Image Super-Resolution Using Nonlocally Centralized Sparse Representation and Fields of Experts Priors

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Introduction: The image Super-Resolution (SR) method essentially is to reconstruct a high-resolution (HR) image from one or more degraded (e.g., noisy, blurred or down-sampled) images. Due to the degradation of the observed images, the concept of sparse coding noise (SCN) was used to recover the original image. Since the image is noisy, so it is necessary to select an appropriate prior model to reconstruct the HR image. In this paper, we proposed a novel image SR reconstruction method, which introduced the Fields of Experts (FoE) priors model into the framework of nonlocally centralized sparse representation (NCSR), then learned the prior information of the whole image. The experimental results show that the proposed approach using FoE prior model is not only reduces the impact of noise, but also improves the quality of the reconstructed image.

Nonlocally Centralized Sparse Representation:

The concept of SCN[1] can be formulated as follow

$$v_\alpha = \alpha_y - \alpha_x \tag{1}$$

In order to calculate v_α, we could have the reasonably estimation of α_x, denoted by β, then the α_y-β becomes a good estimation of v_α. By suppressing the SCN v_α to improve the accuracy of the α_y, there is the following centralized sparse representation (CSR) model

$$\alpha_y = \underset{\alpha}{\operatorname{argmin}} \left\{ \| y - HD \circ \alpha \|_2^2 + \lambda \sum_i \|\alpha_i\|_1 + \gamma \sum_i \|\alpha_i - \beta_i\|_p \right\} \tag{2}$$

where β_i is some reasonably estimation of α_i, γ is the regularization parameter and p can be 1or 2.

In (2), the local sparsity term $\|\alpha_i\|_1$ is used to ensure that only a small number of atoms are selected from the over-complete dictionary D to represent the input image patch. Nevertheless, as long as the given patch is a very sparse representation, the coding coefficients of image patches over the other sub-dictionaries to be 0. Thus we can remove the local sparsity regularization term $\|\alpha_i\|_1$, the sparse coding model in (2) can be rewritten as

$$\alpha_y = \underset{\alpha}{\operatorname{argmin}} \left\{ \| y - HD \circ \alpha \|_2^2 + \gamma \sum_i \|\alpha_i - \beta_i\|_p \right\} \tag{3}$$

where $p=1$, and the estimate β_i is obtained by using the nonlocal redundancy of natural images, so the regularization term $\|\alpha_i-\beta_i\|_p$ will becomes a nonlocally centralized sparsity term. Hence, the (3) was called as nonlocally centralized sparse representation (NCSR) model.

Fields of Experts Priors:

The Fields of Experts (FoE)[2,3] model using the natural image statistics prior to replace Gauss potential function, and the spatial contextual structure for representing the image information will be more widely.

Here we first introduce the Product of Experts (PoE). We found that the responses of linear filters applied to natural images typically exhibit highly kurtotic marginal distributions that resemble a Student-t distribution by the statistical characteristics observation. So Welling et al. proposed the use of Student-t experts. The Product of t-distribution (PoT) model can be formulated as

$$p_{PoE}(x) = \frac{1}{Z(\Theta)} \prod_{i=1}^{N} \phi_i \left(J_i^T x; \alpha_i \right), \Theta = \{\theta_1, \cdots, \theta_N\} \tag{4}$$

where $\theta_i=\{\alpha_i, J_i\}$, $Ji \in R_n$ are filters. $Z(\Theta)$ is the partition function.

The potential function of PoE can be obtained as follow

$$E_{PoE}(x, \Theta) = -\sum_{i=1}^{N} \log \phi_i \left(J_i^T x; \alpha_i \right) \tag{5}$$

So, the probability density of whole image in the FoE model can be formulated as

$$p_{FoE}(x) = \frac{1}{Z(\Theta)} \exp\left(-E_{FoE}(x, \Theta) \right) \tag{6}$$

where

$$E_{FoE}(x, \Theta) = -\sum_k \sum_{i=1}^{N} \log \phi_i \left(J_i^T x_{(k)}; \alpha_i \right) \tag{7}$$

So, the (6) can be equivalent to

$$p_{FoE}(x) = \frac{1}{Z(\Theta)} \prod_k \prod_{i=1}^{N} \phi_i \left(J_i^T x_{(k)}; \alpha_i \right) \tag{8}$$

The important difference with PoE model is that the FoE model takes the product over all neighborhoods k, and establishes a global prior model.

Super-Resolution Reconstruction:

Despite the image super-resolution algorithm based on nonlocally centralized sparse representation have good quality of reconstructed images and eliminates the image artifact, this method not have the appropriate prior model to reduce the noise of the reconstructed images. So we introduced a prior model into this method to improve the effect of image reconstruction in this paper. Hence we proposed the following sparse representation model

$$\alpha_y = \underset{\alpha}{\operatorname{argmin}} \left\{ \| y - HD \circ \alpha \|_2^2 + \gamma \sum_i \|\alpha_i - \beta_i\|_p + \tau P(\alpha) \right\} \tag{9}$$

where τ is regularization parameter, which can ensures the global optimal solution and the convergence of this algorithm. The $P(\alpha)$ is a prior model.

In the paper, we applied the FoE prior model to the SR reconstruction algorithm based on image sparse, so the (9) can be rewritten as

$$\alpha_y = \underset{\alpha}{\operatorname{argmin}} \left\{ \| y - HD \circ \alpha \|_2^2 + \gamma \sum_i \|\alpha_i - \beta_i\|_p + \tau p_{FoE}\left(\alpha_y^{(j-1)} \right) \right\} \tag{10}$$

In this method, the iteration of prior model and the iterative computation of image reconstruction are performed simultaneously. When the reconstructed image is updated to the optimal solution of α_y can be calculated with (10), and then we can reconstruct original image with (11).

$$\hat{x} = D \circ \alpha_y \tag{11}$$

Experimental Results:

In order to prove the efficiency of the proposed approach, we need to verify the effect of image denoising and quality of image SR reconstruction.

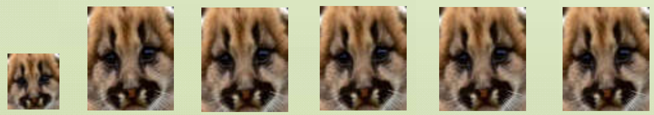
The Table Ⅰ lists the PSNR results of image denoising, while the examples are shown in Fig.1.

The PSNR results of different reconstructed images using different methods as show in Table Ⅱ. The reconstructed images with different algorithms are show in Fig.2.

Fig.1 Denoising results of *Barbara* image. From left to right: original image, noisy image (σ=50), denoised image by NSCR and the proposed method

TABLE I. PSNR(dB) RESULTS OF DENOISE IMAGES

σ	5		10		15		20		50		100	
Lena	38.7394	**38.7423**	35.8466	**36.0123**	34.1176	**34.1345**	32.9494	**33.1034**	28.9028	**28.9716**	25.7143	**25.9158**
Monarch	38.4932	**38.5014**	34.5123	**34.5398**	32.2562	**32.4856**	30.6229	**30.9527**	25.7614	**25.9215**	22.1168	**22.2921**
barbara	38.3746	**38.3901**	35.0032	**35.1962**	33.0639	**33.2147**	31.7771	**31.8045**	26.9898	**27.0267**	23.1811	**23.2695**
boat	**37.3357**	37.3316	**33.9148**	33.7082	32.0809	**32.2413**	**30.7853**	30.7239	26.6651	**26.7659**	**23.6841**	23.5694
cameraman	38.2559	**38.2562**	34.1859	**34.2572**	32.0143	**32.1509**	30.4686	**30.5932**	**26.1541**	26.0283	22.9315	**23.0281**
couple	37.4973	**37.5109**	34.0022	**34.1385**	**31.9985**	31.7962	**30.6025**	30.5364	26.1861	**26.2097**	**23.1530**	23.0169
fingerprint	36.8067	**36.8126**	**32.6830**	32.4213	**30.4468**	30.4351	28.9631	**29.0163**	**24.4819**	24.3681	21.3949	**21.5284**
hill	**37.1768**	37.1624	**33.6920**	33.5109	31.8790	**31.9524**	30.6504	**30.7981**	26.9871	**27.1349**	24.3595	**24.4680**
house	39.9439	**39.9876**	36.7953	**36.8159**	35.0480	**35.1562**	**33.8744**	33.8426	29.6172	**29.6983**	**25.5625**	25.4219
man	**37.8474**	37.8097	34.0470	**34.1845**	31.9809	**32.0659**	30.5886	**30.6549**	**26.6659**	26.5383	24.0162	**24.1325**
peppers	38.1143	**38.1425**	**34.6799**	34.5084	**32.6647**	32.5632	31.1851	**31.2463**	26.5308	**26.8026**	22.8426	**22.9127**
straw	35.8331	**35.8493**	31.4547	**31.4958**	29.0899	**29.2567**	27.4637	**27.5747**	22.4880	**22.5219**	**19.4044**	19.3159
Average	37.8682	**37.8747**	**34.2347**	34.2324	32.2201	**32.2877**	30.8276	**30.9039**	26.4525	**26.4990**	23.1967	**23.2393**



a) LR image b) HR image c) ASDSAR method d) CSR method e) NCSR method f) The proposed

Fig.2 Reconstructed images of Raccoon by different methods

TABLE II. PSNR(dB) RESULTS OF RECONSTRUCTED IMAGES

Images	Butterfly	flower	Girl	Parthenon	Parrot	Raccoon	Bike	Hat	Plants	**Average**
ASDSAR	26.0113	27.6647	31.7700	26.0807	28.7198	27.9919	23.5212	29.5647	31.0595	28.0426
CSR	26.8411	28.0753	32.0383	26.3780	29.4952	28.0198	23.7757	29.9569	31.7779	28.4842
NCSR	26.8195	28.0886	32.0321	26.3690	29.5016	28.0217	23.7812	29.9504	31.8005	28.4850
proposed	**26.9653**	**28.1384**	**32.1349**	**26.4576**	**29.5531**	**28.1591**	**23.8437**	**30.0412**	**31.8513**	**28.5716**

References:

[1] Dong W, Zhang L, Shi G, et al. "Nonlocally Centralized Sparse Representation for Image Restoration," IEEE Transactions on Image Processing A Publication of the IEEE Signal Processing Society,1620-1630,2011.

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[3] Roth S, Black M J. "Fields of Experts," International Journal of Computer Vision,205-229, 2009, 82(2).

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