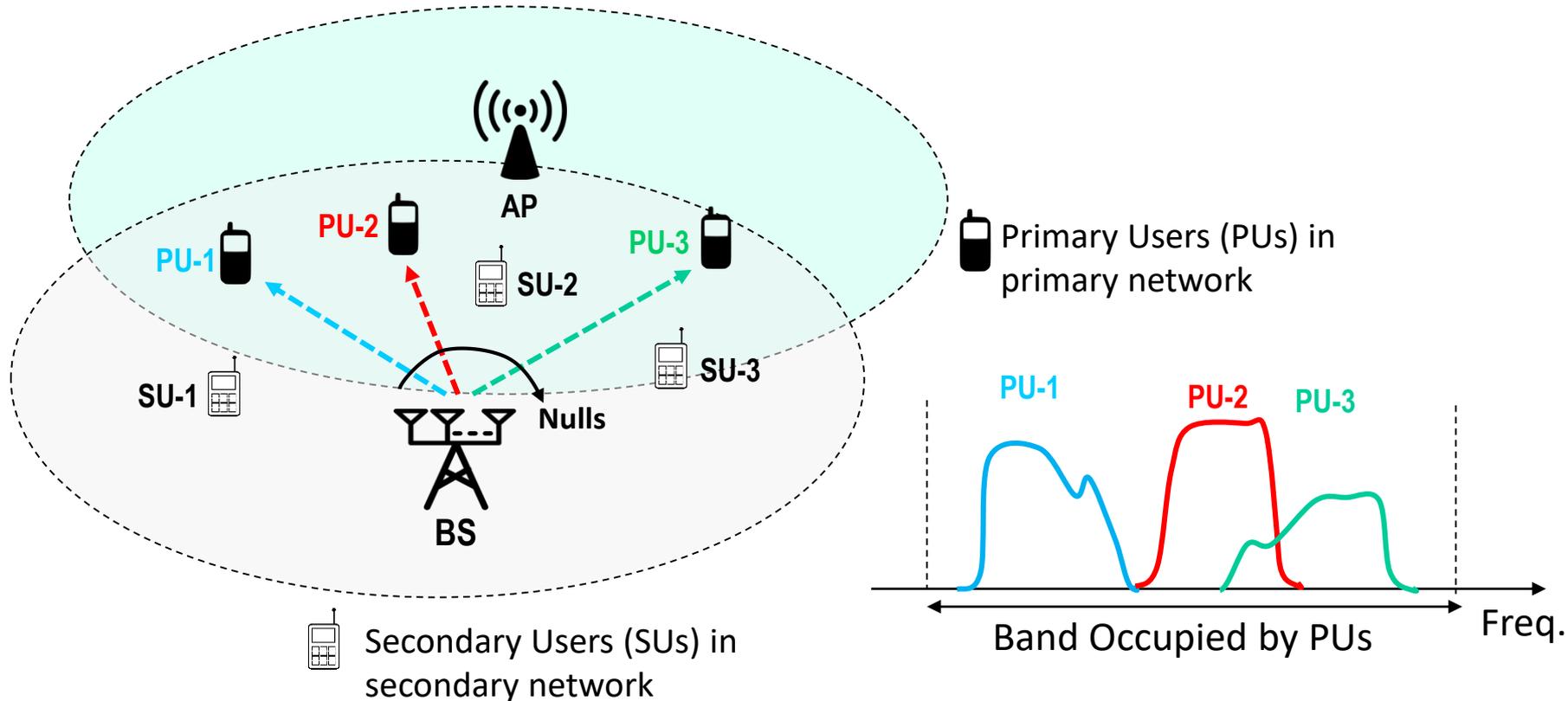
◆ System Model and Proposed Algorithm

- Clustering of Correlated Frequency Bins for Low Complexity Null Steering

◆ Numerical Results

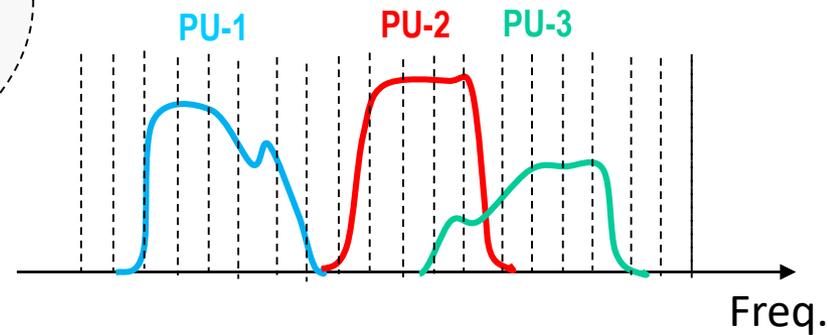
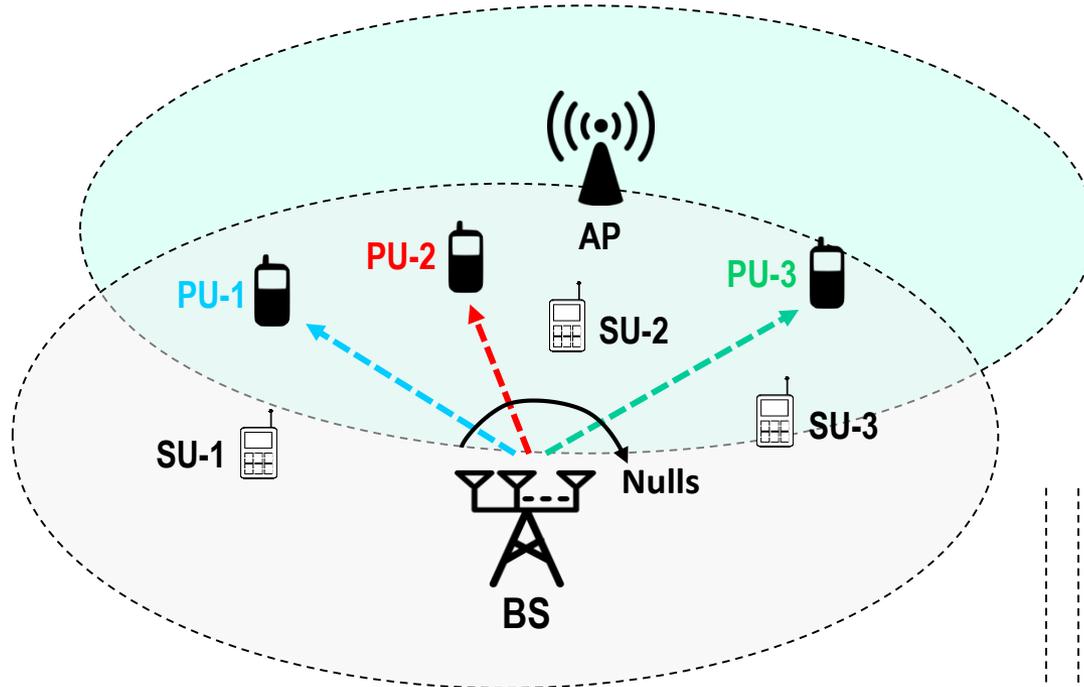
◆ Conclusion

Underlay Spectrum Sharing Networks



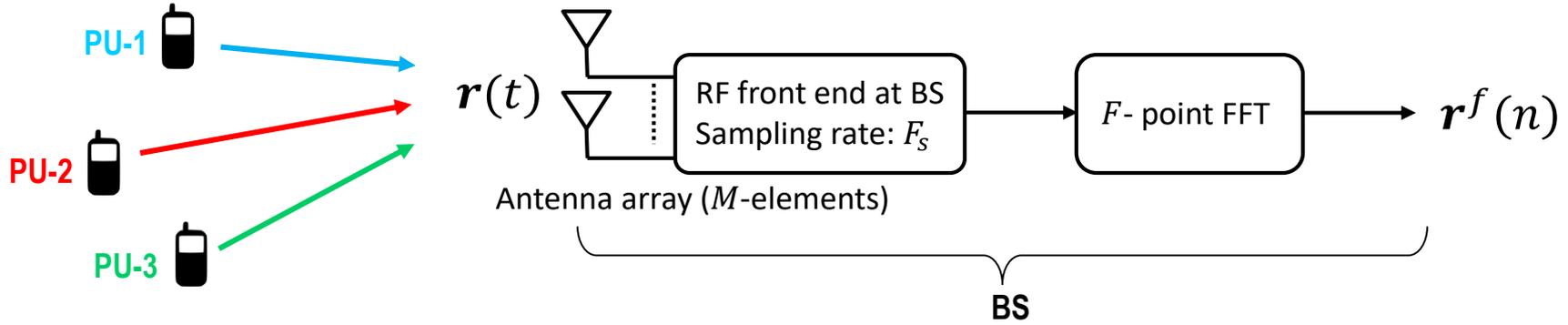
- ◆ Primary Network has existing primary users (PUs) in wideband.
- ◆ Secondary network is deployed in same band.
 - BS needs to eliminate downlink interference to PUs.
 - Use multiple antenna array to steer nulls toward PUs.

Wideband Null Steering



- ◆ Wideband null steering is required from BS, since each PU occupies wideband.
- ◆ BS uses signal received from PUs for null steering.

System Model and Problem Statement



- ◆ $\mathbf{r}^f(n)$: $M \times 1$ received signal vector in bin $f \in \{1, 2, \dots, F\}$ at instant $n = 0, 1, \dots, T - 1$.

- ◆
$$\mathbf{r}^f(n) = \sum_{l=1}^{L_f} \sqrt{p_l^f} x_l^f(n) \mathbf{h}_l^f + \mathbf{w}^f(n)$$

L_f = number of PUs in bin f

p_l^f = transmitted power from PU- l

$x_l^f(n)$ = transmitted symbol from PU- l

$\mathbf{w}^f(n)$: noise vector $\sim CN(0, \mathbf{R}_w^f)$

\mathbf{h}_l^f = $M \times 1$ channel vector between PU- l and BS in bin f

Problem Statement:

Estimate null space matrices \mathbf{U}^f without prior knowledge of \mathbf{h}_l^f , such that

$$(\mathbf{U}^f)^H \mathbf{h}_l^f = \mathbf{0}, \forall l$$

- ◆ Existing solutions use narrowband techniques with fine frequency resolution [1,2].
- ◆ Fine frequency resolution at BS:
 - To obtain spectrum occupancy with high accuracy [3].
 - Due to overdesigned system, e.g., Δf in LTE $<$ coherence BW of extended pedestrian A (EPA) model.
 - Results in large number of bins.
 - Increases complexity.
- ◆ Complexity \propto # occupied bins by PUs.
- ◆ **Proposed approach:**
 - **Exploit correlation in adjacent bins to reduce complexity.**

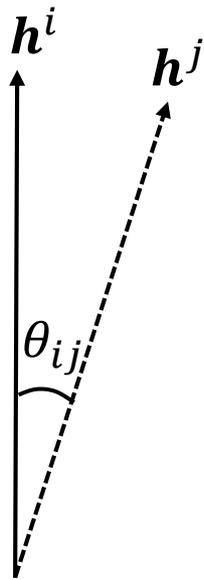
[1] Tsinos et. al., "Blind Opportunistic Interference Alignment in MIMO Cognitive Radio Systems," *Emerg. Sel. Top. Circuits Syst. IEEE J. On*, vol. 3, no. 4, pp. 626–639, Dec. 2013.

[2] Kouassi et. al. "Reciprocity-based cognitive transmissions using a MU MIMO approach," *IEEE ICC*, June 2013

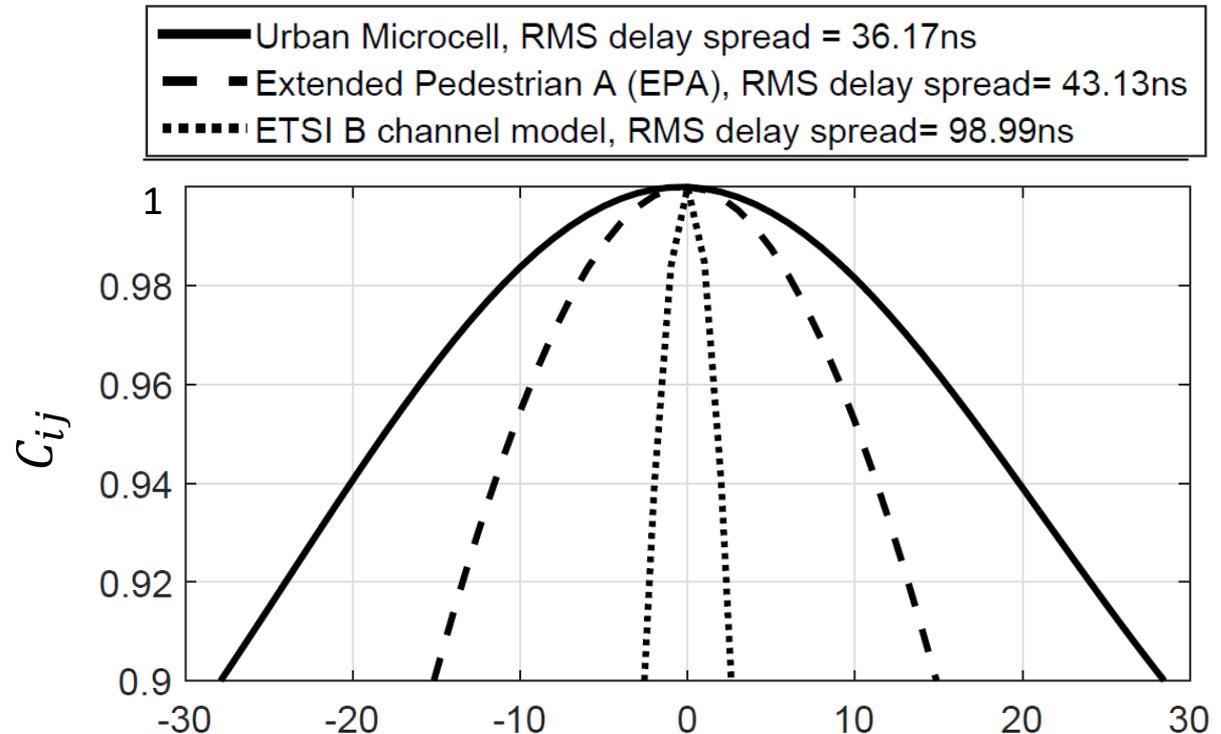
[3] Harjani *et al.*, "Wideband blind signal classification on a battery budget," in *IEEE Commu. Magazine*,, October 2015.

Correlation vs Frequency Separation

- ◆ Correlation in channel vectors in bin i and bin $j = C_{ij} = \cos^2(\theta_{ij})$ [4]



\mathbf{h}^i : Channel vector in bin i
 \mathbf{h}^j : Channel vector in bin j



Frequency bin separation $i - j$
 $F_s = 20$ MHz, $F = 512$, $M = 8$.

[4] Choi et. al., "Interpolation based transmit beamforming for MIMO-OFDM with limited feedback," IEEE TSP, Nov. 2005

Proposed Algorithm

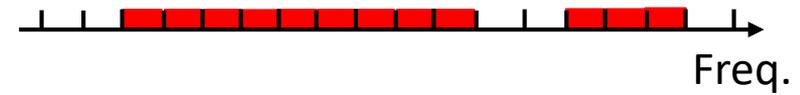
$\mathbf{r}^f(n), f \in F_a$: set of bins with at least one PU



Estimate covariance matrices $\hat{\mathbf{R}}^f, f \in F_a$
using T samples of $\mathbf{r}^f(n), n = 1, \dots, T$



■ Bins in set F_a



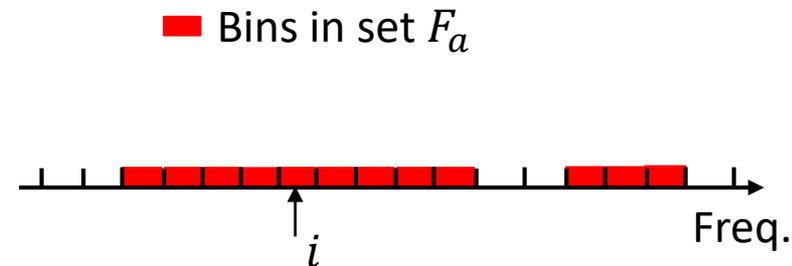
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$i \leftarrow$ Center bin of new cluster.
Compute null space \mathbf{U}^i at bin i

EVD of $\hat{\mathbf{R}}^i - \mathbf{R}_w^i$
 \mathbf{R}_w^i : noise covariance



Proposed Algorithm

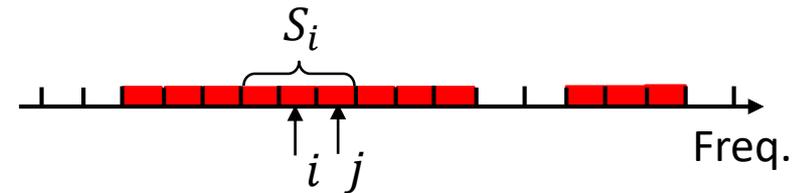
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Form cluster S_i around bin i .
 $S_i \leftarrow j?$ Where $j = i \pm 1, i \pm 2, \dots$

■ Bins in set F_a



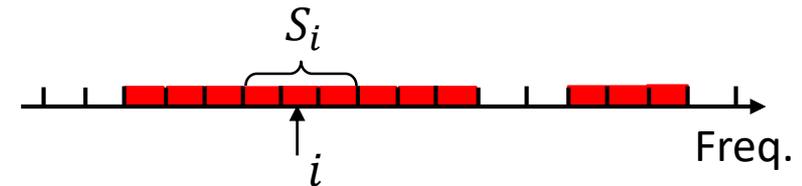
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**Proposed test for clustering
correlated bins (?)**

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**Proposed test for clustering
correlated bins (?)**

Compute common null
space: \mathbf{U}^{S_i}

EVD of $\mathbf{R}^{S_i} = \sum_{j \in S_i} \frac{1}{\hat{p}_r^j} (\hat{\mathbf{R}}^j - \mathbf{R}_w^j)$
 \hat{p}_r^j : Est. received power in bin j

Proposed Algorithm

$\mathbf{r}^f(n), f \in F_a$: set of bins with at least one PU

■ Bins in set F_a
 ■ Clustered bins



Estimate covariance matrices $\hat{\mathbf{R}}^f, f \in F_a$
 using T samples of $\mathbf{r}^f(n), n = 1, \dots, T$

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 Compute null space \mathbf{U}^i at bin i

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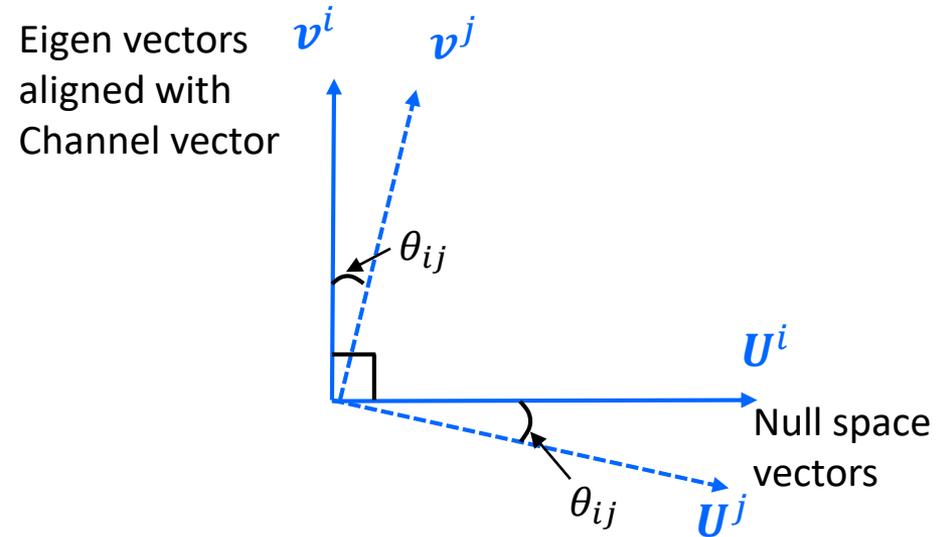
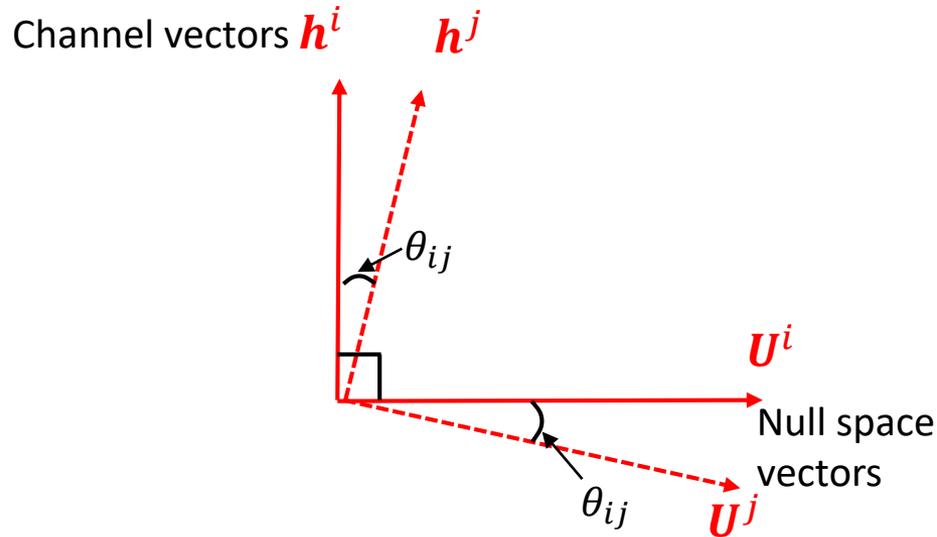
$F_a = F_a \setminus S_i$

Repeat until $F_a = \phi$

S : Set of clusters S_i ,
 null spaces \mathbf{U}^{S_i}

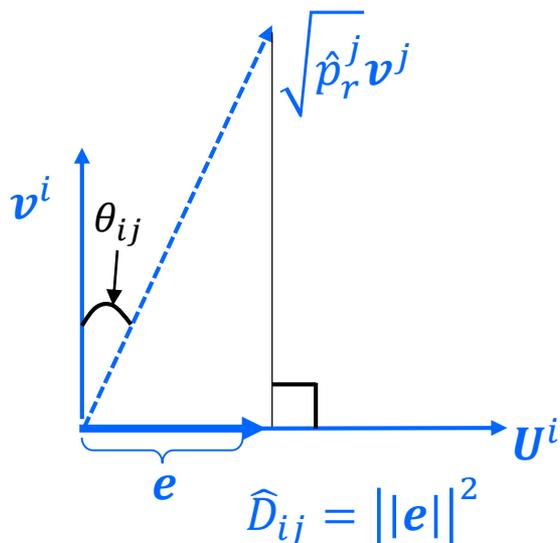
Number of EVD computations = $2 |S|$

Null Spaces in Correlated Bins



- ◆ v^i : Eigen vector of $\widehat{R}^i - R_w^i$ aligned with channel vector h^i
- ◆ Higher $C_{ij} = \cos^2(\theta_{ij}) \Rightarrow$ more aligned null spaces $U^i, U^j \Rightarrow$ Bin j can be clustered with bin i .

Criterion for Clustering bin j with i



$\hat{p}_r^j = \text{Tr}(\hat{\mathbf{R}}^j - \mathbf{R}_w^j)$: Est. received power in bin j

\hat{D}_{ij} : Component of \hat{p}_r^j null space U^i

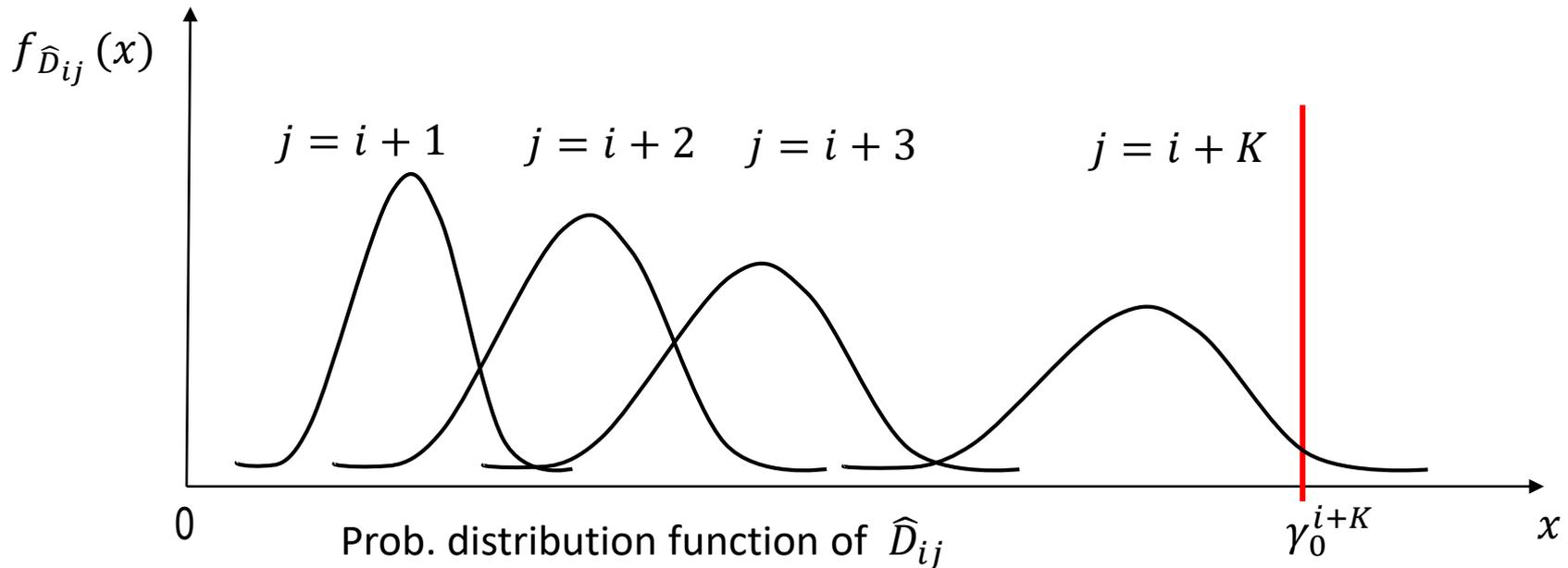
\uparrow Correlation $C_{ij} \Rightarrow \downarrow \theta_{ij} \Rightarrow \downarrow \hat{D}_{ij}$

- ◆ $\hat{D}_{ij} = \text{Tr} \left((U^i)^H (\hat{\mathbf{R}}^j - \mathbf{R}_w^j) U^i \right) \sim N \left((1 - C_{ij})\mu_j, (1 - C_{ij})^2 \sigma_j^2 \right)$.
 - $\mu_j = \text{Tr}(\mathbf{R}^j - \mathbf{R}_w^j)$, $\sigma_j^2 = \frac{\mu_j^2 + \sigma_w^2}{T}$, $\hat{p}_r^j \sim N(\mu_j, \sigma_j^2)$.
- ◆ \hat{D}_{ij} is Gaussian due to non-asymptotic estimation.
- ◆ Cluster j with i ($S_i \leftarrow j$) if $\hat{D}_{ij} \leq \gamma_0^j$.
- ◆ Computations for \hat{D}_{ij} ($\approx 2M^3$ flops) \ll computations for EVD ($\approx 23M^3$ flops) [5].

Selection of Threshold γ_0^j

- ◆ To cluster bins with correlation $C_{ij} \geq 1 - \delta_0$ with probability P_0

$$\widehat{D}_{ij} \leq \gamma_0^j = \delta_0 \sigma_j Q^{-1}(1 - P_0) + \delta_0 \mu_j$$



Larger $\delta_0 \Rightarrow$ larger threshold \Rightarrow more bins will be clustered with bin i

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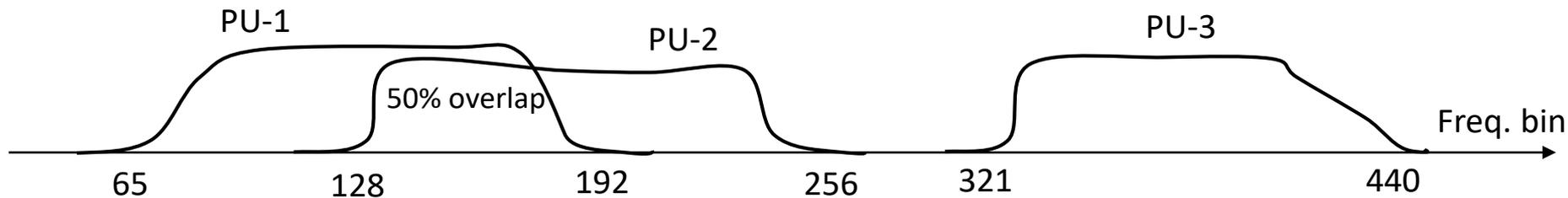
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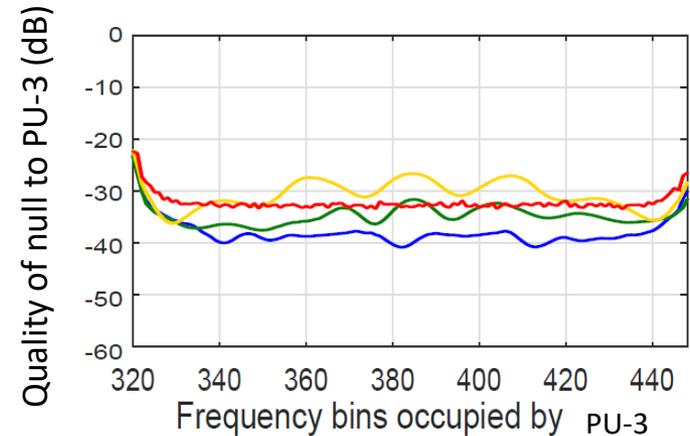
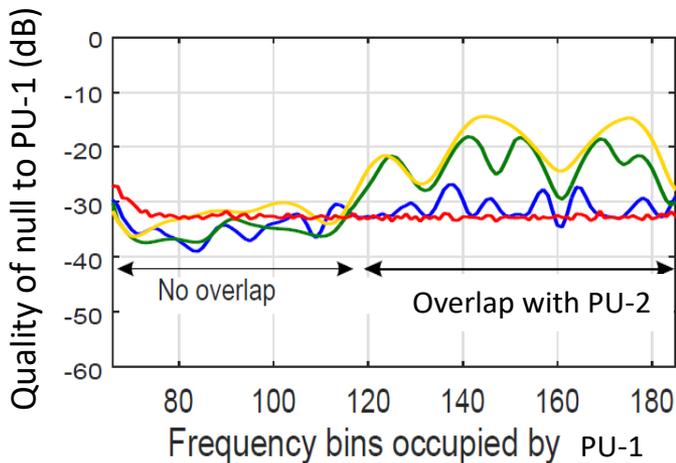
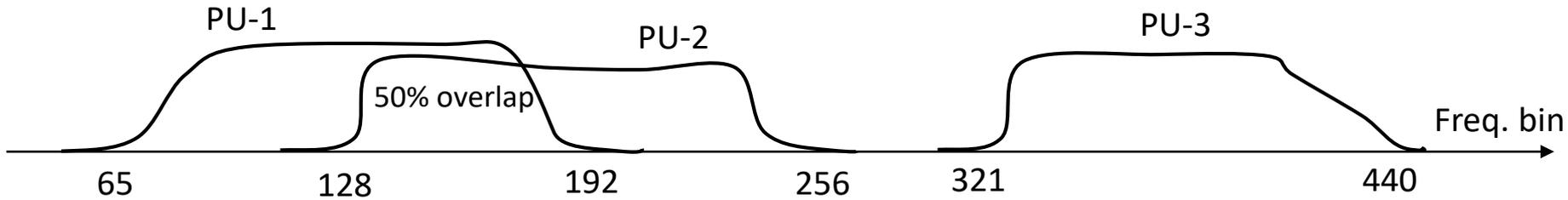
◆ Conclusion

- ◆ BS parameters: $M = 8$ antennas, $F_s = 20\text{MHz}$, $F = 512$ bins.
- ◆ 3 PUs transmitting OFDM signals with bandwidths 5MHz. Average RX SNR = 10dB.



- ◆ Number of occupied bins = 312.
- ◆ # samples to estimate non-asymptotic $\hat{\mathbf{R}}^i: T = 100$.
- ◆ Noise is white Gaussian: $\mathbf{R}_w^i = \mathbf{I}$.
- ◆ Algorithm parameters: $P_0 = 0.95$, $\delta_0 \in \{0.01, 0.05, 0.1\}$.
- ◆ Performance metrics:
 - Quality of null to PU- l : $\left| \left| \mathbf{U}^i \mathbf{h}_l^i \right| \right|^2 / \left| \left| \mathbf{h}_l^i \right| \right|^2$
 - Complexity: number of EVD computations.

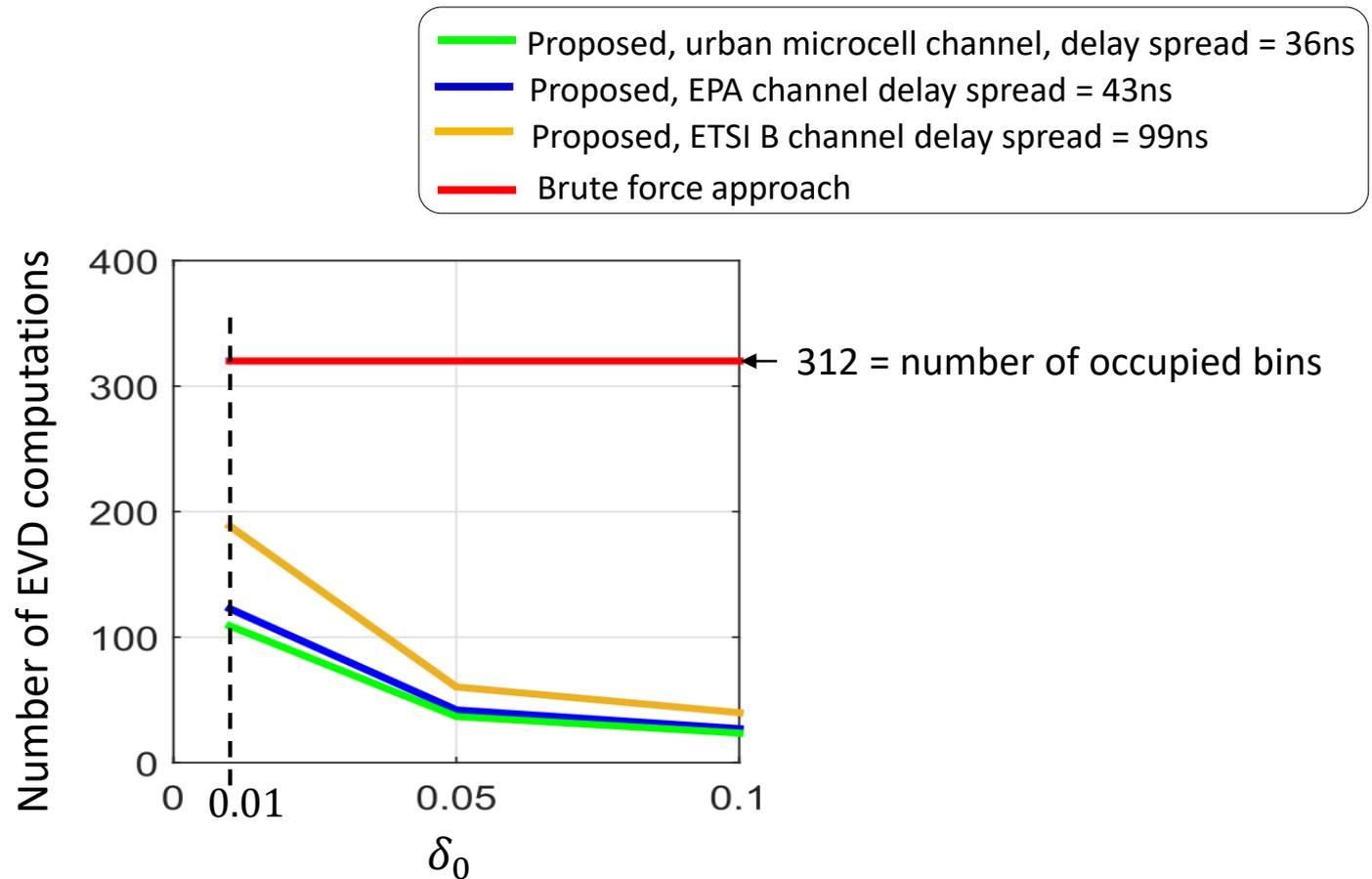
Impact of δ_0 on Quality of Null



Channel: Urban microcell (delay spread 36ns)



- ◆ Use larger δ_0 , e.g., $\delta_0 = 0.1$, in non-overlapped band to get same null quality as brute force method.
- ◆ In overlapped band, smaller clusters (hence $\delta_0 = 0.01$) are required to achieve comparable performance as brute force method.



- Smaller delay spread \Rightarrow more bins per cluster \Rightarrow reduced # EVD computations
- $\delta_0 = 0.1$ for non-overlapped bins reduces complexity up to 1/10.
- $\delta_0 = 0.01$ for overlapped bins will reduce complexity up to 1/3.

- ◆ **Proposed frequency clustering for wideband null steering**
 - Does not require prior knowledge of channels or training.
 - Reduces complexity of wideband null space estimation
 - Number of EVDs reduced by $1/3$ to $1/10$ as compared to brute force.
 - Number of computations depend on embedded correlation in the channels.

Thank you very much!
Questions?

