# Deriving 3D Shape Properties by Using Backward Wavelet Remesher

Hao-Chiang Shao and Wen-Liang Hwang

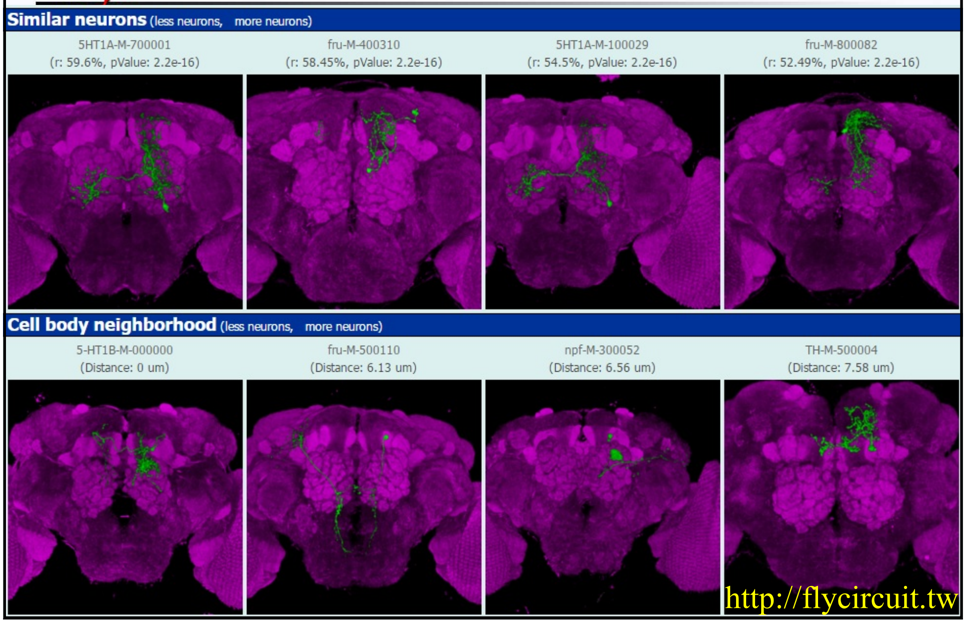
Inst. Information Science, Academia Sinica, Taiwan

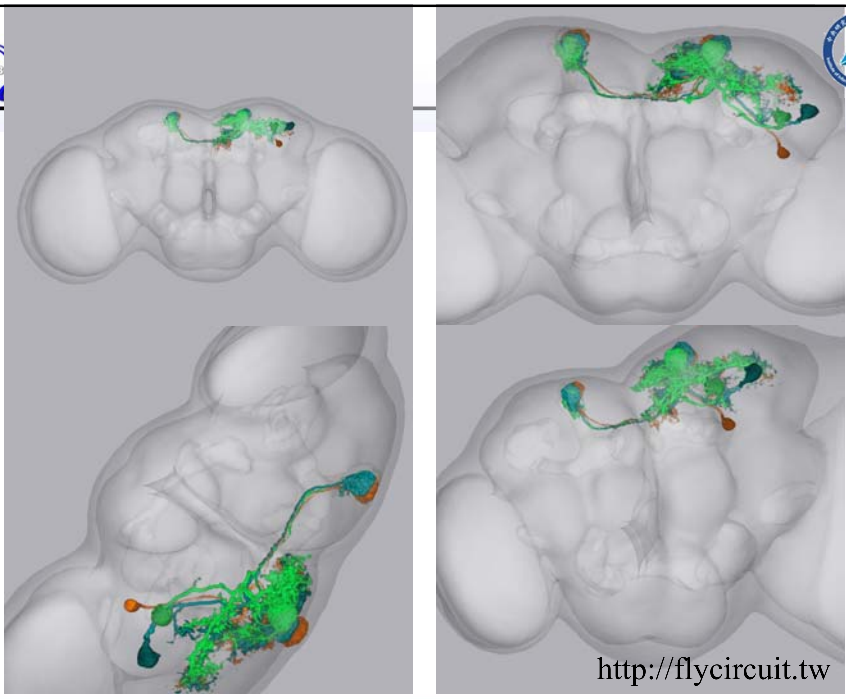
mailto: shao.haochiang@gmail.com

1

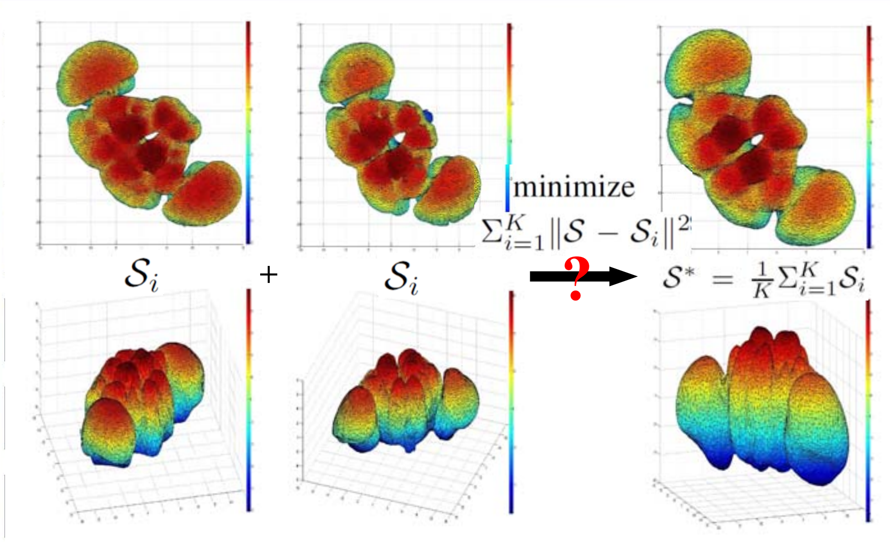
2017/11/14

## Motivation: Brain Image Atlasing





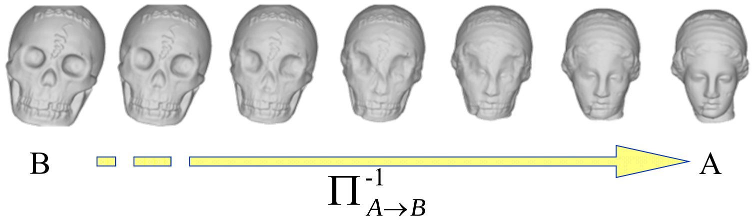

## Motivation: How to derive a 3D average?



## Motivation:

1. How to derive a 3D standard average?
2. How to construct a 2D+Z deformation field?
3. How to define 3D boundary condition?

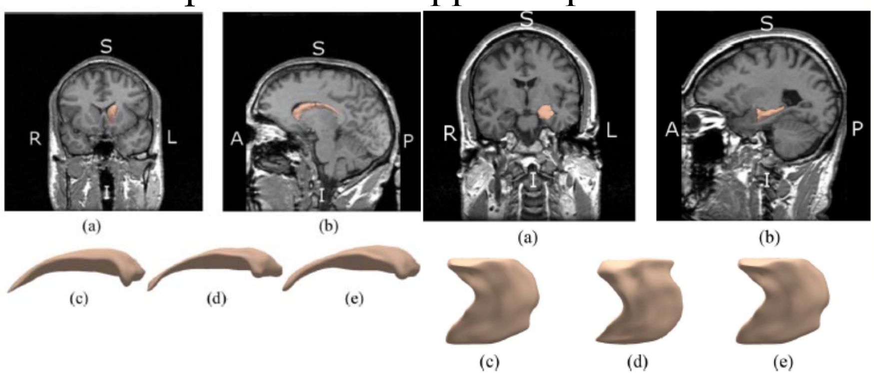
**➔ Analyze 3D shape properties first.**





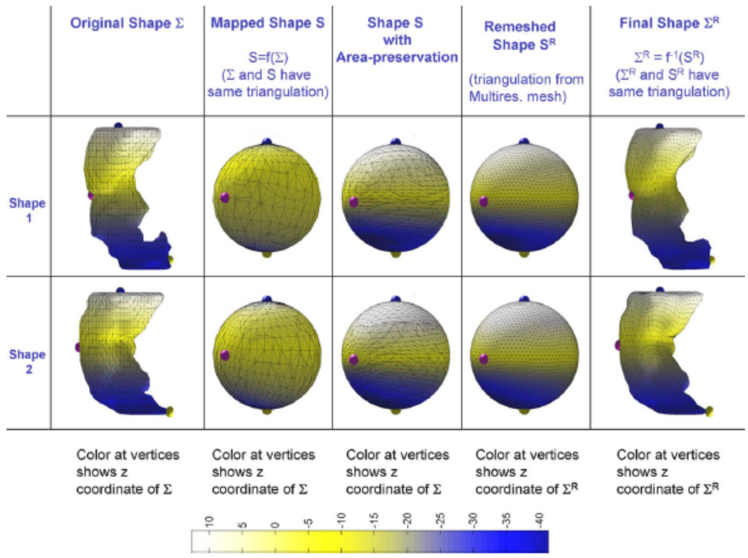

## 3D Shape Analysis:
## An Example (Nain et al.)

- Schizophrenia VS Hippocampus



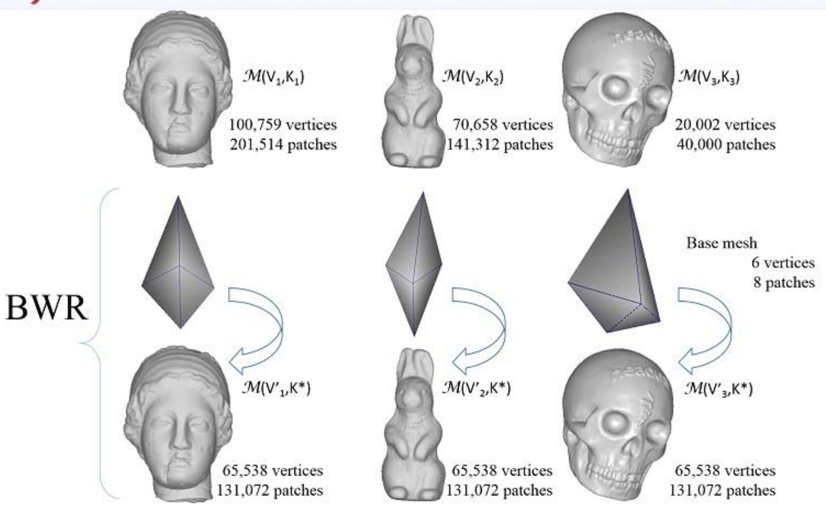

Nain et al., "Multiscale 3-d shape representation and segmentation using spherical wavelets", *IEEE Trans. Med. Imaging,* pp.598-618, vol.26(4), 2007.

## 3D Shape Analysis: An Example (Nain et al.)




7

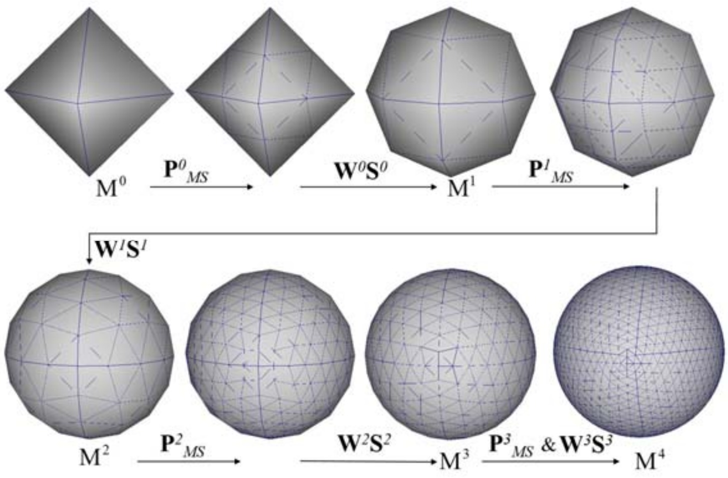
## Our Design Goal & Our Prototype




- Shao et al., "A backward wavelet remesher for level of detail control and scalable coding", IEEE ICIP 2014.

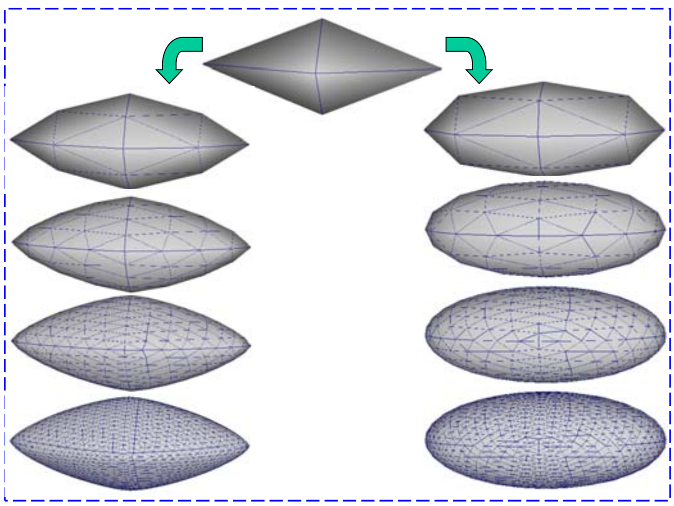
8

## Concept:
Coarse-to-fine (backward) wavelet representation



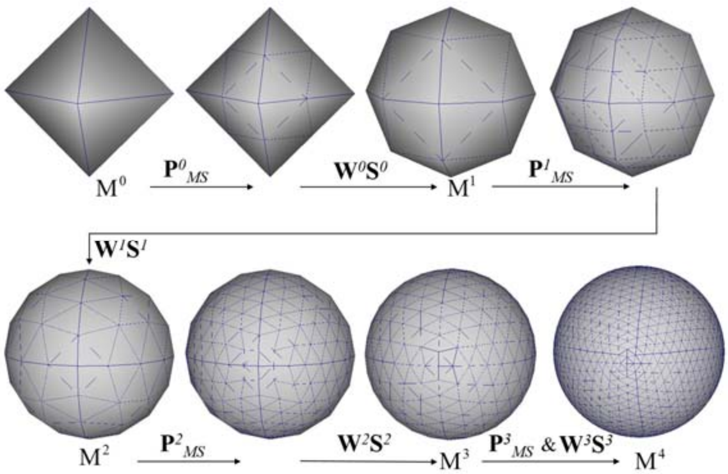

9

## Concept:
Coarse-to-fine (backward) wavelet representation



# Concept:

## Coarse-to-fine (backward) wavelet representation






# Coarse-to-fine (backward) wavelet representation

- 1D DWT

$$\begin{bmatrix} L_x \\ \hline H_x \end{bmatrix} = \begin{bmatrix} A \\ \hline B \end{bmatrix}[x]$$

$$\frac{1}{\sqrt{2}}\cdot\begin{bmatrix}1.5\\3.5\\5.5\\7.5\\9.5\\ \hline -0.5\\-0.5\\-0.5\\-0.5\\-0.5\end{bmatrix}_{10\times1} = \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} &&&&&&&& \\
&& \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} &&&&&& \\
&&&& \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} &&&& \\
&&&&&& \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} && \\
&&&&&&&& \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \hline
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} &&&&&&&& \\
&& \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} &&&&&& \\
&&&& \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} &&&& \\
&&&&&& \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} && \\
&&&&&&&& \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix}_{10\times10} \begin{bmatrix}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\end{bmatrix}_{10\times1}$$

$$\begin{bmatrix} A \\ \hline B \end{bmatrix}^{-1}\begin{bmatrix} L_x \\ \hline H_x \end{bmatrix} = [x]$$

$$= [P \quad Q]\begin{bmatrix} L_x \\ \hline H_x \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\cdot\begin{bmatrix}
\sqrt{2} &&&&& \sqrt{2} &&&& \\
\sqrt{2} &&&&& -\sqrt{2} &&&& \\
& \sqrt{2} &&&&& \sqrt{2} &&& \\
& \sqrt{2} &&&&& -\sqrt{2} &&& \\
&& \sqrt{2} &&&&& \sqrt{2} && \\
&& \sqrt{2} &&&&& -\sqrt{2} && \\
&&& \sqrt{2} &&&&& \sqrt{2} & \\
&&& \sqrt{2} &&&&& -\sqrt{2} & \\
&&&& \sqrt{2} &&&&& \sqrt{2} \\
&&&& \sqrt{2} &&&&& -\sqrt{2}
\end{bmatrix}_{10\times10}\begin{bmatrix}1.5\\3.5\\5.5\\7.5\\9.5\\ \hline -0.5\\-0.5\\-0.5\\-0.5\\-0.5\end{bmatrix}_{10\times1} = \begin{bmatrix}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\end{bmatrix}_{10\times1}$$

## Coarse-to-fine (backward) wavelet representation

- Forward VS inverse transform

$$\begin{bmatrix} L_x \\ H_x \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}[x] \quad \Longleftrightarrow \quad \begin{bmatrix} A \\ B \end{bmatrix}^{-1}\begin{bmatrix} L_x \\ H_x \end{bmatrix} = [P \quad Q]\begin{bmatrix} L_x \\ H_x \end{bmatrix} = [x]$$

- Example:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}
\sqrt{2} & & & & & \sqrt{2} & & & & \\
\sqrt{2} & & & & & -\sqrt{2} & & & & \\
 & \sqrt{2} & & & & & \sqrt{2} & & & \\
 & \sqrt{2} & & & & & -\sqrt{2} & & & \\
 & & \sqrt{2} & & & & & \sqrt{2} & & \\
 & & \sqrt{2} & & & & & -\sqrt{2} & & \\
 & & & \sqrt{2} & & & & & \sqrt{2} & \\
 & & & \sqrt{2} & & & & & -\sqrt{2} & \\
 & & & & \sqrt{2} & & & & & \sqrt{2} \\
 & & & & \sqrt{2} & & & & & -\sqrt{2}
\end{bmatrix}_{10\times10}
\begin{bmatrix} 1.5 \\ 3.5 \\ 5.5 \\ 7.5 \\ 9.5 \\ -0.5 \\ -0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}_{10\times1}
=
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix}_{10\times1}$$

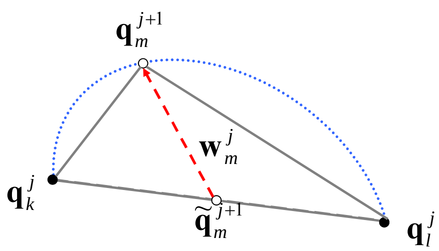
$$[x] = [P \quad Q]\begin{bmatrix} L_x \\ H_x \end{bmatrix} = \begin{bmatrix} P_{upper} & Q_{upper} \\ P_{lower} & Q_{lower} \end{bmatrix}\begin{bmatrix} L_x \\ H_x \end{bmatrix}$$

## Multiresolution Analysis for Surfaces

- Displacement Vector vs. Lazy Wavelet Structure

$$\mathbf{q}_m^{j+1} = \tilde{\mathbf{q}}_m^{j+1} + \mathbf{w}_m^j$$

$$\mathbf{c}^{j+1} = \begin{bmatrix} \mathbf{P}^j & \mathbf{Q}^j \end{bmatrix}\begin{bmatrix} \mathbf{c}^j \\ \mathbf{d}^j \end{bmatrix} = \mathbf{P}^j\mathbf{c}^j + \mathbf{Q}^j\mathbf{d}^j$$




M. Lounsbery, T. DeRose, and J. Warren, **"Multiresolution analysis for surfaces of arbitrary topological type"**, ACM Transactions on Graphics, 16 (1997), pp. 34-73.

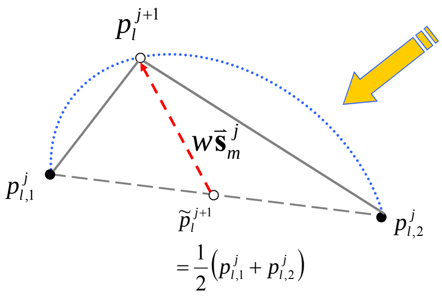
## Multiresolution Analysis for Surfaces

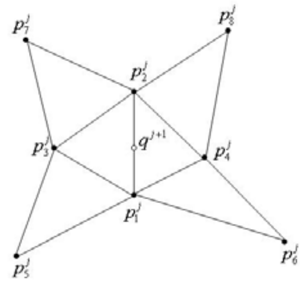
- What will happen if butterfly subdivision scheme is adopted?

$$\mathbf{q}_m^{j+1} = \tilde{\mathbf{q}}_m^{j+1} + \mathbf{w}_m^j$$

$$p_l^{j+1} = \frac{1}{2}\left(p_{l,1}^j + p_{l,2}^j\right) + w\bar{\mathbf{s}}_l$$

$$\bar{s}_l = 2\left(p_{l,3}^j + p_{l,4}^j\right) - \left(p_{l,5}^j + p_{l,6}^j + p_{l,7}^j + p_{l,8}^j\right)$$




N. Dyn, D. Levine, and J. A. Gregory, "A butterfly subdivision scheme for surface interpolation with tension control," ACM transactions on Graphics (TOG), vol. 9, no. 2, pp. 160–169, 1990.



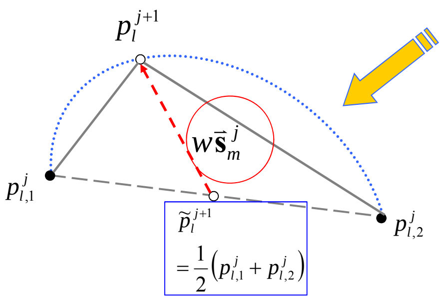
## Multiresolution Analysis for Surfaces

- What will happen if butterfly subdivision scheme is adopted?

$$\mathbf{q}_m^{j+1} = \tilde{\mathbf{q}}_m^{j+1} + \mathbf{w}_m^j$$

$$p_l^{j+1} = \frac{1}{2}\left(p_{l,1}^j + p_{l,2}^j\right) + w\bar{\mathbf{s}}_l$$

$$\bar{s}_l = 2\left(p_{l,3}^j + p_{l,4}^j\right) - \left(p_{l,5}^j + p_{l,6}^j + p_{l,7}^j + p_{l,8}^j\right)$$




$$\mathbf{c}^{j+1} = \begin{bmatrix}\mathbf{P}^j & \mathbf{Q}^j\end{bmatrix}\begin{bmatrix}\mathbf{c}^j \\ \mathbf{d}^j\end{bmatrix}$$

$$= \mathbf{P}^j\mathbf{c}^j + \mathbf{Q}^j\mathbf{d}^j$$

N. Dyn, D. Levine, and J. A. Gregory, "A butterfly subdivision scheme for surface interpolation with tension control," ACM transactions on Graphics (TOG), vol. 9, no. 2, pp. 160–169, 1990.

## Coarse-to-fine (backward) wavelet representation

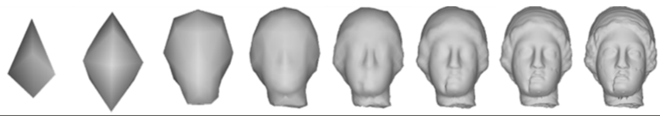
$$[x]=[P \quad Q]\begin{bmatrix} L_x \\ H_x \end{bmatrix} = \begin{bmatrix} P_{upper} & Q_{upper} \\ P_{lower} & Q_{lower} \end{bmatrix}\begin{bmatrix} L_x \\ H_x \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{c}_n^{j+1} \\ \mathbf{c}_m^{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_n^j & \mathbf{0}_{n\times m} \\ \mathbf{P}_m^j & \mathbf{I}_m \end{bmatrix}\begin{bmatrix} \mathbf{c}_n^j \\ \mathbf{d}_m^j \end{bmatrix} \quad \text{(lazy wavelet)}$$

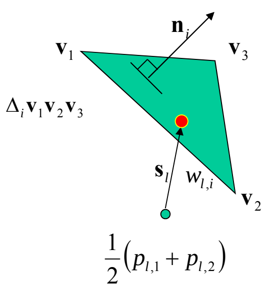
$$\begin{bmatrix} \mathbf{c}_n^{j+1} \\ \mathbf{c}_m^{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_n^j \\ \mathbf{P}_m^j \end{bmatrix}\mathbf{c}_n^j + \begin{bmatrix} \mathbf{0}_{n\times 3} \\ \mathbf{d}_m^j \end{bmatrix}$$

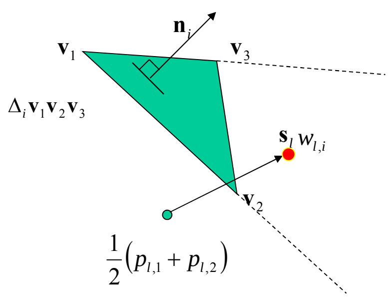
$$\begin{bmatrix} \mathbf{c}_n^{j+1} \\ \mathbf{c}_m^{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_n \\ \mathbf{P}_m^j \end{bmatrix}\mathbf{c}_n^j + \begin{bmatrix} \mathbf{0}_{n\times 3} \\ \mathbf{d}_m^j \end{bmatrix} \quad \text{(interpolating subdivision)}$$

$$\begin{bmatrix} \mathbf{c}_n^{j+1} \\ \mathbf{c}_m^{j+1} \end{bmatrix} = \mathbf{P}_{mid}^j\mathbf{c}_n^j + \begin{bmatrix} \mathbf{0}_{n\times 3} \\ \mathbf{W}^j\mathbf{S}^j + \mathbf{d}^j \end{bmatrix}$$



## How to find "w"?



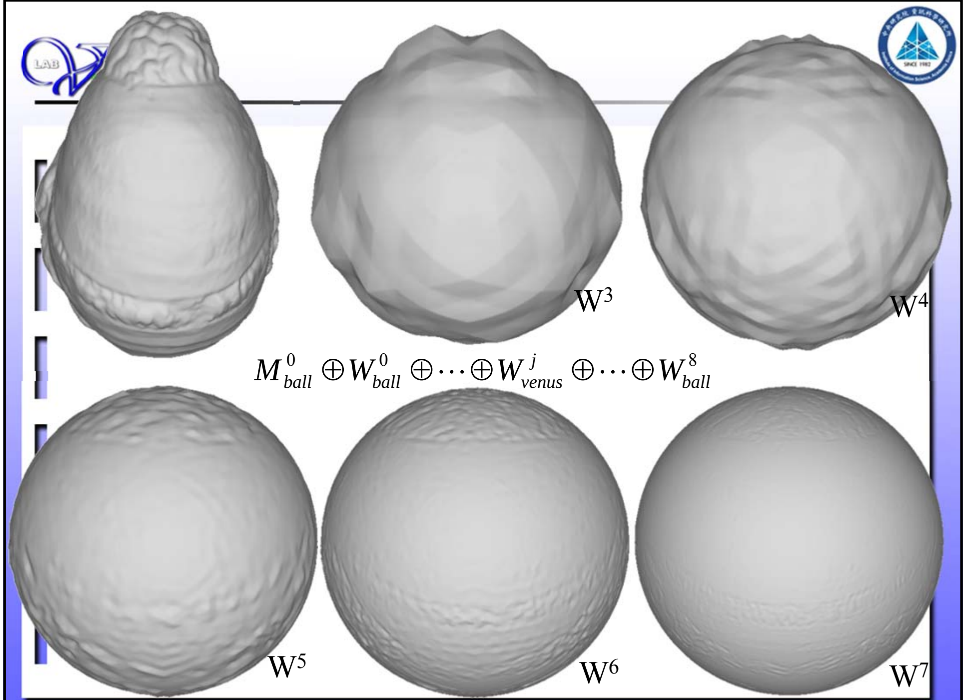

$$w_{l,i} = \frac{\langle \mathbf{n}_i, \mathbf{v}_1\rangle - \langle \mathbf{n}_i, \frac{1}{2}(p_{l,1}+p_{l,2})\rangle}{\langle \mathbf{n}_i, \mathbf{s}_l\rangle}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = (\mathbf{T}^t\mathbf{T})^{-1}\mathbf{T}^t(\mathbf{p}_l - \mathbf{v}_3), \text{ where}$$

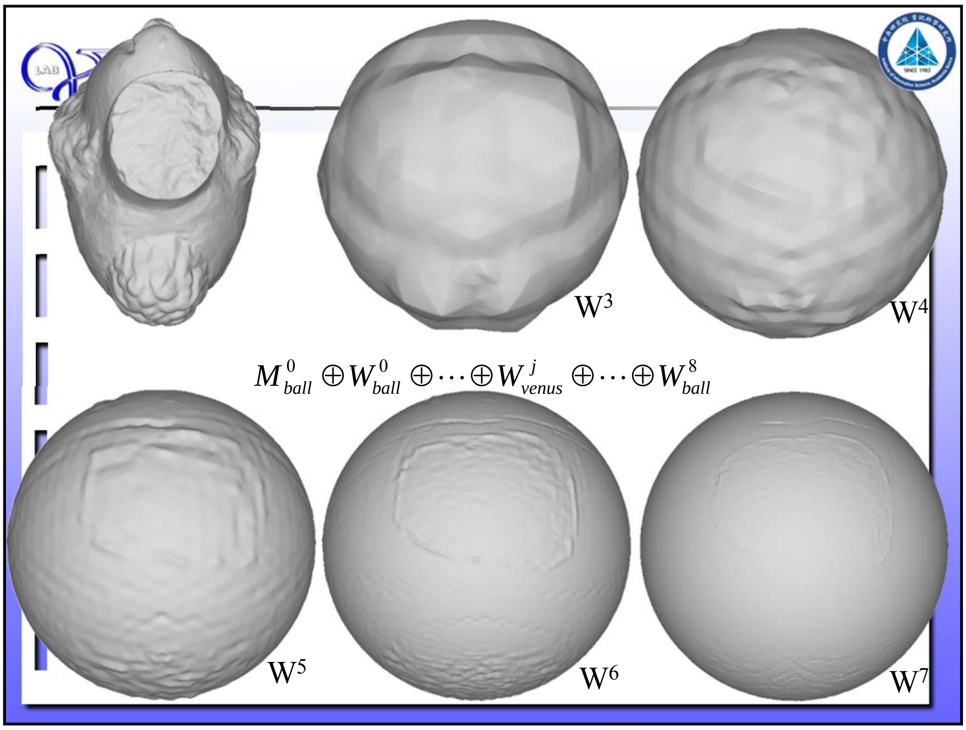
$$\mathbf{T} = \begin{bmatrix} x_1 - x_3 & x_2 - x_3 \\ y_1 - y_3 & y_2 - y_3 \\ z_1 - z_3 & z_2 - z_3 \end{bmatrix}.$$

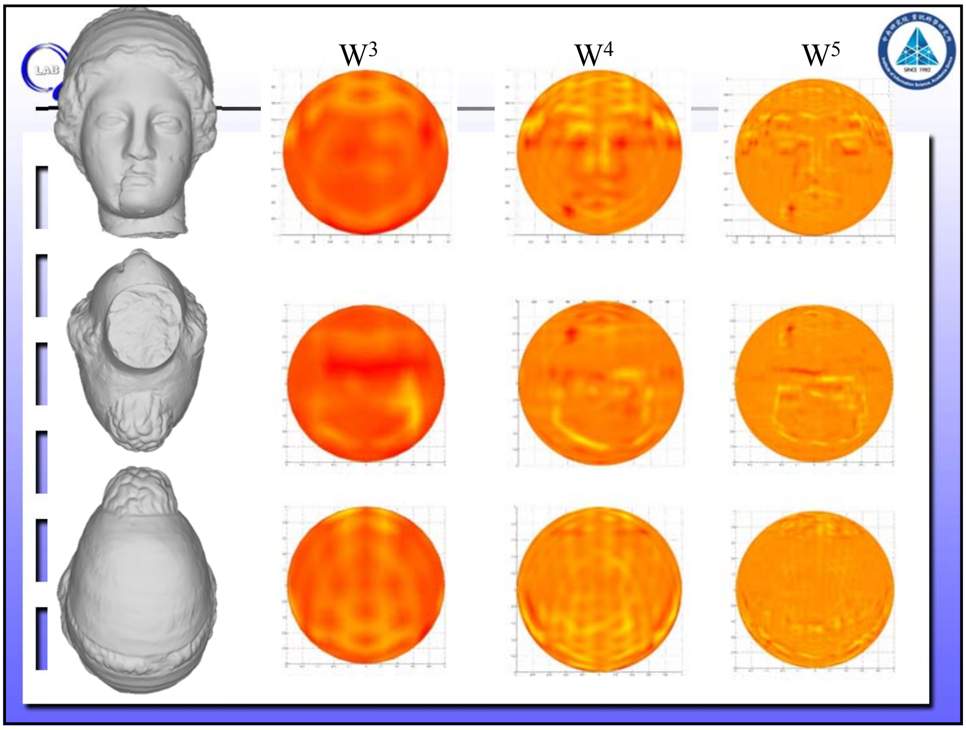
# Experiment Results

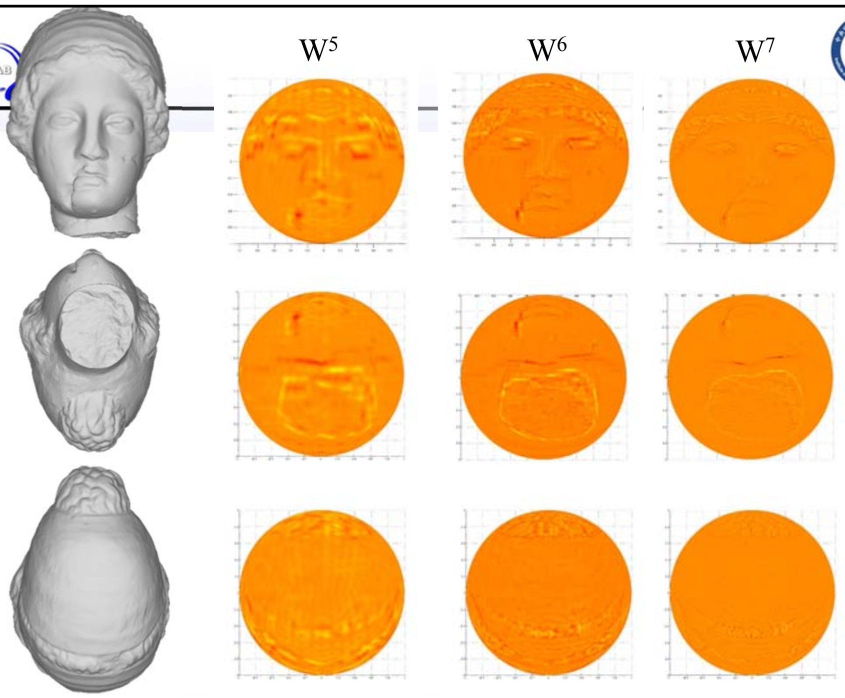
2017/11/14




$$M_{ball}^{0} \oplus W_{ball}^{0} \oplus \cdots \oplus W_{venus}^{j} \oplus \cdots \oplus W_{ball}^{8}$$

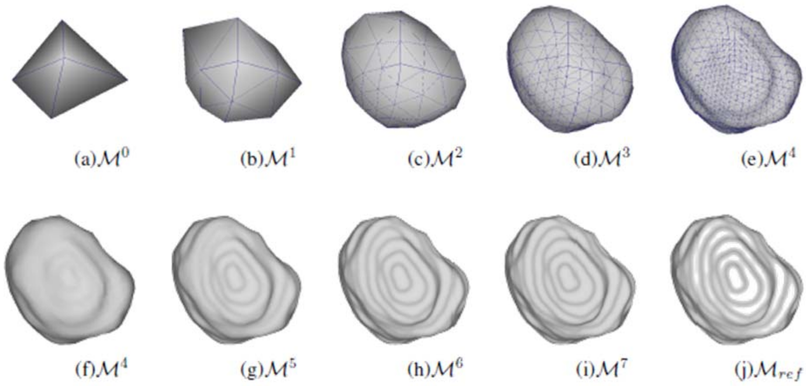

W3
W4
$M_{ball}^{0} \oplus W_{ball}^{0} \oplus \cdots \oplus W_{venus}^{j} \oplus \cdots \oplus W_{ball}^{8}$
W5
W6
W7


W3
W4
W5

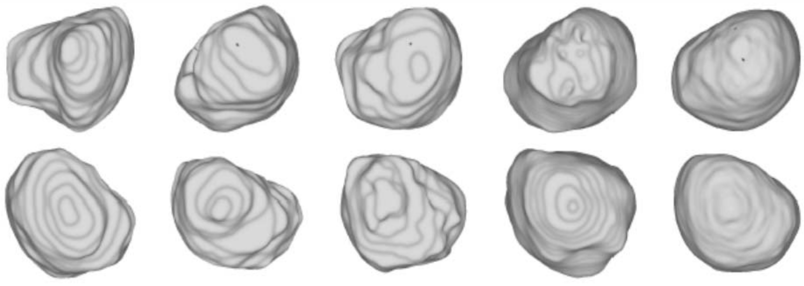



## Multiresolution analysis on Olfactory Glomerulus models

- Based on multiresolution analysis, we can **conditionally** achieve **registration**, **morphing**, and **model-averaging** in a coarse-to-fine manner.



## Multiresolution analysis on Olfactory Glomerulus models



(a)M-01 (b)M-02 (c)M-03 (d)M-04 (e)Average

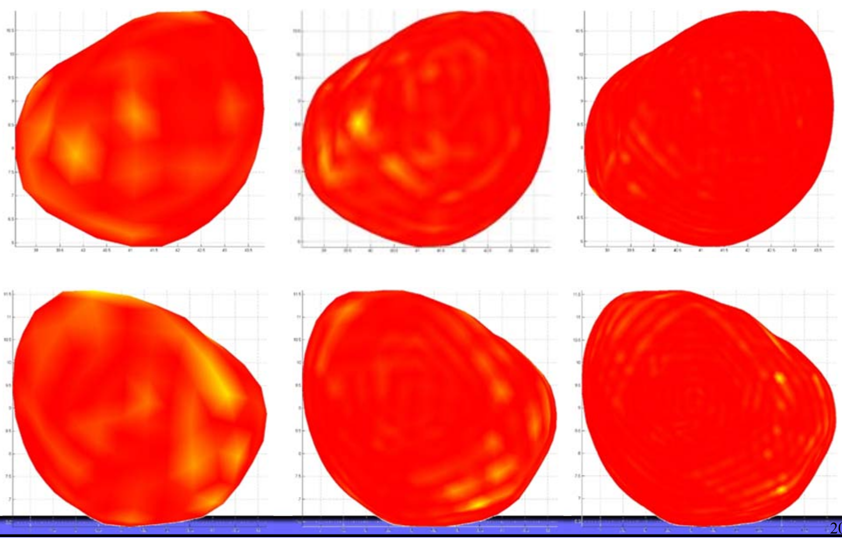
TABLE 1

| | M-01 | M-02 | M-03 | M-04 | M-Avg. |
|---|---|---|---|---|---|
| M-01 | — | 0.0907 | 0.0864 | 0.1102 | 0.0773 |
| M-02 | 0.1145 | — | 0.0796 | 0.1063 | 0.0783 |
| M-03 | 0.0801 | 0.1316 | — | 0.1033 | 0.0791 |
| M-04 | 0.1082 | 0.1192 | 0.1100 | — | 0.0740 |
| M-Avg. | 0.0767 | 0.1019 | 0.0689 | 0.0829 | — |



## Multiresolution analysis on Olfactory Glomerulus models

- Regions with high structural variability are highlighted in yellow.



## Concluding Remarks

- A backward coarse-to-fine method.
- Tiling-invariant → BWR acts as a transformation that can convert input meshes into a standard reference domain.
- Simple: remeshing only.
- Suitable for mesh registration/editing/warping, and also applicable to biomedical applications, such as tracking deformations of a 3D beating heart model.



**Thank you.**

29

2017/11/14

# Multiresolution Analysis on 2-manifold


- M. Eck, T. DeRose, T. Duchamp, H. Hoppe, M. Lounsbery, and W. Stuetzle, **"Multiresolution analysis of arbitrary meshes",** 1995.
- M. Lounsbery, T. DeRose, and J. Warren, **"Multiresolution analysis for surfaces of arbitrary topological type",** ACM Transactions on Graphics, 16 (1997), pp. 34-73.
- I. Guskov, W. Sweldens, and P. Schroder, "Multiresolution signal processing for meshes", 1999.
- Garland and Heckbert, "Surface simplication using quadric error metrics", 1997.
- Guskov et al., "Multiresolution signal processing for meshes", 1999.
- Guskov et al., "Normal meshes", 2000.
- H. Hoppe, "Progressive meshes", 1996.
- Khodakovsky and Guskov, "Compression of normal meshes", 2003.
- Khodakovsky et al., "Progressive geometry compression", 2000.
- Lee et al., "Multiresolution mesh morphing", 1999.
- **Lee et al., "Maps: Multiresolution adaptive parameterization", 1998.**
- Stollnitz, DeRose, and Salesin, "Wavelets for computer graphics: theory and applications", Morgan Kaufmann, 1996.
- Warren and Weimer, "Subdivision methods for geometric design: A constructive approach", Morgan Kaufmann, 2001.
- Schroder and Sweldens, "Spherical wavelets: Efficiently representing functions on the sphere", 1995.
- Dong et al., "Multiscale representation of surfaces by tight wavelet frames with applications to denoising", Applied and Computational Harmonic Analysis, 2015.


30