On the Indistinguishability of Compressed Encryption With Partial Unitary Sensing Matrices

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IEEE Global Conference on Signal and Information Processing (GlobalSIP)
Montreal, Canada

Nov. 15, 2017

This work was supported by the National Research Foundation of Korea (NRF).

CS-based Cryptosystems

- **Security for IoT and M2M**
  - Security issues are major challenges for the Internet-of-Things (IoT) and M2M communications.
  - Security techniques with **low latency**, **low power consumption**, and **low complexity** are required.

- **Compressed Sensing (CS) based Cryptosystems**
  - **Simultaneous sensing** and **encryption**
  - Efficient encryption/decryption
  - Reliability and security
  - Low complexity and low power consumption

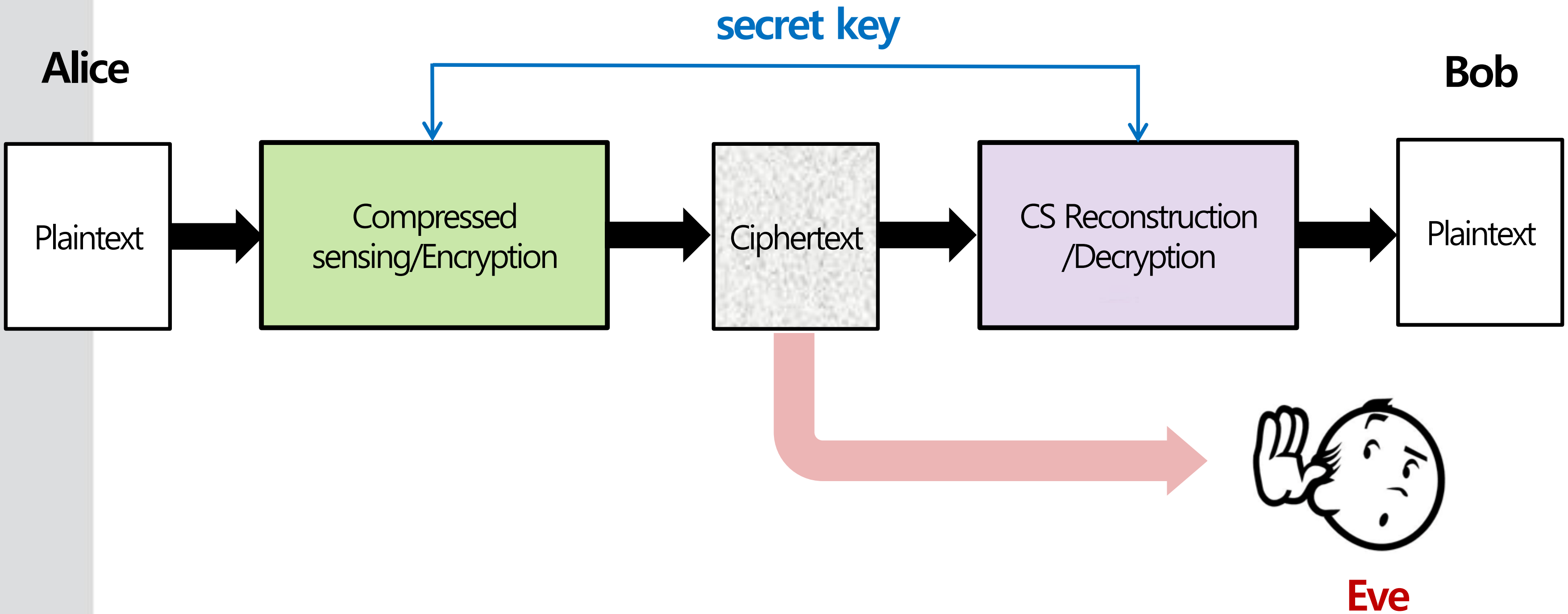
CS-based Cryptosystems

- History
  - **Hint [Candes&Tao'06]**
    : CS measurement samples are *weakly encrypted*.
  - **Kick-off [Rachlin&Baron'08]**
    : CS-based cryptosystems cannot be *perfectly secure*, but can be computationally secure.
  - **Kick-off [Orsdemir *et al.*'08]**
    : Demonstrated that CS-based cryptosystems can be computationally secure.
  - **Gaussian one-time sensing (G-OTS) cryptosystem [Bianchi *et al.*'14]**
    : perfectly secure, as long as each plaintext has constant energy
  - **Random Bernoulli based cryptosystem [Cambareri *et al.*'15]**
    : CS-based cryptosystem for multiclass encryption

  - Many other research works for practical applications
    : smart grids, image encryption, wireless communications, etc.

CS-based Cryptosystems

- Symmetric-key CS-based Cryptosystems

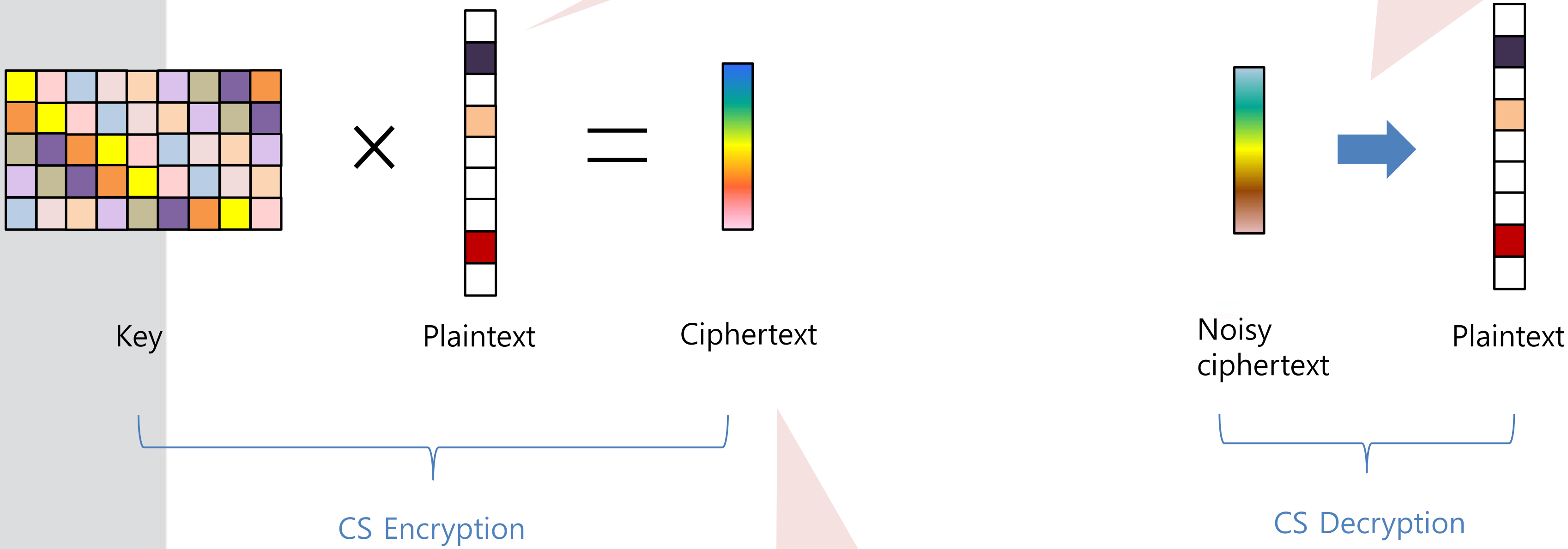


CS-based Cryptosystems

- **CS Encryption/Decryption**

Each plaintext is sparse with respect to an arbitrary basis.

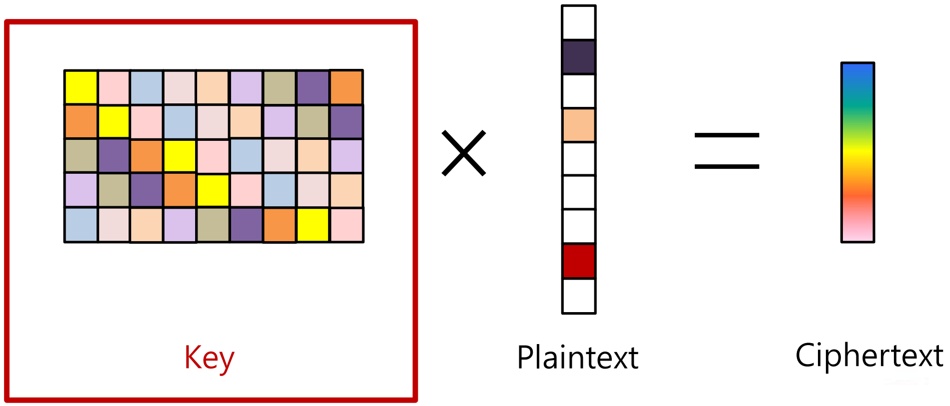
CS recovery algorithms are applied for CS decryption.



Key: M x N matrix
Plaintext: N x 1 vector
Ciphertext: M x 1 vector

CS-based Cryptosystems

- Gaussian One-Time Sensing (G-OTS) Cryptosystem



Φ: **random Gaussian matrix**

One-time sensing is crucial for CPA-security.

– **One-time sensing**: a random Gaussian matrix is used only once, and renewed for each encryption.

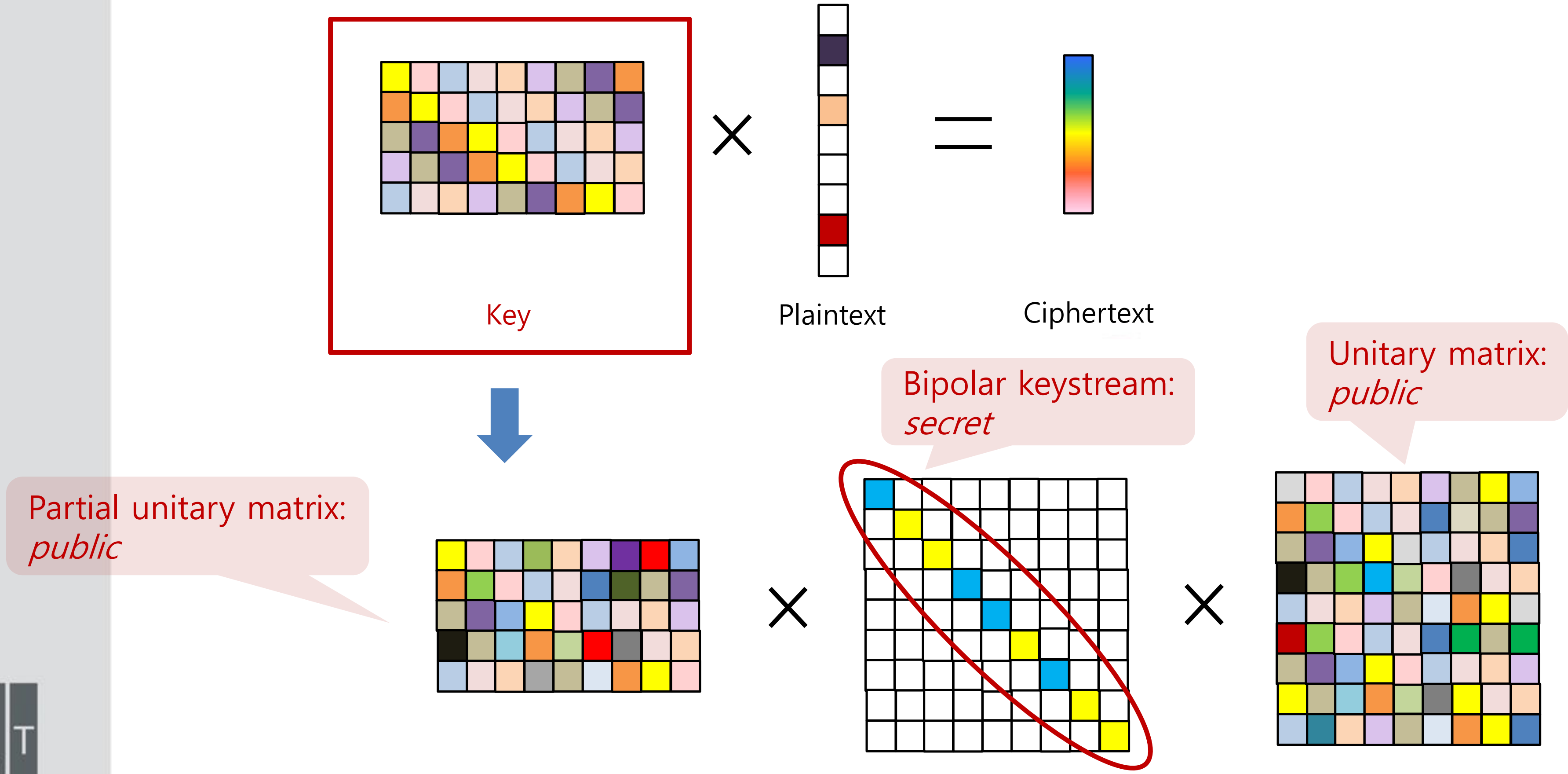
CS-based Cryptosystems

- **Gaussian One-Time Sensing (G-OTS) Cryptosystem**
  - **Pros**
    - The G-OTS cryptosystem reveals only the energy of the plaintext.
    - Thus, it is *perfectly secure,* as long as each plaintext has constant energy.
  - **Cons**
    - Each CS encryption/decryption requires *high complexity* and *processing time* by matrix-vector multiplication with Gaussian distributed elements.
    - M x N Gaussian distributed elements are required for each encryption.

The motivation of this work is to overcome the practical concerns.

Proposed CS-based Cryptosystems

- **Proposed CS encryption**



Proposed CS-based Cryptosystems

- Mathematical Formulation

$$\boxed{\mathbf{\Phi} = \frac{1}{\sqrt{M}}\mathbf{R}_\Omega\mathbf{U} = \frac{1}{\sqrt{MN}}\mathbf{R}_\Omega\mathbf{U}_1\mathrm{diag}(\mathbf{s})\mathbf{U}_2}$$

  - **$\mathbf{U}_1$ = H:** Each entry of $\mathbf{U}_1$ should have unit magnitude.

$$\mathbf{H}(k,t) = \begin{cases} 1, & \text{if } k=0 \text{ or } t=0, \\ (-1)^{d_{k+t-2}}, & \text{otherwise} \end{cases}$$

d is a binary *m*-sequence.

  - **$\mathbf{U}_2$: Unitary matrix**
  - **s: secret bipolar keystream**
    - LFSR-based keystream
    - *Example*: Self-shrinking generator (SSG)

The secret keystream bits can be generated fast and efficiently.

Proposed CS-based Cryptosystems

- **Practical Benefits**
  - **Efficient keystream usage**
    - G-OTS cryptosystem: M x N real-valued elements required for each encryption
    - Proposed cryptosystem: N keystream bits required for each encryption
  - **Fast and efficient keystream generation**: The original keystream can be efficiently generated by an LFSR-based keystream generator.
  - **Fast and efficient CS encryption/decryption**: By employing unitary matrices, matrix-vector multiplications for CS processes can be efficiently implemented.
- **Reliability**
  - **Stable** and **robust CS decryption:** A plaintext with at most K nonzero entries can be decrypted with bounded errors by a legitimate recipient, as long as

$$M = \mathcal{O}(K\log^5 N)$$

Security Analysis

- **Indistinguishability**
  - If a cryptosystem has the indistinguishability, no eavesdropper can learn any partial information about the plaintext from a given ciphertext.
  - The indistinguishability formalizes the notion of **computational security** of a cryptosystem.
  - The indistinguishability is measured by the success probability of an adversary in the **indistinguishability experiment**.

Security Analysis

- **Indistinguishability Experiment (for a CS-based cryptosystem)**

Step 1:	An adversary creates a pair of plaintexts $\mathbf{x}_1$ and $\mathbf{x}_2$ of the same length, and submits them to a CS-based cryptosystem.
Step 2:	The CS-based cryptosystem encrypts a plaintext $\mathbf{x}_h$ by randomly selecting $h \in \{1, 2\}$, and gives a noisy ciphertext $\mathbf{r} = \mathbf{\Phi}\mathbf{x}_h + \mathbf{n}$ back to the adversary.
Step 3:	Given the ciphertext $\mathbf{r}$, the adversary carries out a polynomial time test $\mathcal{D} : \mathbf{r} \rightarrow h' \in \{1, 2\}$, to figure out the corresponding plaintext.
Decision:	The adversary passes the experiment if $h' = h$, or fails otherwise.

- If no adversary passes the indistinguishability experiment in polynomial time with probability significantly better than that of a random guess, the cryptosystem is said to have the indistinguishability.

Security Analysis

- **Total Variation (TV) Distance**

$$d_{\mathrm{TV}}(\mu,\nu) = \sup_{A\subset\Omega}|\mu(A)-\nu(A)|$$

  - μ,ν: probability measures on Ω

– The success probability of an adversary in the indistinguishability experiment

$$p_d \leq \frac{1}{2} + \frac{d_{\mathrm{TV}}(p_1,p_2)}{2}$$

  - $p_1 = \Pr(\mathbf{r}|\mathbf{x}_1)$ and $p_2 = \Pr(\mathbf{r}|\mathbf{x}_2)$

– The TV distance can be a statistical measure for indistinguishability.

Security Analysis

- **Hellinger Distance**

$$d_{\mathrm{H}}(\mu,\nu)=\left[\frac{1}{2}\int_{\Omega}\left(\sqrt{f(x)}-\sqrt{g(x)}\right)^2 dx\right]^{\frac{1}{2}}$$

  - f, g: densities of probability measures μ, ν on Ω

  – For multivariate normal with zero mean,

$$d_{\mathrm{H}}{}^2(p_1,p_2)=1-\frac{\det(\mathbf{C}_1)^{\frac{1}{4}}\det(\mathbf{C}_2)^{\frac{1}{4}}}{\det\left(\frac{\mathbf{C}_1+\mathbf{C}_2}{2}\right)^{\frac{1}{2}}}$$

  - $\mathbf{C}_1$ and $\mathbf{C}_2$: Covariance matrices of $\mathbf{r}$ conditioned on $\mathbf{x}_1$ and $\mathbf{x}_2$

Security Analysis

- **TV and Hellinger distances**

$$d_{\mathrm{H}}{}^{2}(p_1,p_2) \le d_{\mathrm{TV}}(p_1,p_2) \le d_{\mathrm{H}}(p_1,p_2)\sqrt{2-d_{\mathrm{H}}{}^{2}(p_1,p_2)}$$

Theorem: In the proposed CS-based cryptosystem, if each plaintext $\mathbf{x}$ has at most K nonzero elements with constant energy $\mathcal{E}_x$, then

$$d_{\mathrm{H}}(p_1,p_2) \le \sqrt{1-\left(\frac{2\sqrt{K\mu^2(\mathbf{U}_2)\cdot \mathrm{PNR}+1}}{K\mu^2(\mathbf{U}_2)\cdot \mathrm{PNR}+2}\right)^{\frac{M}{4}}}$$

where $\mathrm{PNR}=\frac{\mathcal{E}_x}{M\sigma^2}$ and $\mu(\mathbf{U}_2)$ is the maximum magnitude of the entries of $\mathbf{U}_2$.

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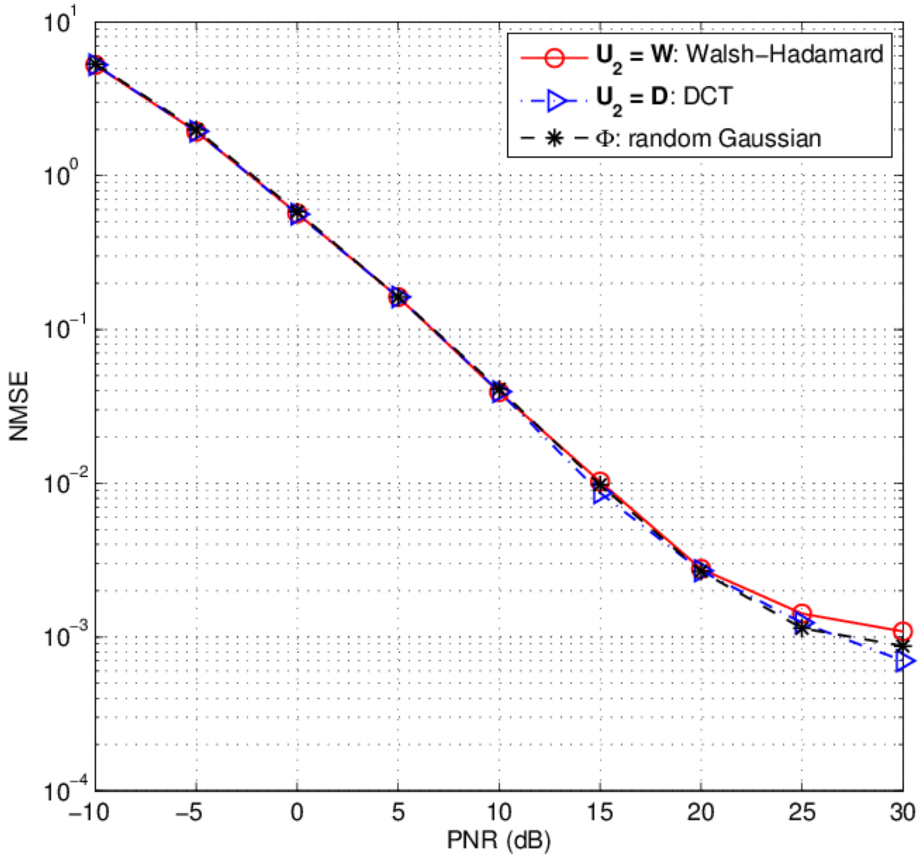
Numerical Results

- **Reliability**

For a legitimate recipient, the proposed CS-based cryptosystem is as reliable as the random Gaussian sensing.

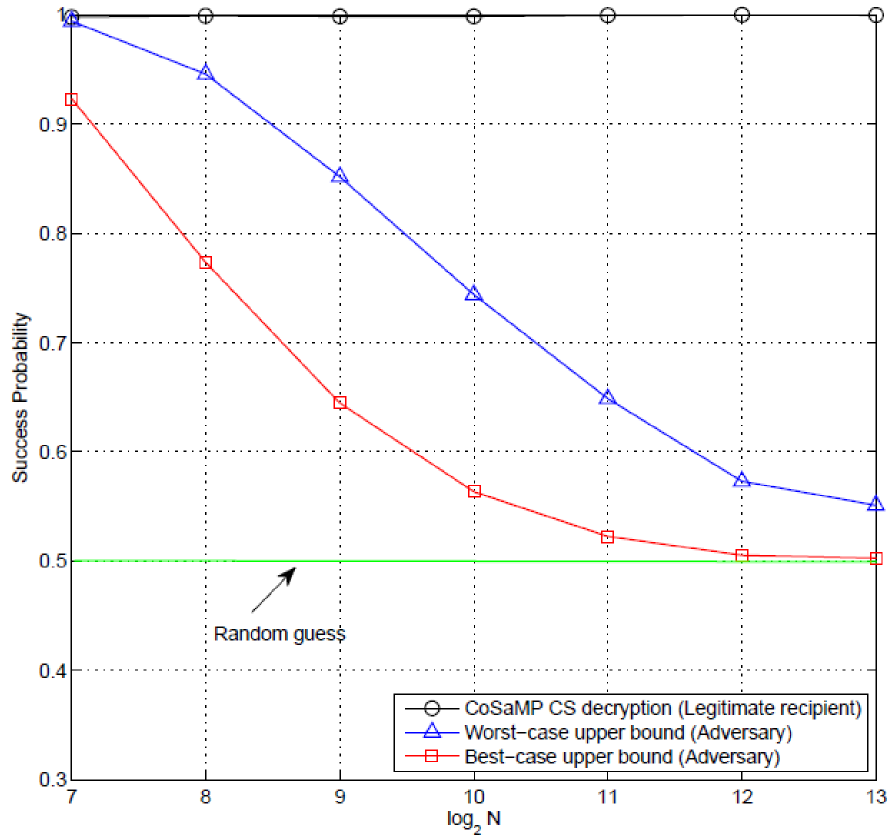


- N = 1024
- M = 48
- K = 4

Numerical Results

- **Success probabilities**

For a given M, the adversary's success probability approaches that of a random guess as N increases.

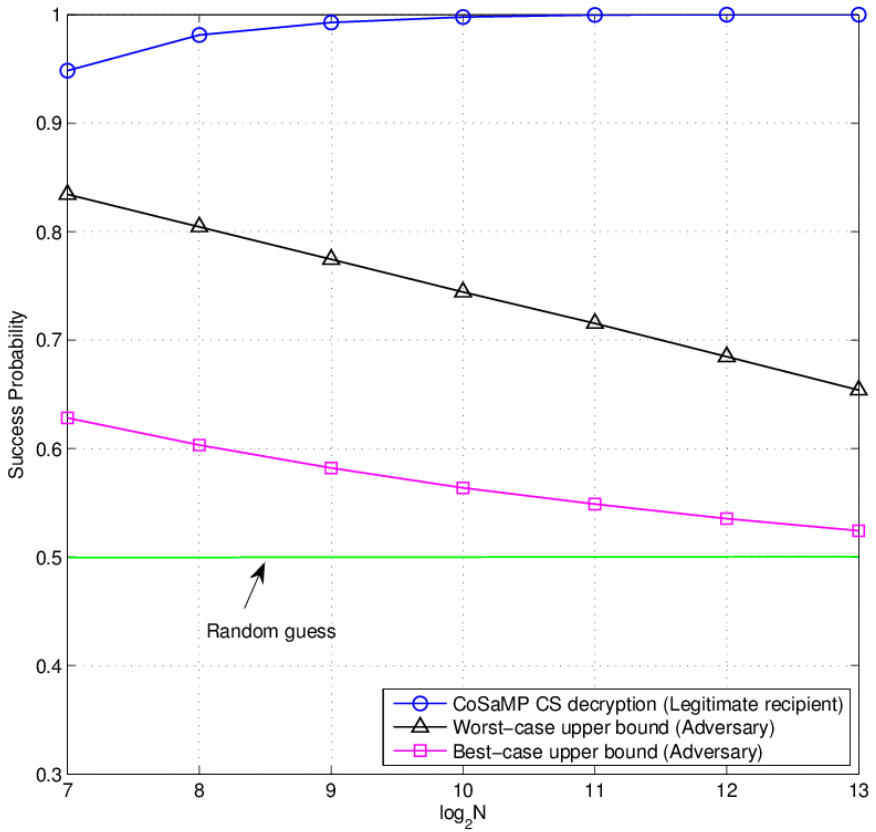


- PNR = 25 dB
- M = 48
- $K = \left\lfloor \frac{8.5M}{\log_2^2 N} \right\rfloor$

Numerical Results

- **Success probabilities**

For a given K, the adversary's success probability approaches that of a random guess as N increases.



- PNR = 25 dB
- K = 4
- $M = \lceil 0.12 K \log_2^2 N \rceil$

Conclusions

- **Proposed CS-based cryptosystem**
  - CS-based cryptosystem with partial unitary matrices embedding a secret bipolar keystream
  - Theoretically guarantees **reliable decryption** for a legitimate recipient.
  - Demonstrates the potential of **computational security** against an eavesdropper, if the keystream is sufficiently long with low compression and sparsity ratios.
  - Practical benefits
    - **Efficient usage of cryptographic primitives** by embedding a short keystream.
    - **Fast and efficient keystream generation** by LFSR-based keystream generators.
    - **Fast and efficient CS encryption/decryption** by employing unitary matrices.

Thank You!