Multiband TDOA Estimation from Sub-Nyquist Samples with Distributed Sensing Nodes

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TECHNISCHE UNIVERSITÄT
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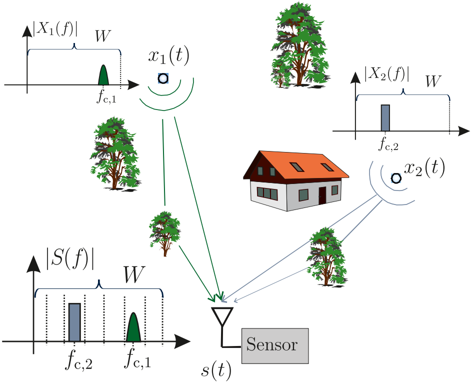
Introduction: multiband spectrum sensing

Sensing scenario

- Wide frequency band of interest W
- N communication sub-bands
  - a bandwidth of $B = W/N$
  - central frequency f_n^c
- K out of N are occupied by

$$x_k(t) = \bar{x}_k(t)e^{\jmath 2\pi f_k t},$$

  - $f_k \in \{f_n^c\}_{n=1}^N$ - unknown
  - $\bar{x}_k(t)$ are w.s.s. and unknown



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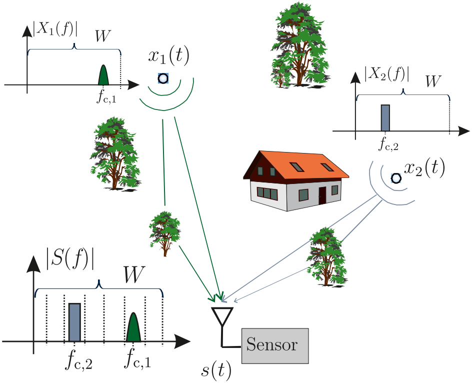
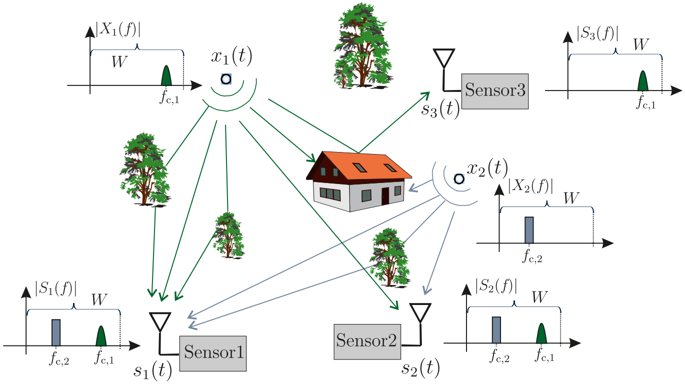


Sensing task

- Acquire $s(t) = \sum_{k=1}^{K} x_k(t)$ at a sub-Nyquist rate (on the order of KB rather than W)
- Find out which sub-bands are occupied, i.e., estimate f_k.

Introduction: multiband sensing and TDOA estimation



- **Multiple** distributed time-synchronized wideband sensing **nodes** that exchange data with each other (or some centralized processing unit)

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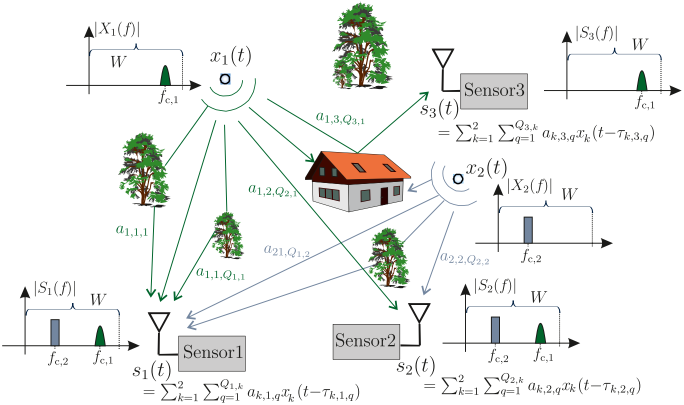

- **Multiple** distributed time-synchronized wideband sensing **nodes** that exchange data with each other (or some centralized processing unit)
- $Q_{k,p} \leq Q$ multipath components in the propagation channel from k-th source to p-th sensor, each with an amplitude $a_{k,p,q}$ and a delay $\tau_{k,p,q}$

Multiband sensing and TDOA Estimation: problem formulation

Source signals:

- to each source signal $x_k(t)$ corresponds an autocorrelation function $R_{kk}(\tau)$

$$R_{kk}(\tau) = \mathbb{E}\{x_k(t)x_k^*(t-\tau)\} = \bar{r}_{kk}(\tau)e^{\jmath 2\pi f_k \tau},$$

  where $\bar{r}_{kk}(\tau) = \mathbb{E}\{\bar{x}_k(t)\bar{x}_k^*(t-\tau)\}$ is the baseband autocorrelation function.
- we assume that $R_{k_1 k_2}(\tau) = \mathbb{E}\{x_{k_1}(t)x_{k_2}^*(t-\tau)\} \equiv 0 \; \forall \; k_1 \neq k_2$.

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Received signals:

- the noise-free signal at the p-th sensor $s_p(t) = \sum_k \sum_{q=1}^{Q_{p,k}} a_{k,p,q} x_k(t-\tau_{k,p,q}) = \sum_k x_{k,p}(t)$

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- for any pair of sensors (p_1, p_2), we can calculate a cross-correlation function $R_{p_1,p_2}(\tau)$

$$R_{p_1,p_2}(\tau) = \mathbb{E}\{s_{p_1}(t)s_{p_2}^*(t-\tau)\} = \sum_k \sum_{q_1,q_2} \tilde{a}_{k,q_1,q_2}^{(p_1,p_2)} R_{kk}(\tau - \underbrace{\tilde{\tau}_{k,q_1,q_2}^{(p_1,p_2)}}_{\text{relative delay}}) = \sum_k \bar{r}_k^{(p_1,p_2)}(\tau)e^{\jmath 2\pi f_k \tau}$$

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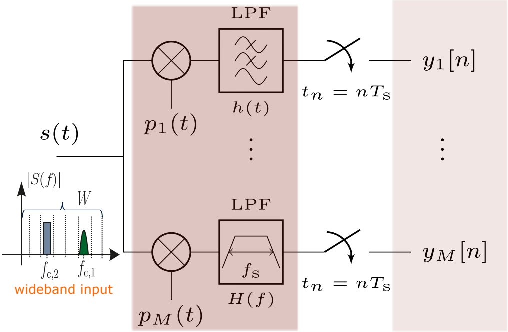
Sensing task:

- detect which sub-bands are active
- estimate **relative autocorrelations** $\bar{r}_k^{(p_1,p_2)}(\tau)$

} from sub-Nyquist samples of $s_p(t)$

Sub-Nyquist receiver system: the MWC[1]



Modulated Wideband Converter:

- M sampling channels
- analog compression in each channel
  - mixing with periodic sequences $p_m(t)$
  - low-pass filtering with cut-off $f_s/2$
- sampling at f_s $\rightarrow$ M low-rate digital outputs $y_m[n]$
- total sampling rate of Mf_s vs. W

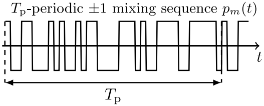
[1] M. Mishali and Y. C. Eldar. "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals." *IEEE Journal of Selected Topics in Signal Processing* vol. 4, no. 2, 2010, pp. 375-391.

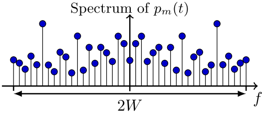
MWC operation in frequency domain

- analog mixing: $p_m(t)$ is T_p-periodic $\rightarrow$ it can be represented by Fourier series

$$p_m(t) = \sum_{\ell=-\infty}^{\infty} c_{m,\ell} e^{\jmath 2\pi \ell f_\mathrm{p} t}, \text{ where}$$

$$c_{m,\ell} = \frac{1}{T_\mathrm{p}} \int_{T_\mathrm{p}} p_m(t) e^{-\jmath 2\pi \ell f_\mathrm{p} t} \mathrm{d}t$$ – weighted "Dirac-comb"





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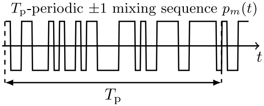

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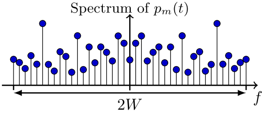
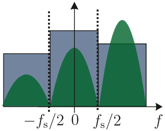


- multiplication in time $\leftrightarrow$ convolution in frequency:



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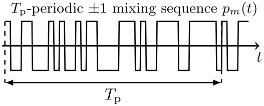

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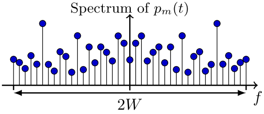

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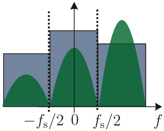
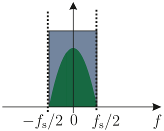


- low-pass filtering: only the narrowband part of the mixture around the origin is kept

- low-rate sampling with $f_\mathrm{p} = f_\mathrm{s} \geq B$: each digital output contains all spectral parts of the original signal (rearranged and differently weighted)



MWC operation for time-delay estimation

- consider m-th digital output at p-th sensor[1]:

$$\underbrace{y_{p,m}(f)}_{\text{DTFT of } y_{p,m}[nT_{\mathrm{s}}]} = \underbrace{[c_{m,-L_0}, \ldots, c_{m,0}, \ldots c_{m,L_0}]}_{\boldsymbol{c}_m} \underbrace{\begin{bmatrix} S_p(f + L_0 f_{\mathrm{p}}) \\ \ldots \\ S_p(f) \\ \ldots \\ S_p(f - L_0 f_{\mathrm{p}}) \end{bmatrix}}_{\boldsymbol{z}_p(f),\, f \in \mathcal{F}_{\mathrm{s}} = [-f_{\mathrm{s}}/2, f_{\mathrm{s}}/2]} = \boldsymbol{c}_m \boldsymbol{z}_p(f)$$

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Estimation: joint recovery

- consider the cross-correlation between the i-th output of p_1-th sensor and the j-th output of p_2-th sensor:

$$r_{i,j}^{(p_1,p_2)}[\tau_\nu] = \mathbb{E}\{y_{p_1,i}[t_n] y^*_{p_2,j}[t_n - \tau_\nu]\} = \sum_{k=1}^{K} \underbrace{w_{i,j,\ell_k}}_{c_{i,\ell_k} c^*_{j,\ell_k}} \sum_{q_1,q_2} \tilde{a}^{(p_1,p_2)}_{k,q_1,q_2} \underbrace{\bar{r}_{kk}[\tau_\nu - \tilde{\tau}^{(p_1,p_2)}_{n,q_1,q_2}]}_{\bar{r}_k^{(p_1,p_2)}[\tau_\nu]}$$

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- concatenate all cross-correlations $r_{i,j}^{(p_1,p_2)}[\tau_\nu]$ together into one vector $\boldsymbol{r}_{\mathbf{y}}^{(p_1,p_2)}[\tau_\nu]$:

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  - $\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}[\tau_\nu]$ is a K-sparse vector of length L that contains unknown relative autocorrelation functions $\bar{r}_k^{(p_1,p_2)}[\tau_\nu]$ at the positions with indices ℓ_k
  - support of $\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}$ defines the central frequencies of the active sub-bands: $\forall \ell_k \in \mathcal{S}\, \exists f_k = \ell_k f_\mathrm{p}$

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  - $\boldsymbol{W}$ is a matrix comprised of elements $w_{i,j,\ell}$ such that its ℓ-th column is

$$\boldsymbol{w}_\ell = \Big[\underbrace{w_{1,1,\ell}, \ldots, w_{1,M,\ell}}_{M}, \underbrace{w_{2,1,\ell}, \ldots, w_{2,M,\ell}}_{M}, \ldots, \underbrace{w_{M,1,\ell}, \ldots, w_{M,M,\ell}}_{M}\Big]^\mathrm{T}$$

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$$\boldsymbol{r}_{\mathbf{y}}^{(p_1,p_2)}[\tau_\nu] = \boldsymbol{W}\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}[\tau_\nu]$$

- typical sparse recovery problem $\rightarrow$ can be solved for each τ_ν independently
- we can apply the CTF block from (2)

(2) M. Mishali and Y. C. Eldar, "Blind multiband signal reconstruction: compressed sensing for analog signals," *IEEE Transactions on Signal Processing*, vol. 57, no. 3, pp. 993–1009, 2009.

Estimation: joint recovery

$$\boldsymbol{r}_{\mathbf{y}}^{(p_1,p_2)}[\tau_\nu] = \boldsymbol{W}\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}[\tau_\nu]$$

- typical sparse recovery problem $\rightarrow$ can be solved for each τ_ν independently
- we can apply the CTF block from (2)

Once the support $\mathcal{S}$ of $\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}[\tau_\nu]$ is found we obtain

- the central frequencies of the active sub-bands

$$f_k = \ell_k f_{\mathrm{p}},\ \ell_k \in \mathcal{S}$$

- the discrete baseband relative autocorrelation functions $\bar{r}_k^{(p_1,p_2)}[\tau_\nu]$ as

$$\left(\boldsymbol{r}_{\mathbf{z}}^{(p_1,p_2)}\right)_{\mathcal{S}}[\tau_\nu] = \boldsymbol{W}_{\mathcal{S}}^{\dagger}\boldsymbol{r}_{\mathbf{y}}^{(p_1,p_2)}[\tau_\nu],$$

  where for some vector $\boldsymbol{a}$ and matrix $\boldsymbol{A}$ the notation $\boldsymbol{a}_{\mathcal{S}}$ and $\boldsymbol{A}_{\mathcal{S}}$ means taking the entries of $\boldsymbol{a}$ and the columns of $\boldsymbol{A}$ indexed by $\mathcal{S}$, respectively.

(2) M. Mishali and Y. C. Eldar, "Blind multiband signal reconstruction: compressed sensing for analog signals," *IEEE Transactions on Signal Processing*, vol. 57, no. 3, pp. 993–1009, 2009.

Estimation: two-step recovery

Alternatively, we can

- first estimate the support $\mathcal{S}$ and the corresponding low-rate sequences $\bar{x}_{k,p}[t_n]$ from the outputs of individual sensors
- compute $\bar{r}_k^{(p_1,p_2)}[\tau_\nu]$ for each k independently

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- first estimate the support $\mathcal{S}$ and the corresponding low-rate sequences $\bar{x}_{k,p}[t_n]$ from the outputs of individual sensors
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Estimation procedure:

- collect all M outputs $y_{p,m}[t_n]$ of the p-th sensor together into one vector $\boldsymbol{y}_p[t_n]$:

$$\boldsymbol{y}_p[t_n] = \underbrace{[\boldsymbol{c}_1, \cdots, \boldsymbol{c}_M]^{\mathrm{T}}}_{\boldsymbol{C}} \boldsymbol{z}_p[t_n]$$

- find the support of $\boldsymbol{z}_p[t_n]$ from $\boldsymbol{y}_p[t_n]$
  - the support of $\boldsymbol{z}_p[t_n]$ is also $\mathcal{S}$
  - as before, we can find it either for each t_n or by applying the CTF block
- estimate the individual low-rate sequences $\bar{x}_{k,p}[t_n]$ via $(\boldsymbol{z}_p)_{\mathcal{S}}[t_n] = \boldsymbol{C}_{\mathcal{S}}^{\dagger} \boldsymbol{y}_p[t_n]$
- obtain baseband relative autocorrelations $\bar{r}_k^{(p_1,p_2)}[\tau_\nu]$

$$\bar{r}_k^{(p_1,p_2)}[\tau_\nu] = \mathbb{E}\{\bar{x}_{k,p_1}[t_n] \bar{x}_{k,p_2}[t_n - \tau_\nu]\}$$

Numerical example: simulation setup

Sensing scenario:

- frequency band of $W = 3.9$ GHz is split into $N = 135$ communication channels
- W is occupied by $K = 3$ BPSK modulated signals $x_k(t)$
  - bandwidth of $B = 20$ MHz
  - carrier f_k chosen uniformly at random from $\{f_n^{\mathrm{c}}\}_{n=1}^{N}$

Sensors:

- sensors operate with $M = 20$ sampling channels
- the sampling rate is $f_{\mathrm{s}} = 28$ MHz
- $p_m(t)$ are generated as pseudo-random $\{\pm 1\}$ piece-wise constant functions
- total sampling rate at each sensor is 560 MHz, which is 14% of the Nyquist rate

Propagation parameters:

- number of multipath components is $Q_{k,p} = 2$
- time delays $\tau_{k,p,q}$ and amplitudes $a_{k,p,q}$ are chosen uniformly at random from $[\frac{N_{\mathrm{T}}}{100}, \frac{9N_{\mathrm{T}}}{100}]$ and $[0.6, 1]$
  - N_{T} is the sensing time in samples

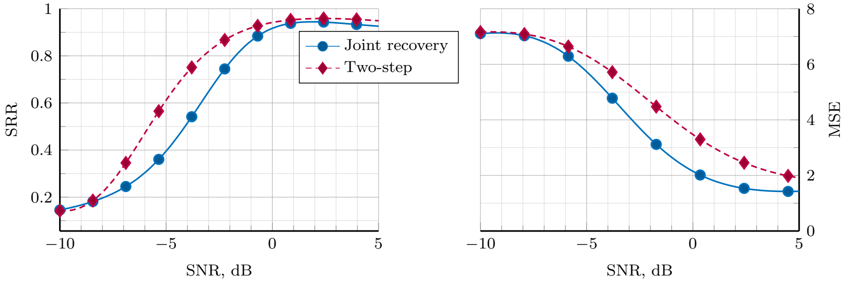
Performance metrics:

- support recover rate (SRR): $|\hat{\mathcal{S}} \cap \mathcal{S}|/K$
- mean square error (MSE) between the true and the estimated relative autocorrelation functions:

$$\frac{1}{K N_{\mathrm{T}}} \sum_{k=1}^{K} \sum_{\nu=1}^{N_{\mathrm{T}}} \frac{|\bar{r}_k^{(p_1,p_2)}[\tau_\nu] - \hat{\bar{r}}_k^{(p_1,p_2)}[\tau_\nu]|^2}{|\bar{r}_k^{(p_1,p_2)}[\tau_\nu]|^2}$$

Numerical example: performance vs SNR

$N_T = 500$ samples

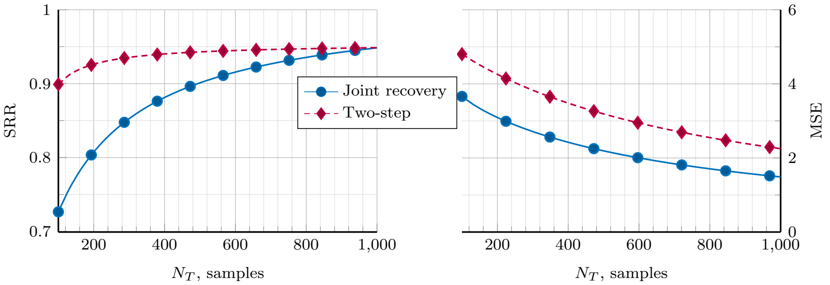


(a) SRR **(b)** MSE

- two-step recovery approach provides somewhat higher SRR rate
- joint estimation method provides slightly better accuracy in terms of $\bar{r}_k^{(p_1,p_2)}[\tau_\nu]$ recovery

Numerical example: performance vs sensing time

SNR $= 0$ dB



(a) SRR **(b)** MSE

- the same tendency
- two-step approach is less sensitive to sensing time duration for support recovery

Conclusions

- we considered the task of relative autocorrelation estimation of multiple unknown transmitters from the sub-Nyquist samples of wideband multiband signals obtained by a network of spatially distributed sensing nodes.
- we showed that the central frequencies and the relative autocorrelation functions of the individual transmissions can be estimated from the low-rate outputs of different sensors and proposed two estimation methods
  - joint recovery of the frequency support and the relative autocorrelation functions
  - two-step approach
- both proposed methods allow for central frequency and relative autocorrelation estimation from sub-Nyquist samples
  - the joint recovery yields an improved accuracy of the latter while being more sensitive with respect to the sensing time

Thank you! Questions?