Iterative Optimization for Max-Min SINR in Dense Small-Cell Multiuser MISO SWIPT System

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15 December 2015

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Outline

- Background and Motivation
- System Model & Problem Formulation
- Proposed Solution
- Numerical Results

Motivation

- **Dense small-cell deployment** has been identified as one of the 'big pillars' to support the much needed $1,000\times$ increase in data throughput for the 5G wireless networks

- While there is a major concern with the energy consumption of such a dense small-cell deployment, recent advances in wireless power transfer allow the emitted energy in the radio frequency (RF) signals to be harvested and recycled.

- The **simultaneous wireless information and power transfer (SWIPT)** from a BS to its UEs is viable in a dense small-cell environment because of the close BS-UE proximity.

Motivation (contd.)

- In such multicell network with SWIPT, the joint design of transmit beamformers at the base stations (BSs) and receive power splitting (PS) ratios at the users (UEs) is a nonconvex challenging problem.
- The semidefinite programming relaxation (SDR) may even fail to locate a feasible solution due to inevitable rank-one matrix constraints.
- We have therefore, proposed a new iterative optimization approach that offers **maximized minimum SINR** among all UEs.

System Model

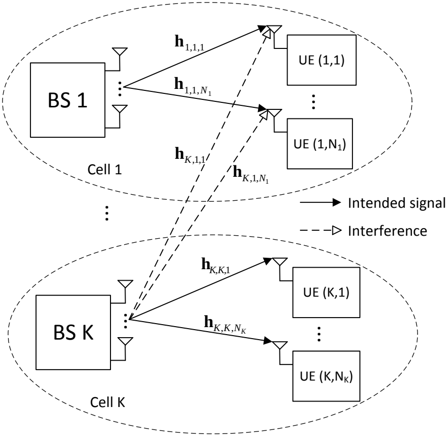


Figure: Downlink multiuser multicell interference scenario in a dense network consisting of K small cells and N_k single-antenna UEs in cell k.

Received Signal

$$y_{k,n} = \mathbf{h}_{k,k,n}^H \mathbf{w}_{k,n} x_{k,n} + \mathbf{h}_{k,k,n}^H \sum_{\bar{n} \in \mathcal{N}_k \setminus \{n\}} \mathbf{w}_{k,\bar{n}} x_{k,\bar{n}} + \sum_{\bar{k} \in \mathcal{K} \setminus \{k\}} \mathbf{h}_{\bar{k},k,n}^H \sum_{\bar{n} \in \mathcal{N}_{\bar{k}}} \mathbf{w}_{\bar{k},\bar{n}} x_{\bar{k},\bar{n}} + z_{k,n}^a$$

- The first term in is the intended signal for UE (n,k), the second term is the intracell interference from within cell k, and the third term is the intercell interference from other cells $\bar{k} \in \mathcal{K} \setminus \{k\}$.
- $\mathbf{w}_{k,n} \in \mathbb{C}^{M \times 1}$ is the beamforming vector by BS $k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ for its UE (k,n).
- By BS k and UE (k,n), we mean the BS that serves cell k and the UE $n \in \mathcal{N}_k \triangleq \{1, \ldots, N_k\}$ of the same cell, respectively.

Power Splitting

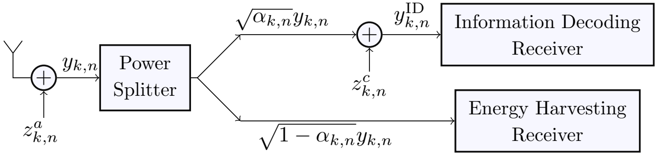
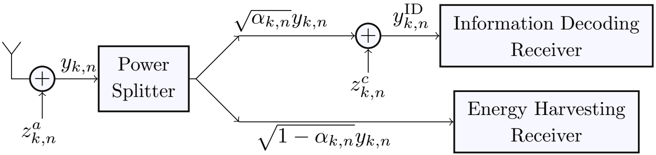


Figure: PS-based receiver structure at UE (k, n).

- The power splitter (PS) divides the received signal $y_{k,n}$ into two parts in the proportion of $\alpha_{k,n} : 1 - \alpha_{k,n}$, where $\alpha_{k,n} \in (0, 1)$ is termed as the PS ratio for UE (k, n).
- The first part $\sqrt{\alpha_{k,n}}y_{k,n}$ forms an input to the information decoding (ID) receiver. The second part $\sqrt{1-\alpha_{k,n}}y_{k,n}$ of the received signal is processed by an energy harvesting (EH) receiver.

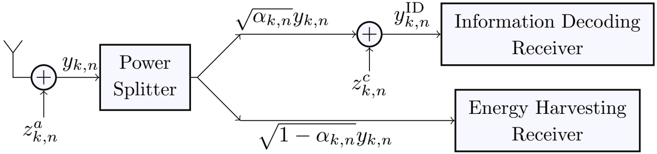
SINR at ID receiver



$$\mathsf{SINR}_{k,n} = f_{k,n}(\mathbf{w}, \alpha_{k,n})$$

$$\triangleq \frac{\alpha_{k,n}|\mathbf{h}^H_{k,k,n}\mathbf{w}_{k,n}|^2}{\underbrace{\alpha_{k,n}\sum_{\bar{n}\in\mathcal{N}_k\setminus\{n\}}|\mathbf{h}^H_{k,k,n}\mathbf{w}_{k,\bar{n}}|^2}_{\text{intracell interference}} + \underbrace{\alpha_{k,n}\sum_{\bar{k}\in\mathcal{K}\setminus\{k\}}\sum_{\bar{n}\in\mathcal{N}_{\bar{k}}}|\mathbf{h}^H_{\bar{k},k,n}\mathbf{w}_{\bar{k},\bar{n}}|^2}_{\text{intercell interference}} + \alpha_{k,n}\sigma_a^2 + \sigma_c^2}. \quad (1)$$

Harvested Energy by EH Receiver



$$E_{k,n} \triangleq \zeta_{k,n}(1-\alpha_{k,n})\left(\sum_{\bar{k}\in\mathcal{K}}\sum_{\bar{n}\in\mathcal{N}_{\bar{k}}}|\mathbf{h}^{H}_{\bar{k},k,n}\mathbf{w}_{\bar{k},\bar{n}}|^2+\sigma_a^2\right) \qquad (2)$$

◇ $\zeta_{k,n} \in (0,1)$ is the energy harvesting efficiency.

Max-Min SINR Problem

$$\max_{\substack{\mathbf{w}_{k,n}\in\mathbb{C}^{M\times 1},\\ \alpha_{k,n}\in(0,1),\\ \forall\ k\in\mathcal{K},\ n\in\mathcal{N}_k}} F(\mathbf{w},\boldsymbol{\alpha}) \triangleq \min_{k\in\mathcal{K},n\in\mathcal{N}_k} f_{k,n}(\mathbf{w},\alpha_{k,n}) \quad (3a)$$

$$\text{s.t.} \quad \sum_{n\in\mathcal{N}_k} \|\mathbf{w}_{k,n}\|^2 \le P_k^{\max},\ \forall k\in\mathcal{K} \quad (3b)$$

$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k} \|\mathbf{w}_{k,n}\|^2 \le P^{\max} \quad (3c)$$

$$E_{k,n}(\mathbf{w},\alpha_{k,n}) \ge e_{k,n}^{\min},\ \forall k\in\mathcal{K}, n\in\mathcal{N}_k, \quad (3d)$$

- $P_k^{\max}$ is the transmit power budget of each BS k. $P^{\max}$ is the budget for total transmit power of the network.
- $e_{k,n}^{\min}$ is the target harvested energy.
- (3) is a nonconvex nonsmooth (due to minimization operator) optimization function subject to nonconvex constraint (3d).

Semidefinite Programming (SDP)

By defining $\mathbf{W}_{k,n} \triangleq \mathbf{w}_{k,n}\mathbf{w}_{k,n}^H \succcurlyeq \mathbf{0}$ and $\mathbf{H}_{k,k,n} \triangleq \mathbf{h}_{k,k,n}\mathbf{h}_{k,k,n}^H$,

$$\max_{\substack{\mathbf{w}_{k,n}\in\mathbb{C}^{M\times M}\\ \alpha_{k,n}\in(0,1),\gamma}} \quad \gamma \tag{4a}$$

$$\text{s.t.} \quad \frac{1}{\gamma}\text{Tr}\{\mathbf{H}_{k,k,n}\mathbf{W}_{k,n}\} - \sum_{\bar{k}\in\mathcal{K}\setminus\{k\}}\sum_{\bar{n}\in\mathcal{N}_{\bar{k}}}\text{Tr}\{\mathbf{H}_{\bar{k},k,n}\mathbf{W}_{\bar{k},\bar{n}}\} - \sum_{\bar{n}\in\mathcal{N}_k\setminus\{n\}}\text{Tr}\{\mathbf{H}_{k,k,n}\mathbf{W}_{k,\bar{n}}\} \geq \sigma_a^2 + \frac{\sigma_c^2}{\alpha_{k,n}}, \tag{4b}$$

$$\sum_{n\in\mathcal{N}_k}\text{Tr}\{\mathbf{W}_{k,n}\} \leq P_k^{\max}, \ \forall k\in\mathcal{K} \tag{4c}$$

$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k}\text{Tr}\{\mathbf{W}_{k,n}\} \leq P^{\max} \tag{4d}$$

$$\sum_{\bar{k}\in\mathcal{K}}\sum_{\bar{n}\in\mathcal{N}_{\bar{k}}}\text{Tr}\{\mathbf{H}_{\bar{k},k,n}\mathbf{W}_{\bar{k},\bar{n}}\} \geq \frac{e_{k,n}^{\min}}{\zeta_{k,n}(1-\alpha_{k,n})} - \sigma_a^2, \ \forall k,n \tag{4e}$$

$$\mathbf{W}_{k,n} \succcurlyeq \mathbf{0}, \ \forall k\in\mathcal{K}, n\in\mathcal{N}_k \tag{4f}$$

$$\text{rank}(\mathbf{W}_{k,n}) = 1, \ \forall k\in\mathcal{K}, n\in\mathcal{N}_k. \tag{4g}$$

Semidefinite Relaxation (SDR)

- By fixing γ and further *ignoring* the difficult rank-one constraint (4g), (4) becomes a feasibility (convex) semidefinite relaxation (SDR) (4b)–(4f).
- The optimal value of γ can be found via a bisection search.
- If $\text{rank}(\mathbf{W}^{\star}_{k,n}) = 1,\ \forall k \in \mathcal{K}, n \in \mathcal{N}_k$, the rank-one constraint (4g) is automatically satisfied.
- **Problem:** $\text{rank}(\mathbf{W}^{\star}_{k,n}) > 1$ for some (k, n) in more than 38% of the time. Thus, solving SDR is not adequate to recover optimal beamforming vectors. Only provides upper bound.
- **Existing Approach:** Randomization *, however, the generated solutions are not guaranteed to be even close to the actual optimum of problem

* N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," IEEE Trans. Signal Process., vol. 54, no. 6, pp. 2239Ű2251, Jun. 2006.

Dealing with Rank-1 Constraints

- Denoting $\lambda_{\max}\{\cdot\}$ as a maximum eigenvalue of a matrix, we can replace the rank-one matrix constraints (4g) by a single reverse convex constraint.

$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k}\left[\mathrm{Tr}\{\mathbf{W}_{k,n}\}-\lambda_{\max}\{\mathbf{W}_{k,n}\}\right]\le 0,\ \forall k,n. \quad (5)$$

- If (5) holds then $\mathrm{Tr}\{\mathbf{W}_{k,n}\}-\lambda_{\max}\{\mathbf{W}_{k,n}\}=0$ for all $k\in\mathcal{K}$ and $n\in\mathcal{N}_k$, which means that each $\mathbf{W}_{k,n}$ has exactly one nonzero eigenvalue.
- (5) is a reverse convex constraint because the function $\lambda_{\max}\{\cdot\}$ is convex on the set of Hermitian matrices.

Dealing with Rank-1 Constraints (contd.)

- Our aim is thus to make $\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k}[\mathrm{Tr}\{\mathbf{W}_{k,n}\}-\lambda_{\max}\{\mathbf{W}_{k,n}\}]$ as small as possible.
- To this end, we incorporate the reverse convex constraint (5) into the objective as a penalty function.

$$\min_{\substack{\mathbf{W}_{k,n}\in\mathbb{C}^{M\times M}\\ \alpha_{k,n}\in(0,1)}} \quad \tilde{F}(\mathbf{W}) \triangleq \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k}\mathrm{Tr}\{\mathbf{W}_{k,n}\}-\lambda_{\max}\{\mathbf{W}_{k,n}\}$$

$$\text{s.t.} \quad (4b)-(4f). \tag{6}$$

Dealing with Rank-1 Constraints (contd.)

- Since the subgradient of $\lambda_{\max}\{\mathbf{W}_{k,n}\}$ is $\mathbf{w}_{k,n}^{\max}(\mathbf{w}_{k,n}^{\max})^{H\dagger}$, we have

$$\lambda_{\max}\{\mathbf{X}_{k,n}\} \geq \lambda_{\max}\{\mathbf{W}_{k,n}\} + (\mathbf{w}_{k,n}^{\max})^{H}(\mathbf{X}_{k,n} - \mathbf{W}_{k,n})\mathbf{w}_{k,n}^{\max}, \ \forall k, n \tag{7}$$

- for any $\mathbf{X}_{k,n} \succcurlyeq \mathbf{0}$.
- $\mathbf{w}_{k,n}^{\max}$ is the unit-norm eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}\{\mathbf{W}_{k,n}\}$.

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†H. D. Tuan, P. Apkarian, S. Hosoe, and H. Tuy, "D.C. optimization approach to robust control: Feasibility problems," Int. J. Contr, vol. 73, no. 2, pp. 89Ű-104, Feb. 2000.

Dealing with Rank-1 Constraints (contd.)

Given some feasible $\mathbf{W}_{k,n}^{(\kappa)}$ of (6) at iteration κ with the corresponding maximum eigenvalue $\lambda_{\max}\{\mathbf{W}_{k,n}^{(\kappa)}\}$ and unit-norm eigenvector $\mathbf{w}_{k,n}^{\max,(\kappa)}$,

$$\begin{aligned}\tilde{F}^{(\kappa)}(\mathbf{W}) \triangleq \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k} & \mathrm{Tr}\{\mathbf{W}_{k,n}\} - \lambda_{\max}\{\mathbf{W}_{k,n}^{(\kappa)}\} \\ & - (\mathbf{w}_{k,n}^{\max,(\kappa)})^H(\mathbf{W}_{k,n} - \mathbf{W}_{k,n}^{(\kappa)})\mathbf{w}_{k,n}^{\max,(\kappa)} \\ & \geq F(\mathbf{W}),\ \forall \mathbf{W}\end{aligned} \tag{8}$$

Thus, the following SDP

$$\min_{\substack{\mathbf{W}_{k,n}\in\mathbb{C}^{M\times M}\\ \alpha_{k,n}\in(0,1)}} \tilde{F}^{(\kappa)}(\mathbf{W}) \quad \text{s.t.} \quad (4b)-(4f). \tag{9}$$

is a convex majorant minimization of the nonconvex program (6).

Dealing with Rank-1 Constraints (contd.)

Given some feasible $\mathbf{W}_{k,n}^{(\kappa)}$ of (6) at iteration κ with the corresponding maximum eigenvalue $\lambda_{\max}\{\mathbf{W}_{k,n}^{(\kappa)}\}$ and unit-norm eigenvector $\mathbf{w}_{k,n}^{\max,(\kappa)}$,

$$\begin{aligned}\tilde{F}^{(\kappa)}(\mathbf{W}) &\triangleq \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k} \mathrm{Tr}\{\mathbf{W}_{k,n}\} - \lambda_{\max}\{\mathbf{W}_{k,n}^{(\kappa)}\} \\ &- (\mathbf{w}_{k,n}^{\max,(\kappa)})^H(\mathbf{W}_{k,n} - \mathbf{W}_{k,n}^{(\kappa)})\mathbf{w}_{k,n}^{\max,(\kappa)} \\ &\geq F(\mathbf{W}),\ \forall \mathbf{W}\end{aligned} \tag{10}$$

Thus, the following SDP

$$\min_{\substack{\mathbf{W}_{k,n}\in\mathbb{C}^{M\times M}\\ \alpha_{k,n}\in(0,1)}} \tilde{F}^{(\kappa)}(\mathbf{W}) \quad \text{s.t.} \quad (4b)-(4f). \tag{11}$$

is a convex majorant minimization of the nonconvex program (6).

Dealing with Rank-1 Constraints (contd.)

Program (11) can be further simplified to:

$$\min_{\substack{\mathbf{W}_{k,n}\in\mathbb{C}^{M\times M}\\ \alpha_{k,n}\in(0,1)}} \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}_k} \mathrm{Tr}\{\mathbf{W}_{k,n}\} - (\mathbf{w}_{k,n}^{\max,(\kappa)})^H \mathbf{W}_{k,n}\mathbf{w}_{k,n}^{\max,(\kappa)}$$

$$\text{s.t.} \quad (4b)-(4f). \qquad (12)$$

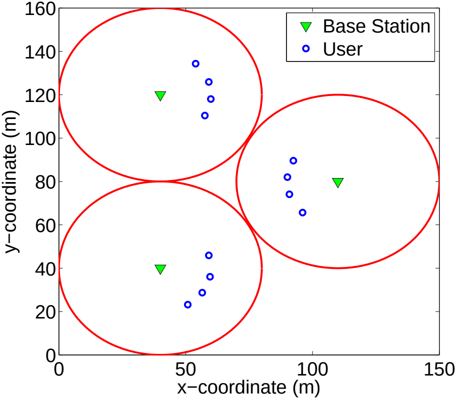
With (12), we then propose to use a bisection search in an outer loop to find the optimal value of γ.

Proposed Algorithm

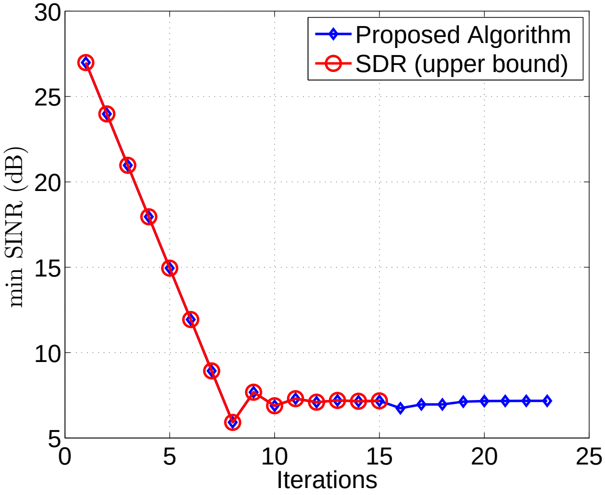
- We choose the initial solution $(\mathbf{W}_{k,n}^{(0)}, \alpha_{k,n}^{(0)})$ as the optimal solution $(\mathbf{W}_{k,n}^{\star}, \alpha_{k,n}^{\star})$ of SDR (4a)-(4f).
- The Optimization stage ensures a rank-one solution. In the Optimization stage, the inner loop optimizes $\mathbf{W}_{k,n}, \alpha_{k,n}, \ \forall k \in \mathcal{K}, n \in \mathcal{N}_k$ for a given value of γ by solving exactly one convex SDP (12) in each iteration. The inner loop terminates at the convergence of the objective function in (12) or equivalently $\tilde{F}(\mathbf{W})$ (maximum of 2 iterations required).
- Once $\tilde{F}(\mathbf{W})$ converges, we determine the rank of the optimized beamforming matrices $\mathbf{W}_{k,n}^{(\kappa)}$. If $\text{Tr}\{\mathbf{W}_{k,n}^{(\kappa)}\} \approx \lambda_{\text{max}}\{\mathbf{W}_{k,n}^{(\kappa)}\}$, i.e., $\text{rank}(\mathbf{W}_{k,n}^{(\kappa)}) = 1, \ \forall k \in \mathcal{K}, n \in \mathcal{N}_k$, we update $\gamma_{\text{lo}} := \gamma$, and otherwise we set $\gamma_{\text{hi}} := \gamma$. The outer loop optimizes γ via a simple bisection search.

Multicell Network Topology

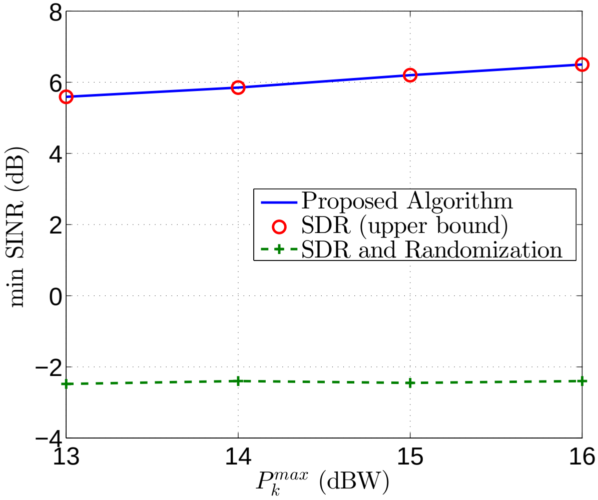
◇ Cell radius = 40 m, BS-UE distance = 20 m, $\zeta = 0.5$, $P^{\max} = 22$ dBW, $P_k^{\max} = 16$ dBW, $e^{\min} = -20$ dBm, $\delta = 1$, $\sigma_a^2 = \sigma_c^2 = -90$ dBm, Rician fading channel with Rician factor = 10 dB



Convergence of the Proposed Algorithm



Comparison with Randomization and Upper Bound SDR



Findings

- We observe that solving an SDR fails to deliver a rank-one solution in 38.3% of the time on average while the proposed Algorithm 1 always deliver a rank-one solution. In our simulations, we establish that a matrix is only of rank one if the magnitude of its second largest eigenvalue is less than $\rho = 1/200$ of that of its largest eigenvalue. Since this criterion is much more relaxed than conventionally where ρ is much smaller, it ensures that a rank-one matrix is not mistaken.
- The optimal solution provided by our SDP-based spectral optimization achieves the theoretical bound.